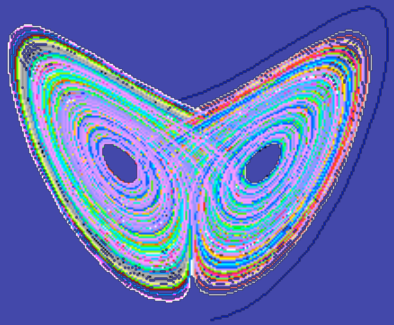


Lorenz Lecture – AGU'05

MPI für PKS, Dresden, 27 Feb. 2006

The Earth as a Complex System, and a Simple Way of Looking at It

Michael Ghil

Ecole Normale Supérieure, Paris, &
University of California, Los Angeles

Work with *D. Dee* (NASA Goddard), *V. Keilis-Borok* (IGPP, UCLA, & MITP, Moscow), *A. Mullhaupt* (Wall Street), *P. Pestiaux* (TotalFina, France), *A. Saunders* (UCLA), & *I. Zaliapin* (IGPP, UCLA, & MITP, Moscow).

Edward Norton Lorenz
born May 23, 1917

Jule Gregory Charney
January 1, 1917 – June 16, 1981

Motivation

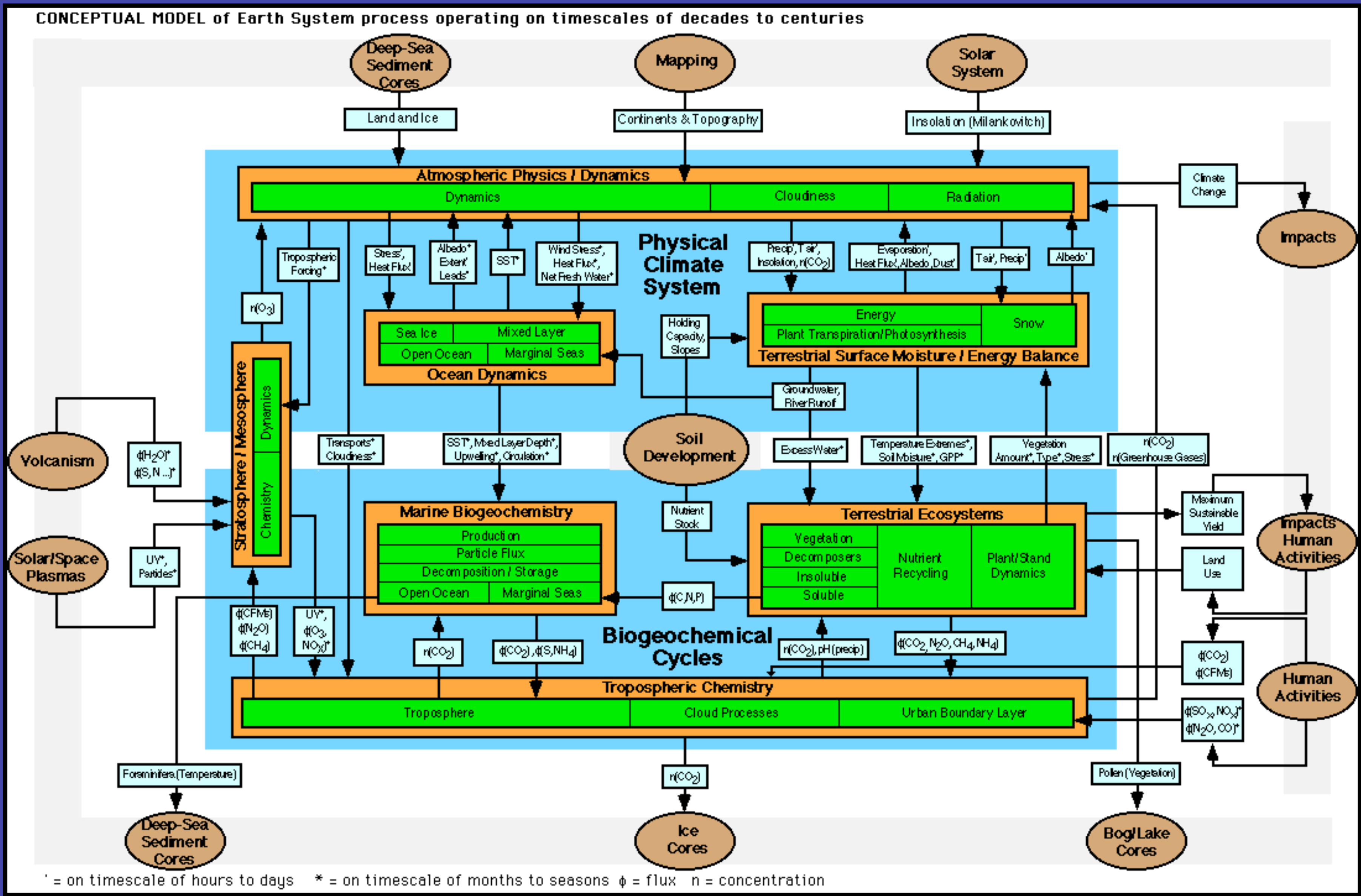
1. ***Components***
   - solid earth (crust, mantle)
   - fluid envelopes (atmosphere, ocean, snow & ice)
   - living beings on and in them (fauna, flora, people)

2. ***Complex feedbacks***
   - positive and negative
   - nonlinear - small pushes, big effects?

3. ***Approaches***
   - reductionist
   - holistic

4. ***What to do?*** - Let's see!

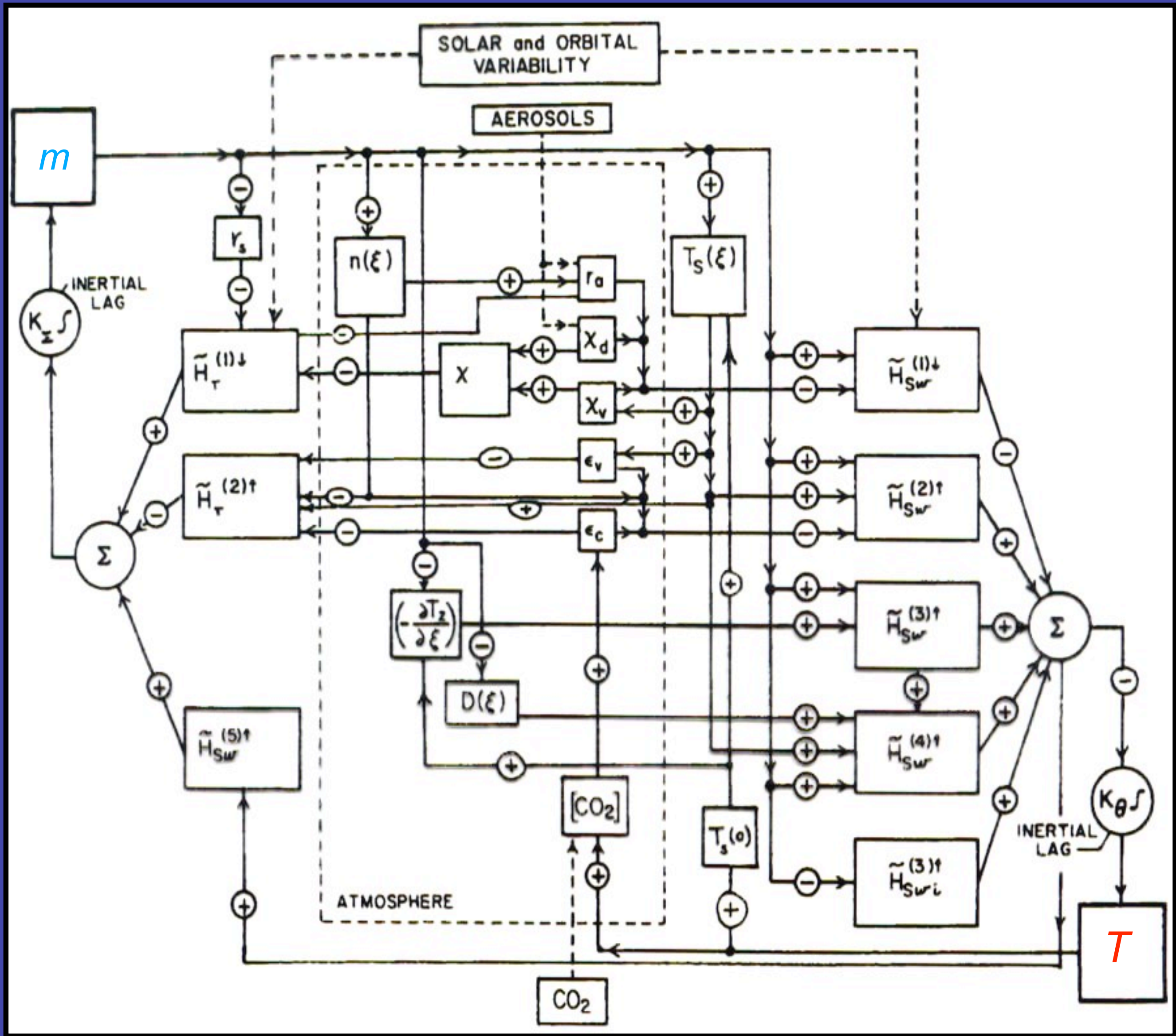
F. Bretherton's "horrendogram" of Earth System Science



Earth System Science Overview, *NASA Advisory Council*, 1986

The climate system on long time scales

“Ambitious” diagram



Flow diagram showing feedback loops contained in the dynamical system for ice-mass *m* and ocean temperature variations *T*.

Constants for ODE & PDE models are poorly known. Mechanisms and effective delays are easier to ascertain.

B. Saltzman, Climatic system analysis, *Adv. Geophys.*, **25**, 1983

Introduction

Binary systems Examples: Yes/No, True/False (ancient Greeks)

Classical logic (*Tertium not datur*)

Boolean algebra (19th cent.)

Propositional calculus (20th cent.)

(syllogisms as trivial examples)

Genes*: on/off*

Descriptive – Jacob and Monod (1961)

Mathematical *genetics – L. Glass, S. Kauffman, M. Sugita (1960s)*

Symbolic dynamics of differentiable dynamical systems (DDS): S. Smale (1967)

Switches*:* on/off, 1/0

Modern *computation* (EE & CS)

- cellular automata (CAs) J. von Neumann (1940s, 1966), S. Ulam, Conway (the game of life), S. Wolfram (1970s, '80s)
- spatial increase of complexity – infinite number of channels
- conservative logic Fredkin & Toffoli (1982)
- kinetic logic: importance of distinct delays to achieve temporal increase in complexity (synchronization, operating systems & parallel computation), R. Thomas (1973, 1979,…)

Introduction (cont.)

M.G.'s immediate motivation:

Climate dynamics – complex interactions
(reduce to binary), C. Nicolis (1982)

Joint work on developing and applying BDEs to climate dynamics
with D. Dee, A. Mullhaupt & P. Pestiaux (1980s)
& with A. Saunders (late 1990s)
Work of L. Mysak and associates (early 1990s)

Recent applications to solid-earth geophysics
(*earthquake modeling and prediction*)
with V. Keilis-Borok and I. Zaliapin

Recent applications to the biosciences
(*genetics and micro-arrays*)
Oktem, Pearson & Egiazarian (2003) *Chaos*
Gagneur & Casari (2005) *FEBS Letters*

Outline

What for BDEs?

- life is sometimes too complex for ODEs and PDEs

What are BDEs?

- formal models of complex feedback webs
- classification of major results

Applications to climate modeling

- paleoclimate – Quaternary glaciations
- interdecadal climate variability in the Arctic
- ENSO – interannual variability in the Tropics

Applications to earthquake modeling

- colliding-cascades model of seismic activity
- intermediate-term prediction

Concluding remarks

- bibliography
- future work

What are BDEs?

Short answer:

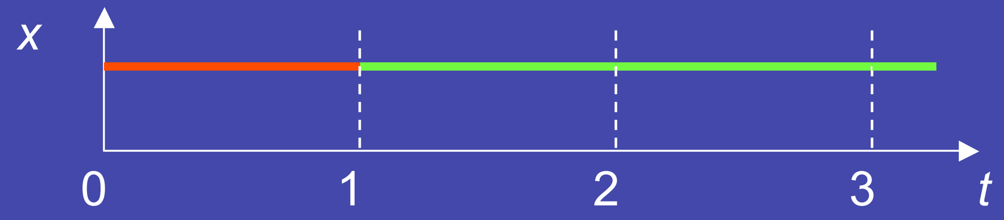
Maximum simplification of nonlinear dynamics
(non-differentiable time-continuous dynamical system)

Longer answer:

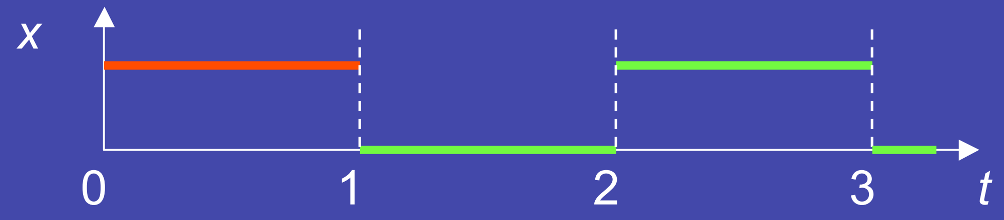
1) $x \in B = \{0,1\}$

$x(t) = x(t-1)$

(simplest EBM: $x = T$)



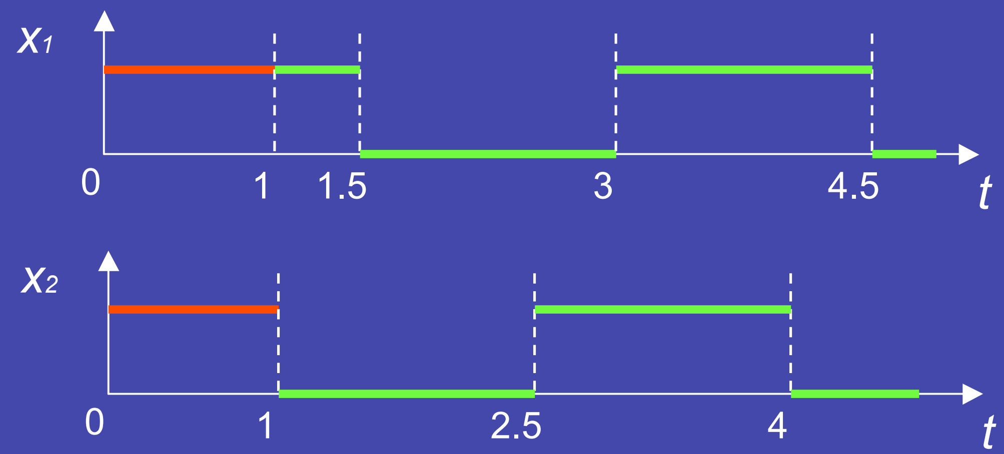
2) $x(t) = \overline{x}(t-1)$



3) $x_1, x_2 \in B = \{0,1\}; 0 < \theta \leq 1$

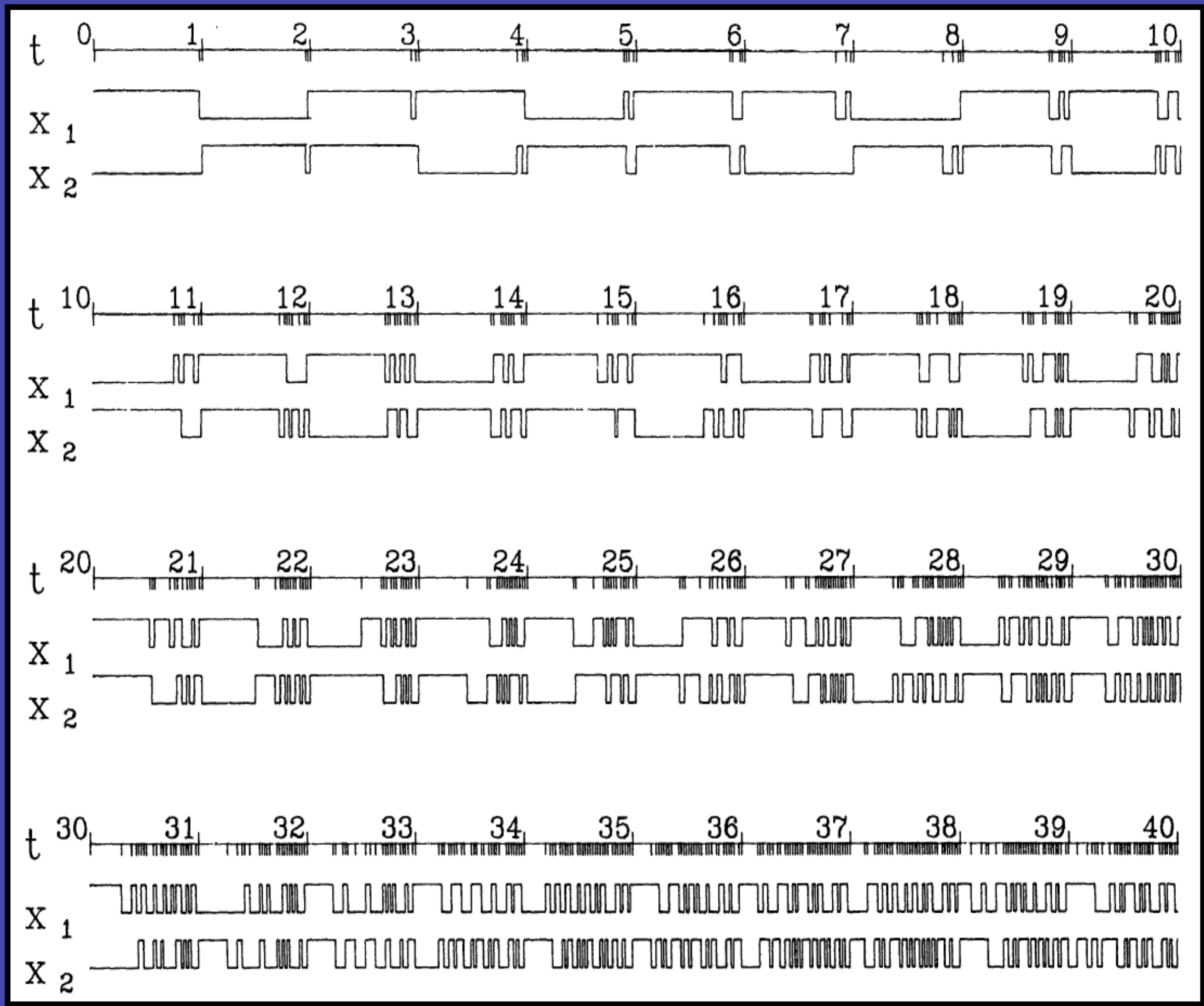
$$\begin{cases} x_1(t) = x_2(t-\theta), \theta = 1/2 \\ x_2(t) = \overline{x}_1(t-1) \end{cases}$$

Eventually periodic with
a period $= 2(1+\theta)$
(simplest OCM: $x_1 = m$, $x_2 = T$)



$$\begin{cases} x_1(t) = x_2(t-\theta) \\ x_2(t) = x_2(t-1)\nabla x_1(t-\theta) \end{cases}$$

θ is irrational



Increase in complexity!

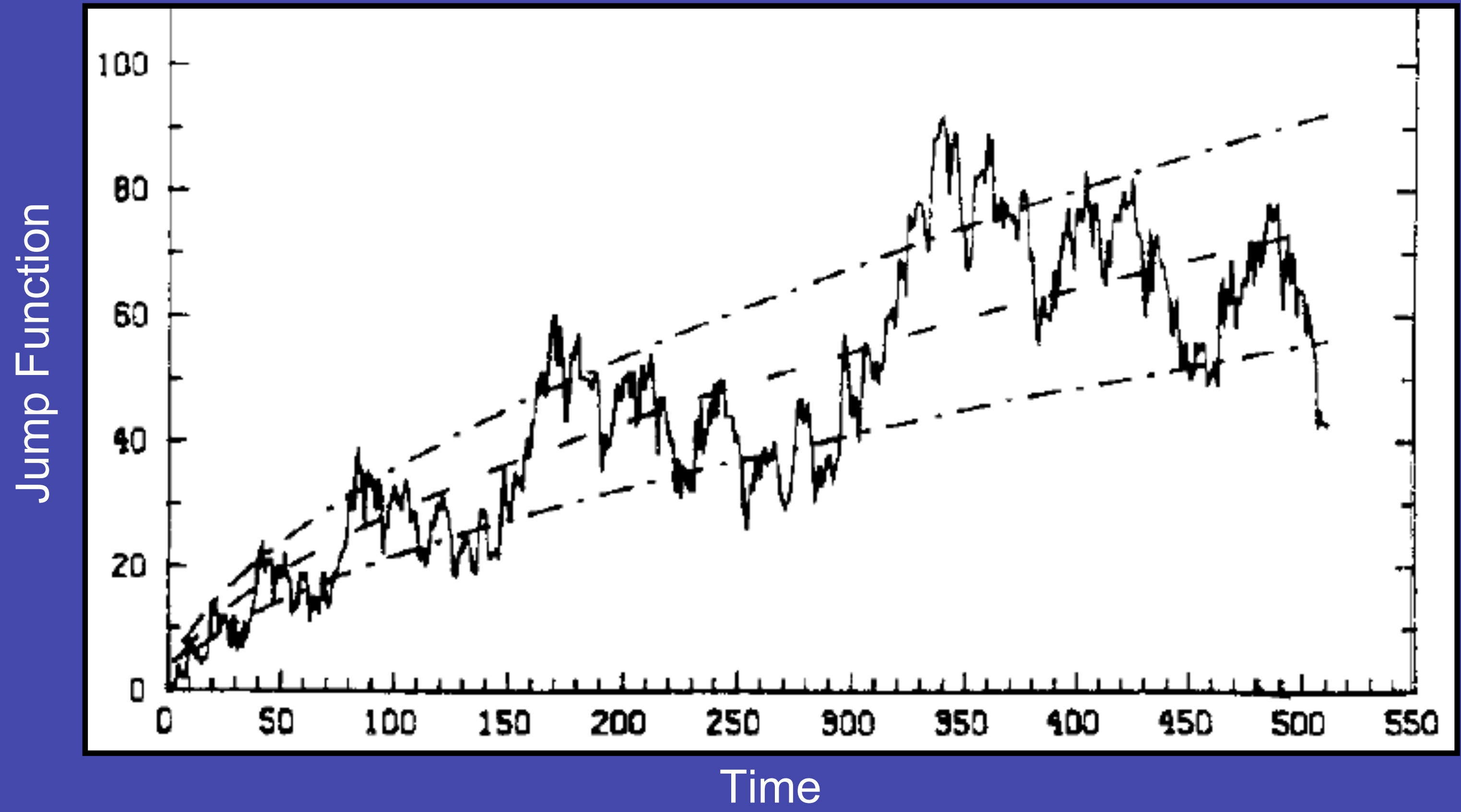
Evolution: biological, cosmogonic, historical

But how much?

Dee & Ghil, *SIAM J. Appl. Math*. (1984), **44**, 111-126

Aperiodic solutions with *increasing complexity*

$$x(t) = x(t-1)\nabla x(t-\theta), \qquad \theta = \frac{\sqrt{5}-1}{2} = \text{"golden ratio"}$$

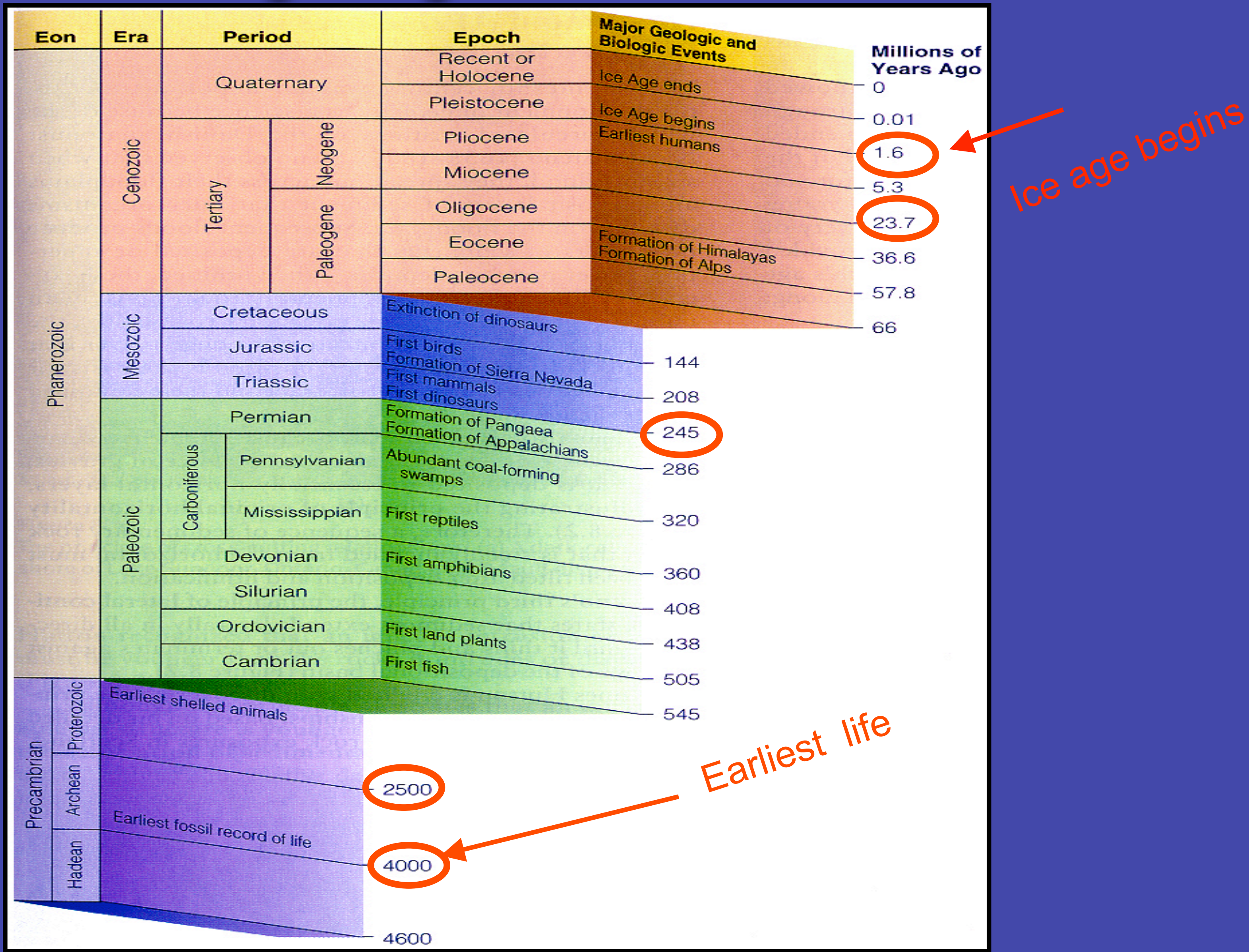


Theorem:

Conservative BDEs with irrational delays have aperiodic solutions with a *power-law increase in complexity.*

N.B. Log-periodic behavior!

The geological time scale

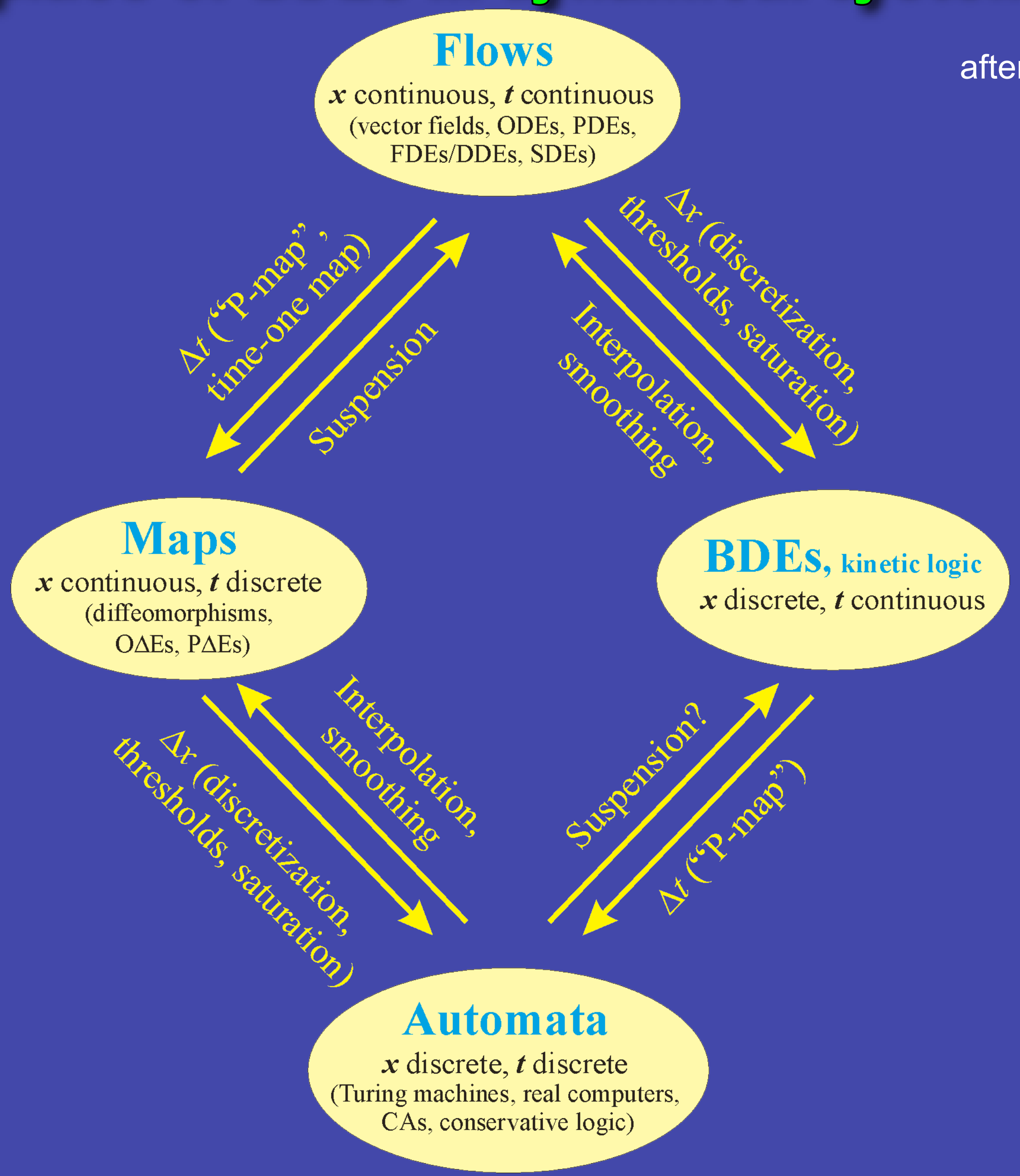


http://www.yorku.ca/esse/veo/earth/image/1-2-2.JPG

Density of events $\cong \log(t)$

The place of BDEs in dynamical system theory

after *A. Mullhaupt* (1984)



Classification of BDEs

Definition: A BDE is *conservative* if its solutions are immediately periodic, i.e. no transients; otherwise it is *dissipative*.

Remark: Rational vs. irrational delays.

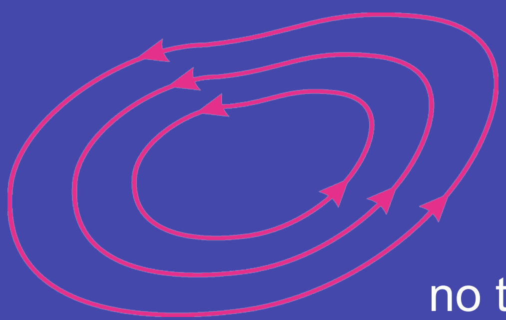
Example:

1) Conservative $x(t) = \overline{x}(t-1)$

2) Dissipative $x(t) = x(t-1) \wedge x(t-\theta)$

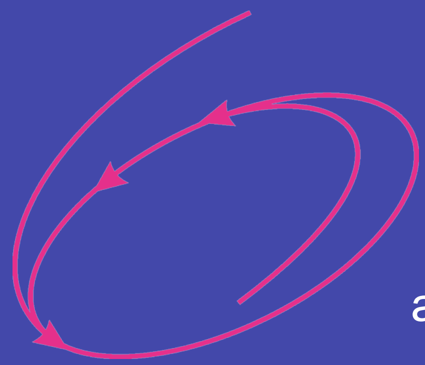
Analogy with ODEs

Conservative – Hamiltonian



no transients

Dissipative – limit cycle



attractor

M. Ghil & A. Mullhaupt, *J. Stat. Phys*., **41**, 125-173, 1985

Examples. Convenient shorthand for scalar 2nd order BDEs

$$x = y \circ z \Leftrightarrow x(t) = x(t-1) \circ x(t-\theta)$$

1. Conservative

$$x = y \nabla z = y \oplus z = y + z (\mathrm{mod}\, 2)$$

$$x = y \Delta z = 1 \oplus y \oplus z$$

Remarks: i) Conservative $\Leftrightarrow$ linear (mod 2)
ii) $\exists$ few conservative connections (~ ODEs)

2. Dissipative

$$x = y \wedge z \,\tilde{\Rightarrow}\, x \to 0$$

$$x = y \vee z \,\tilde{\Rightarrow}\, x \to 1$$

Theorem

Conservative $\Leftrightarrow$ reversible
$\Leftrightarrow$ invertible

A. Mullhaupt, Ph.D. Thesis, May 1984, CIMS/NYU
M. Ghil & A. Mullhaupt, *J. Stat. Phys*., **41**, 125-173, 1985

Classification of BDEs

Structural stability & bifurcations

Theorem

BDEs with periodic solutions only are structurally stable,and conversely

Remark. They are dissipative.

Meta-theorems, by example.

The asymptotic behavior of

$$x(t) = x(t-\theta) \wedge \overline{x}(t-\tau)$$

is given by

$$x(t) = x(t-\theta)$$

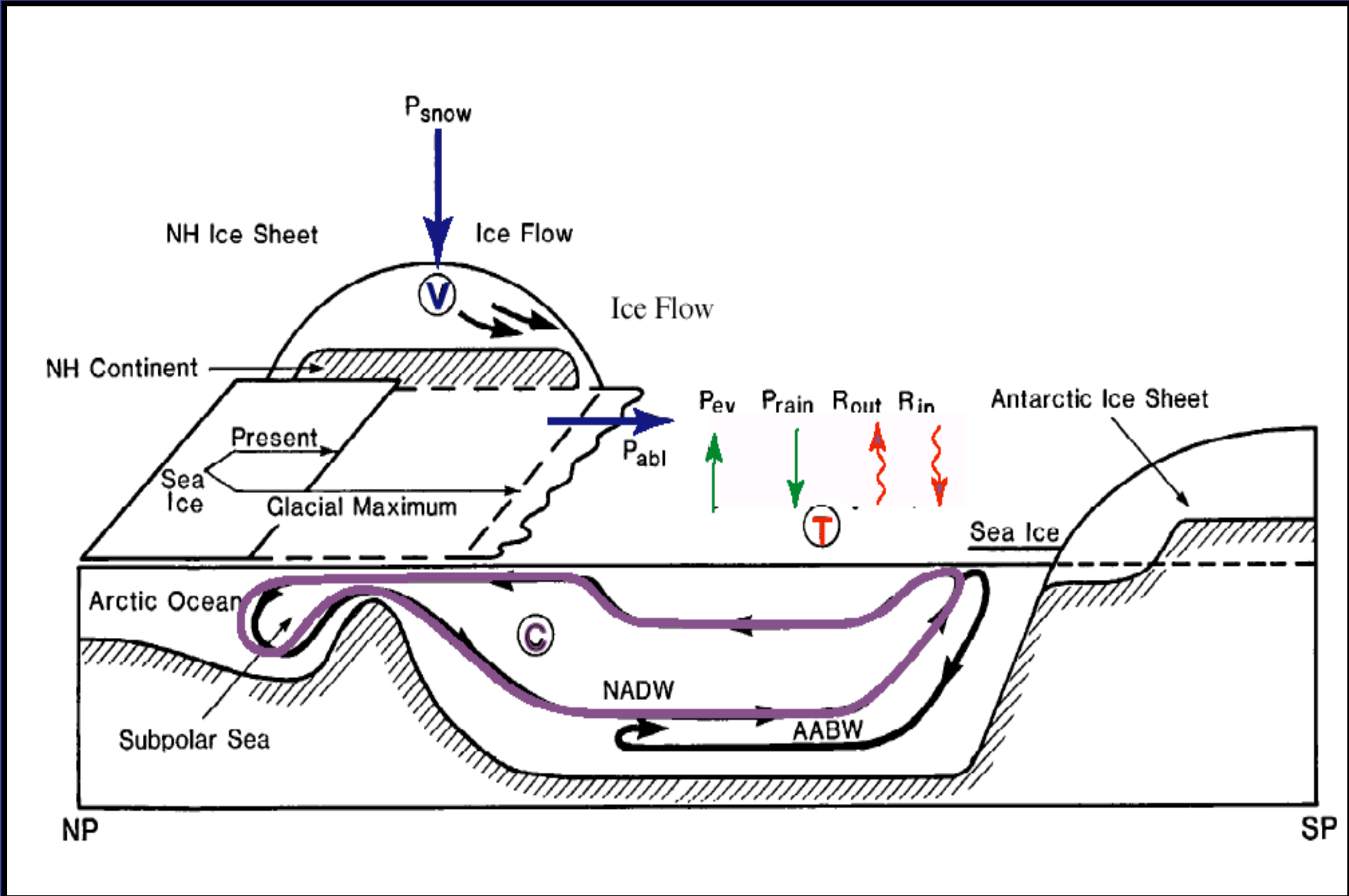
Hence, if $\tau < \theta = 1$ then solutions are asymptotically periodic;

if however $\theta < \tau = 1$ then solutions tend asymptotically to 0.

Therefore, as θ passes through τ, one has Hopf bifurcation.

Paleoclimate application

Thermohaline circulation and glaciations



Logical variables

T - global surface temperature;

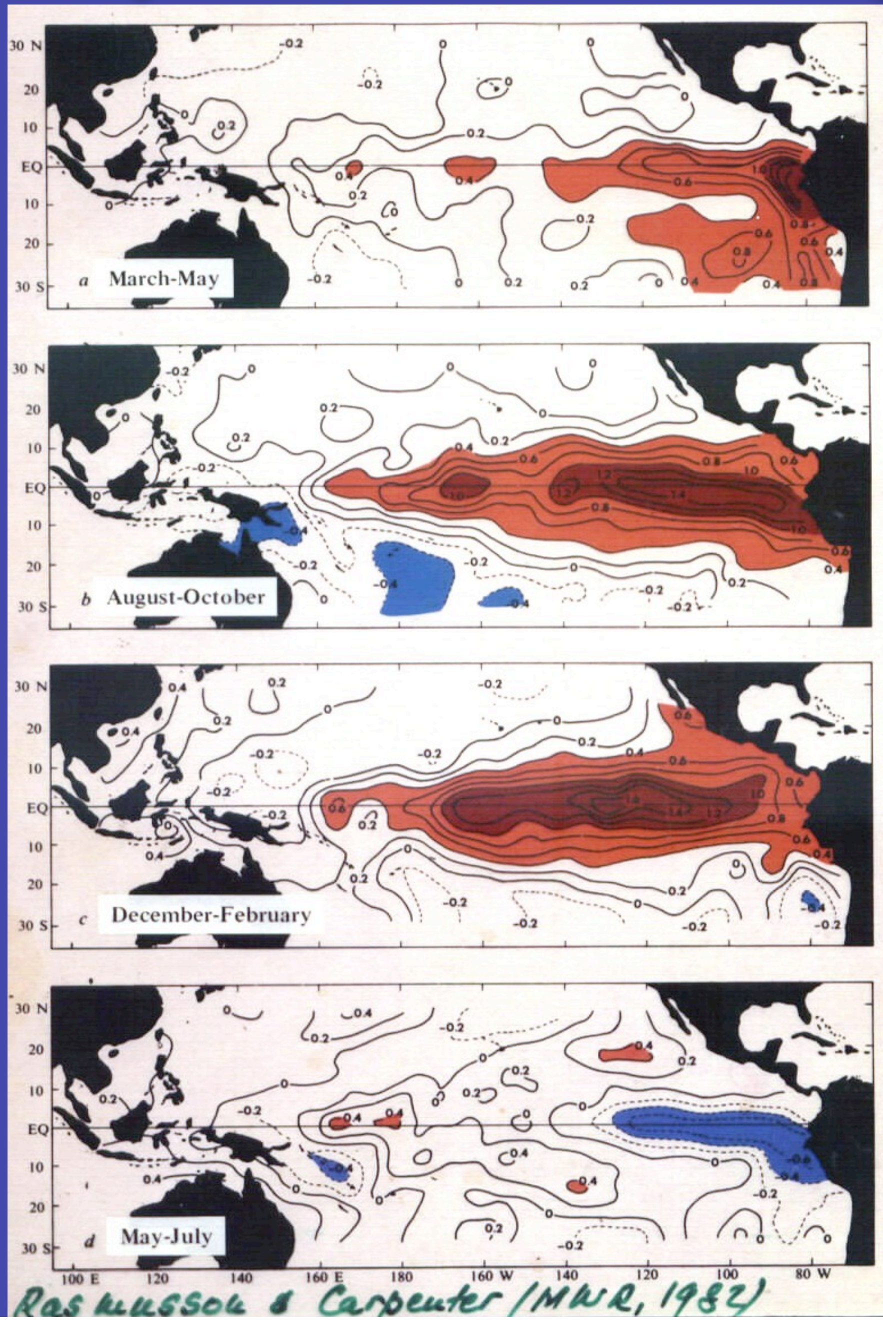
V_N - NH ice volume, V_N = V;

V_S - SH ice volume, V_S = 1;

C - deep-water circulation index

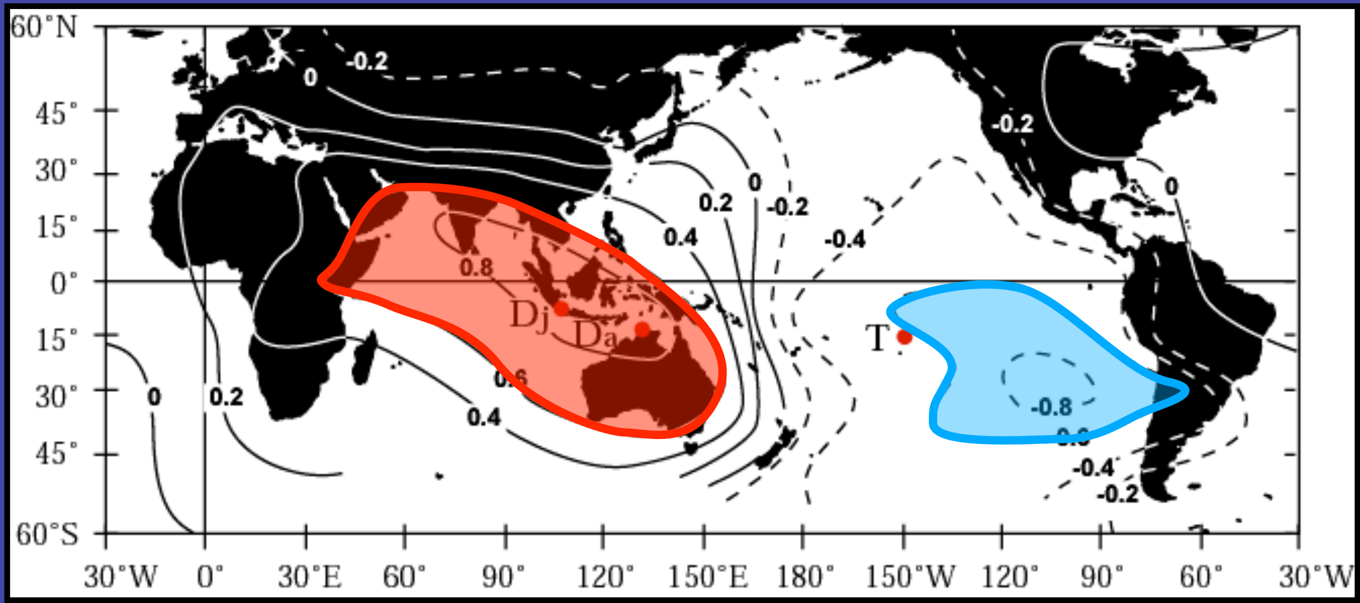
M. Ghil, A. Mullhaupt, & P. Pestiaux, *Climate Dyn.*, **2**, 1-10, 1987.

Spatio-temporal evolution of ENSO episode



Scalar time series that capture ENSO variability

The large-scale Southern Oscillation (SO) pattern associated with El Niño (EN), as originally seen in surface pressures



Neelin (2006) *Climate Modeling and Climate Change*, after Berlage (1957)

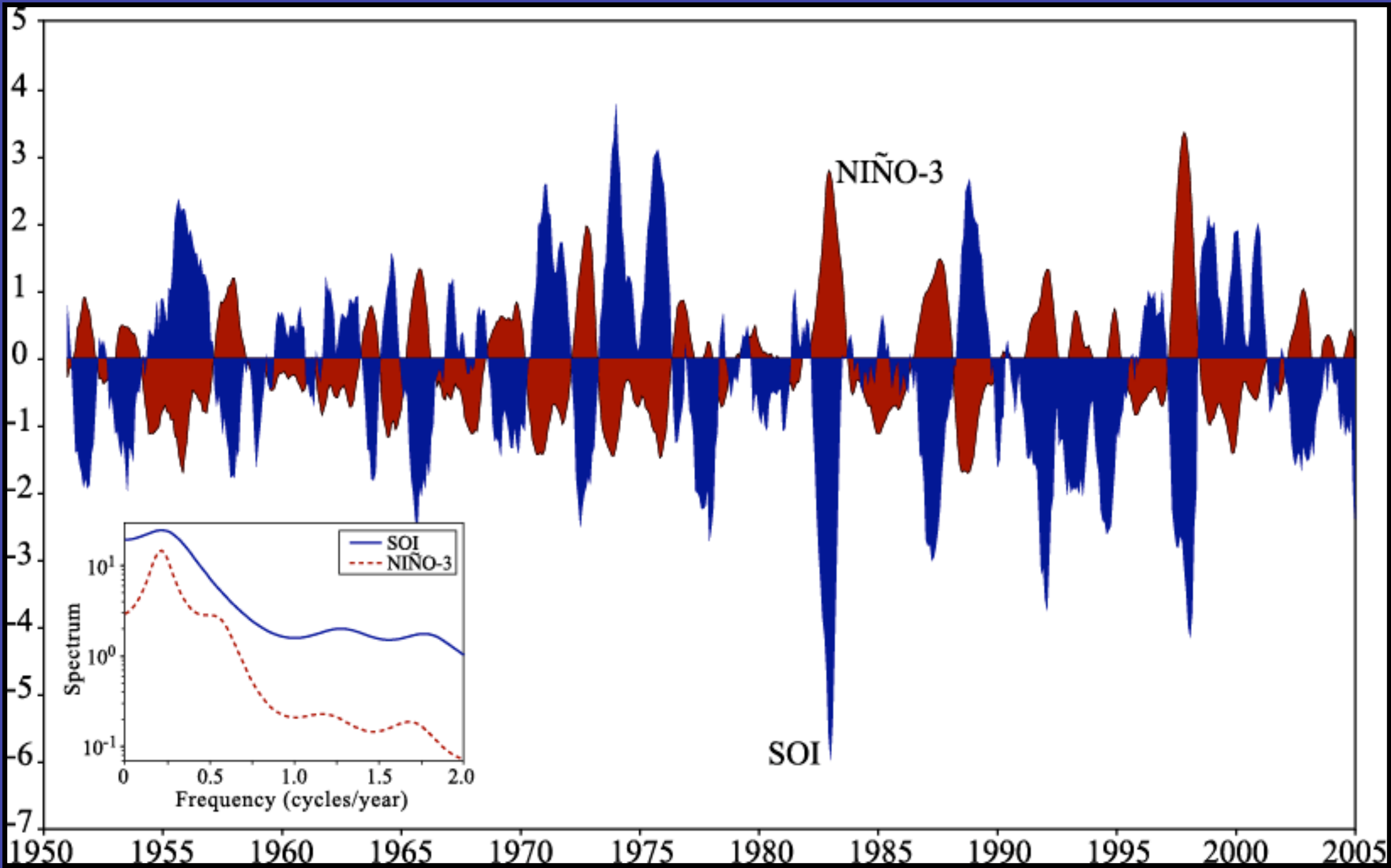
Southern Oscillation:

The seesaw of sea-level pressures p_s between the two branches of the Walker circulation

Southern Oscillation Index (SOI) = normalized difference between p_s at Tahiti (T) and p_s at Darwin (Da)

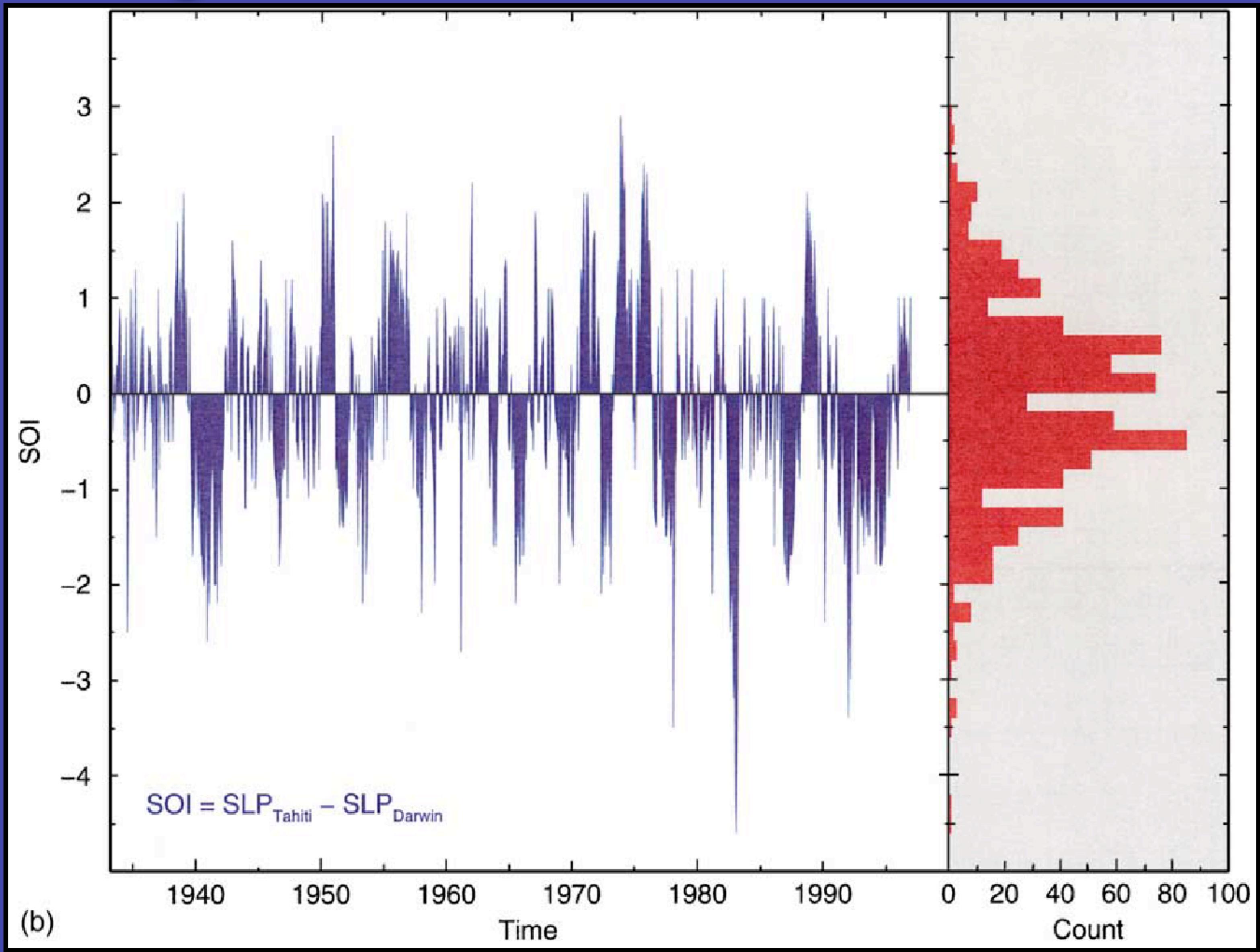
Scalar time series that capture ENSO variability

Time series of ***atmospheric pressure***
and ***sea surface temperature*** (SST) indices



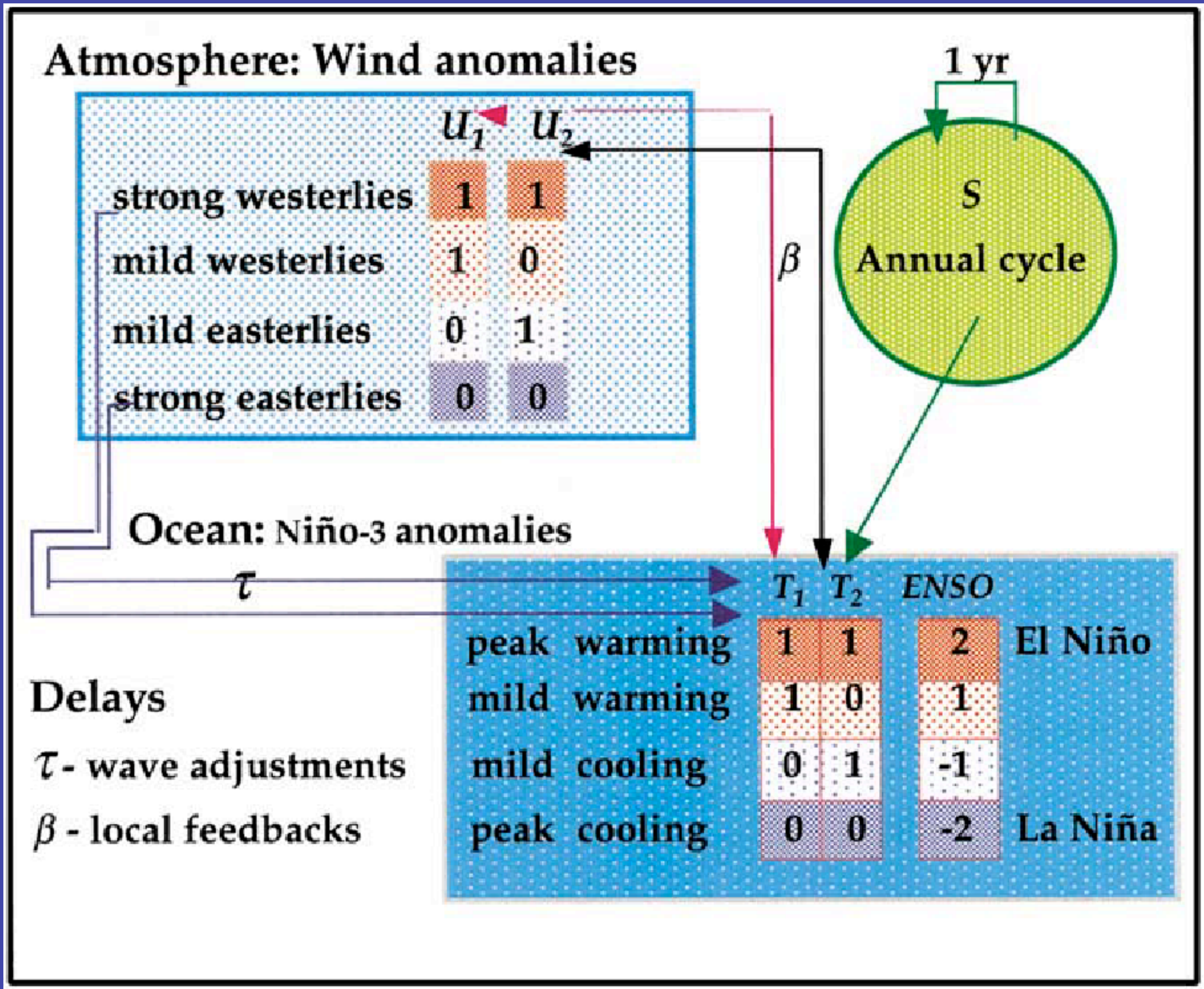
Data courtesy of NCEP's Climate Prediction Center
Neelin (2006) *Climate Modeling and Climate Change*

Histogram of size distribution for ENSO events



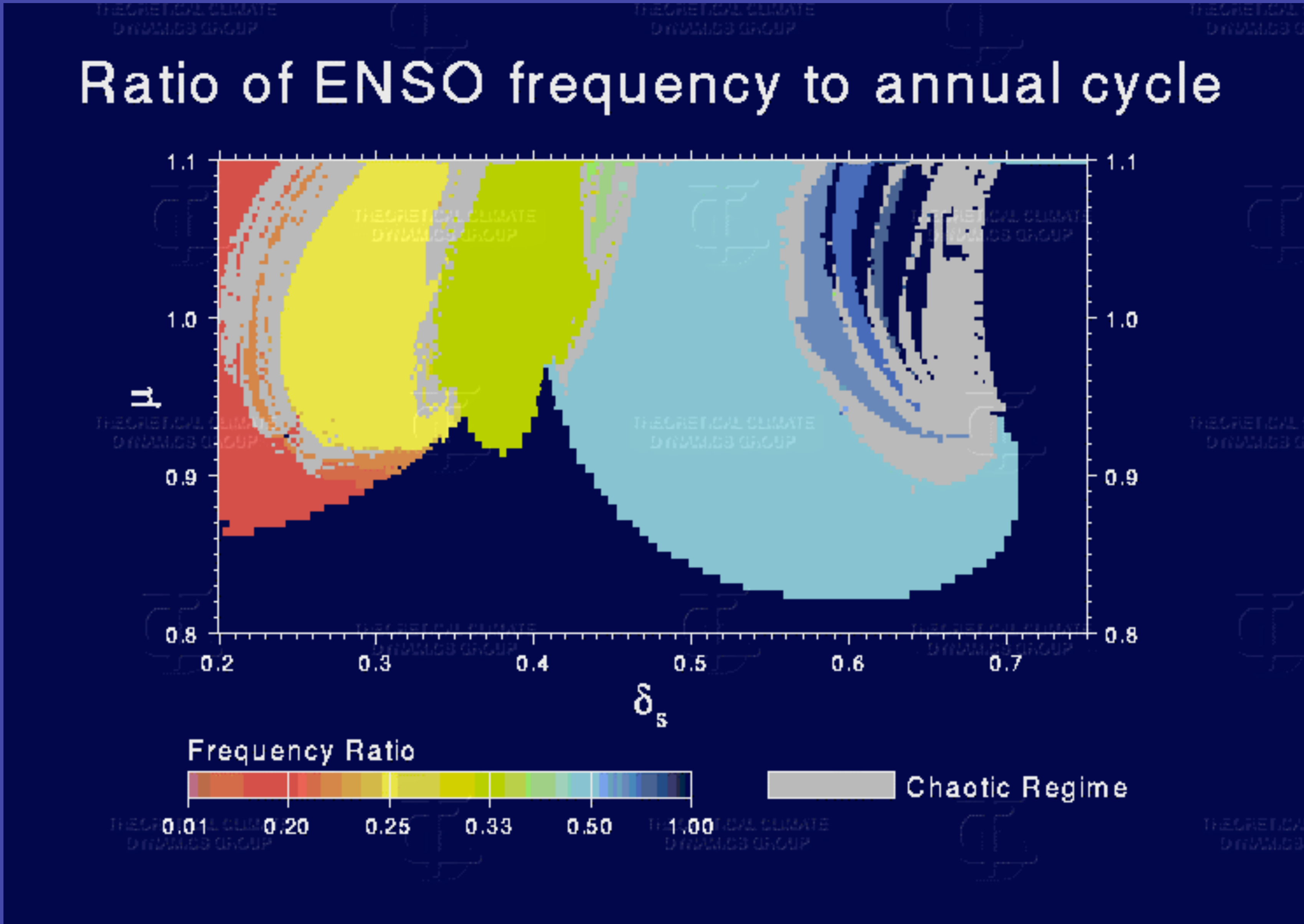
A. Saunders & M. Ghil, *Physica D,* **160**, 54–78, 2001
(courtesy of Pascal Yiou)

BDE Model for ENSO: Formulation



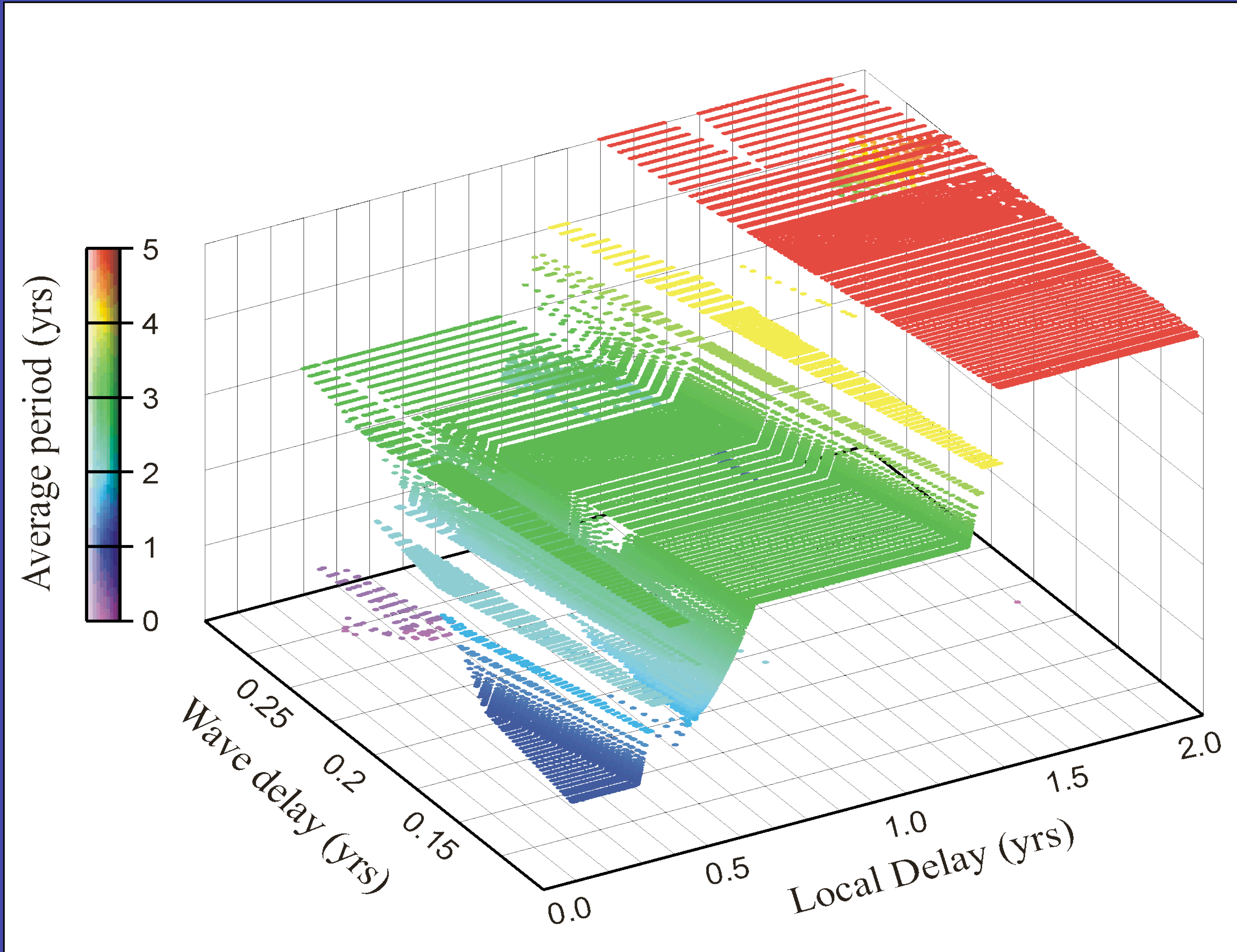
A. Saunders & M. Ghil, *Physica D*, **160**, 54–78, 2001

Devil's Bleachers in a 1-D ENSO Model



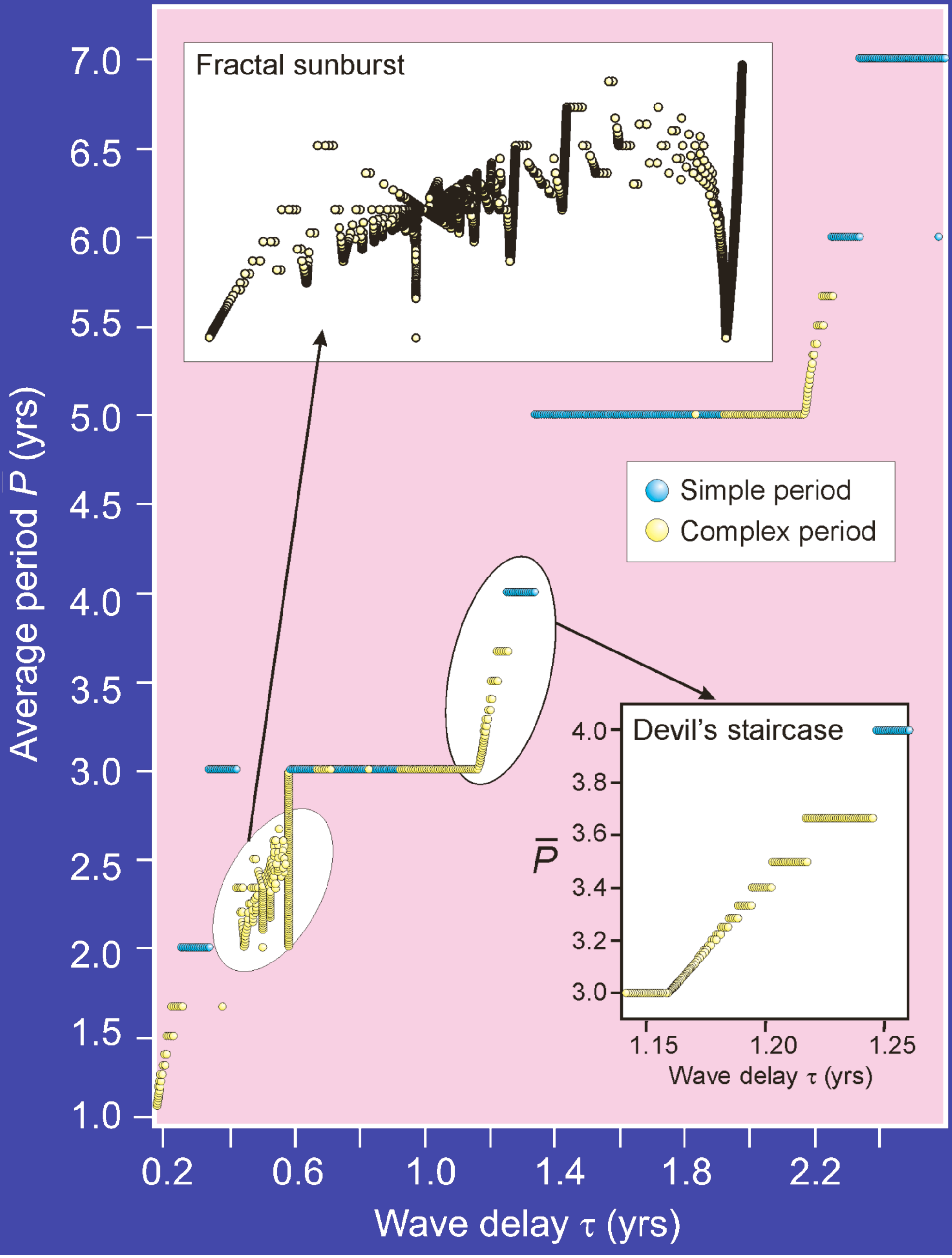
F.-F. Jin, J.D. Neelin & M. Ghil, *Physica D*, **98**, 442-465, 1996

Devil's Bleachers in the BDE Model of ENSO

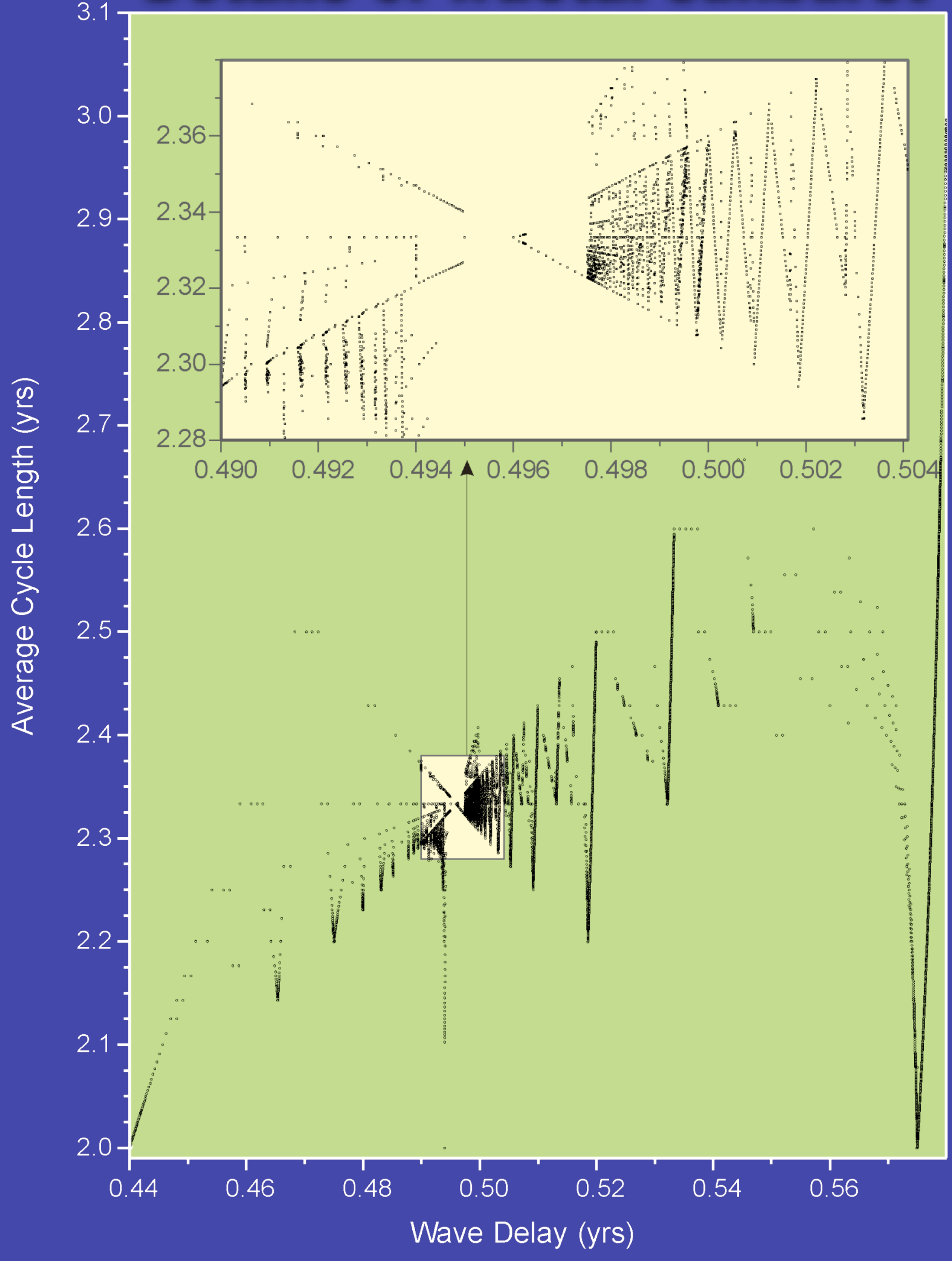


A. Saunders & M. Ghil, *Physica D,* **160**, 54–78, 2001

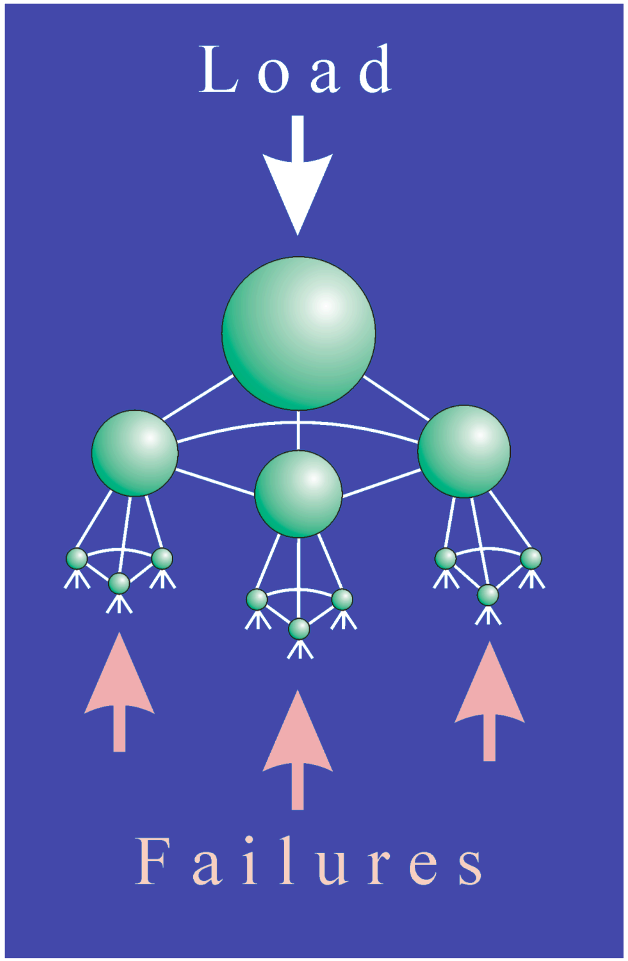
Devil's staircase and fractal sunburst



Details of fractal sunburst



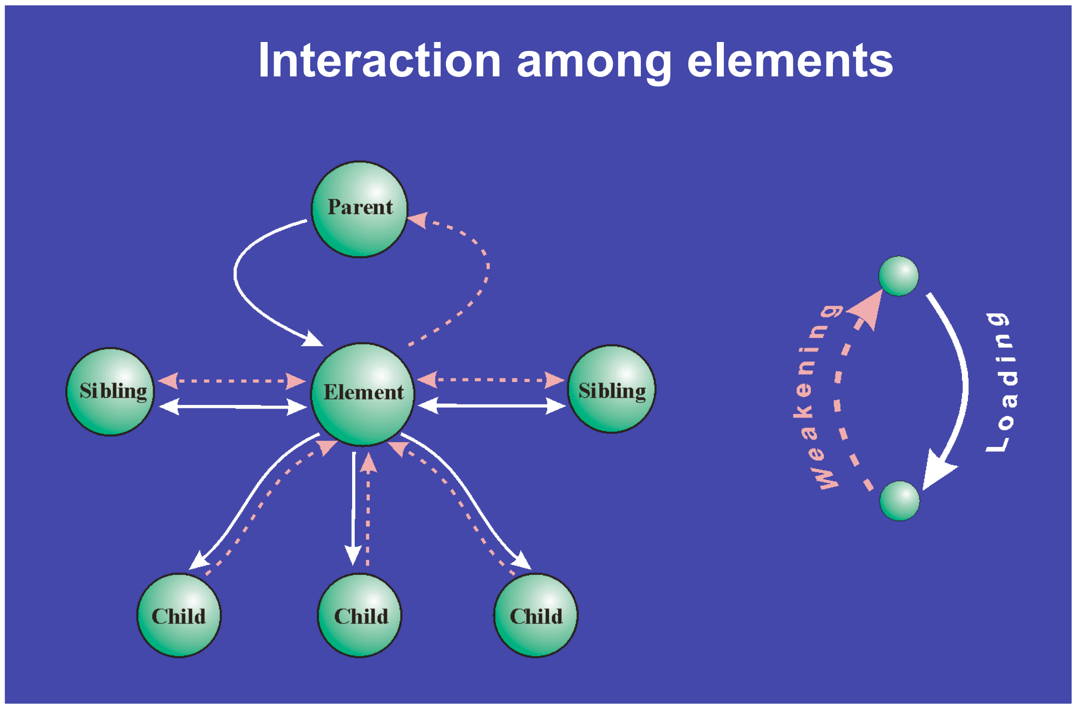
Colliding-Cascade Model



1. **Hierarchical** structure

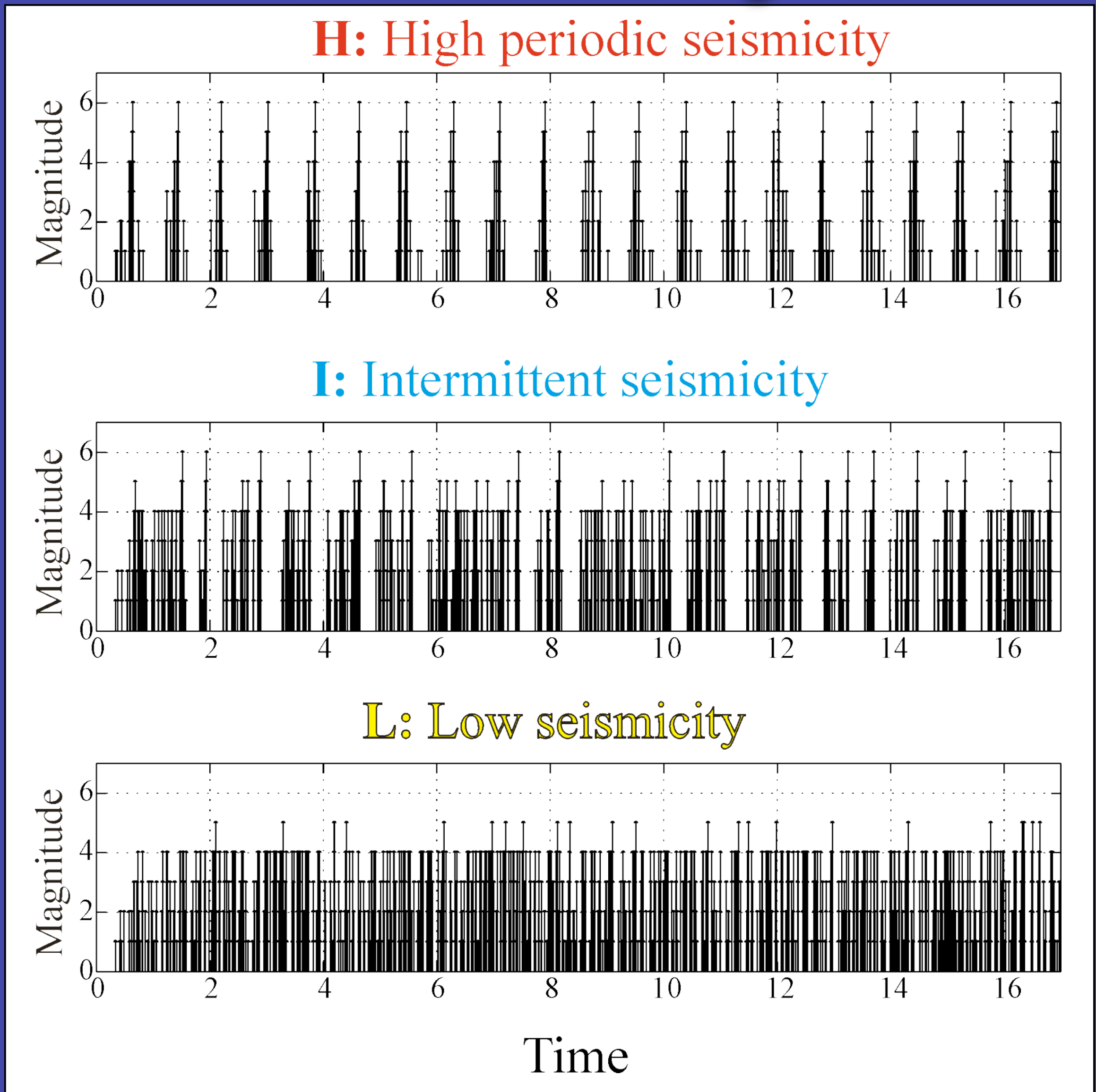
2. **Loading** by external forces

3. Elements' ability to **fail & heal**



A. Gabrielov, V. Keilis-Borok, W. Newman, & I. Zaliapin (2000a, b, *Phys. Rev. E*; *Geophys. J. Int.*)

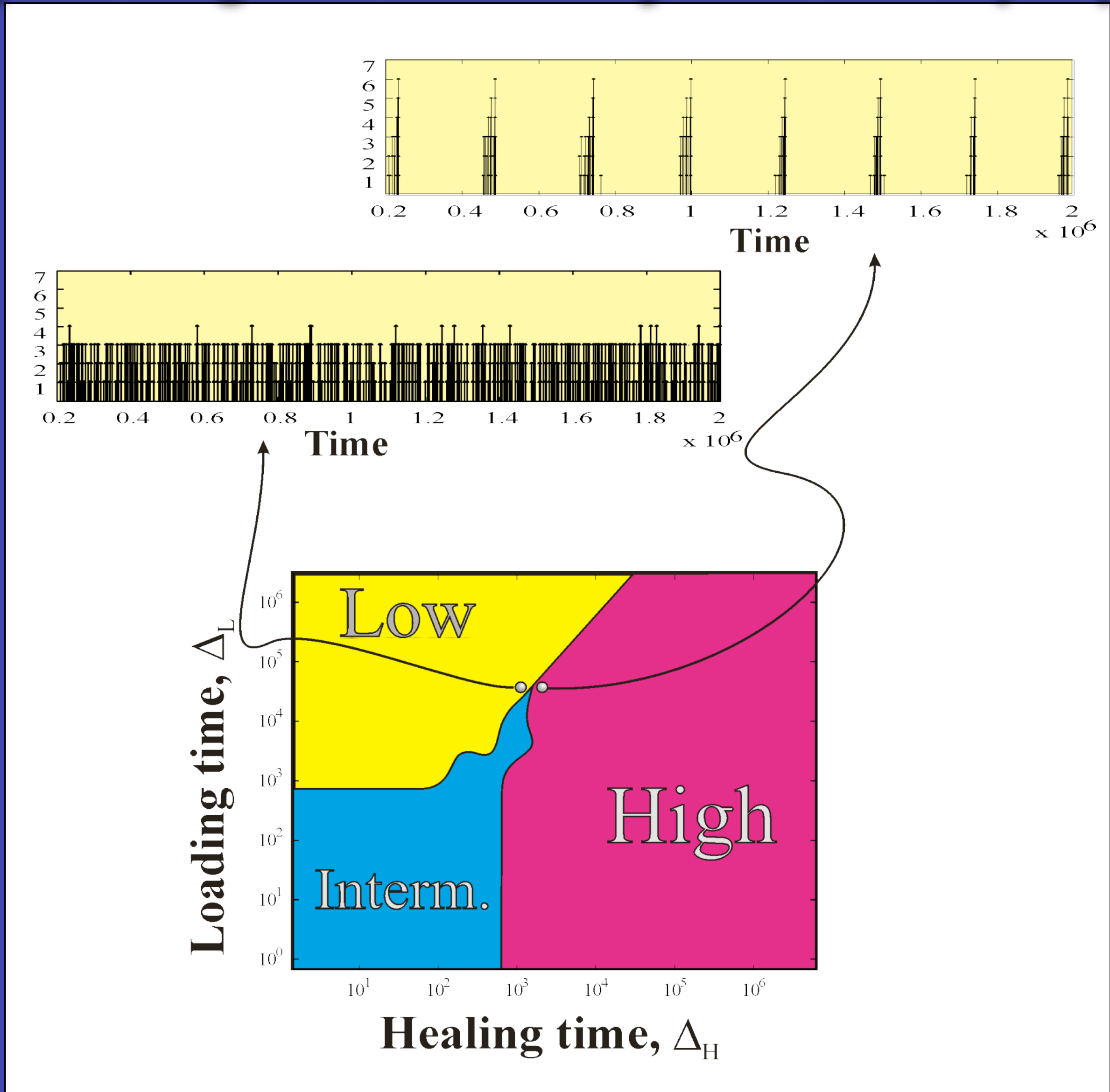
BDE model of colliding cascades: Three seismic regimes



I. Zaliapin, V. Keilis-Borok & M. Ghil (2003a, *J. Stat. Phys.*)

BDE model of colliding cascades

Regime diagram: Instability near the triple point

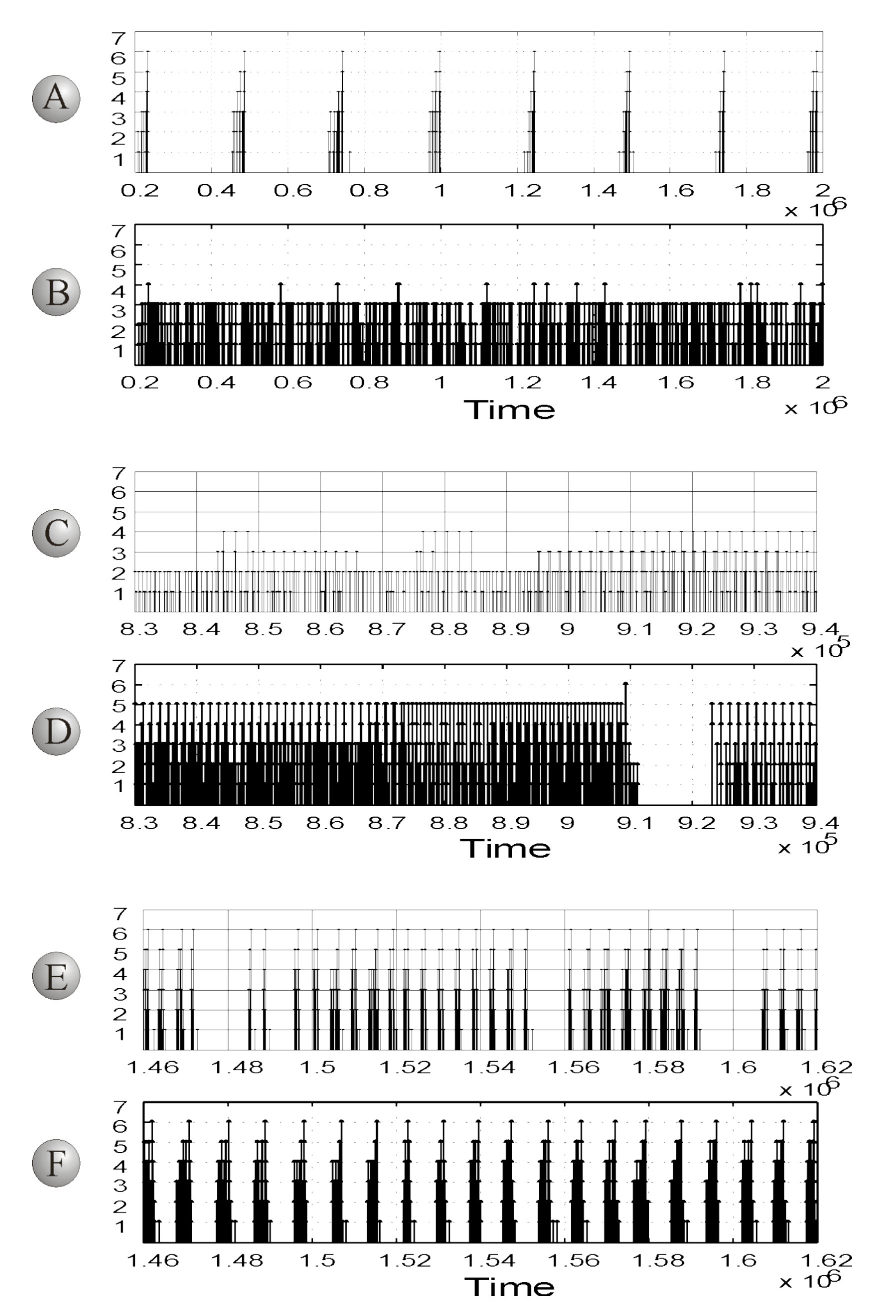


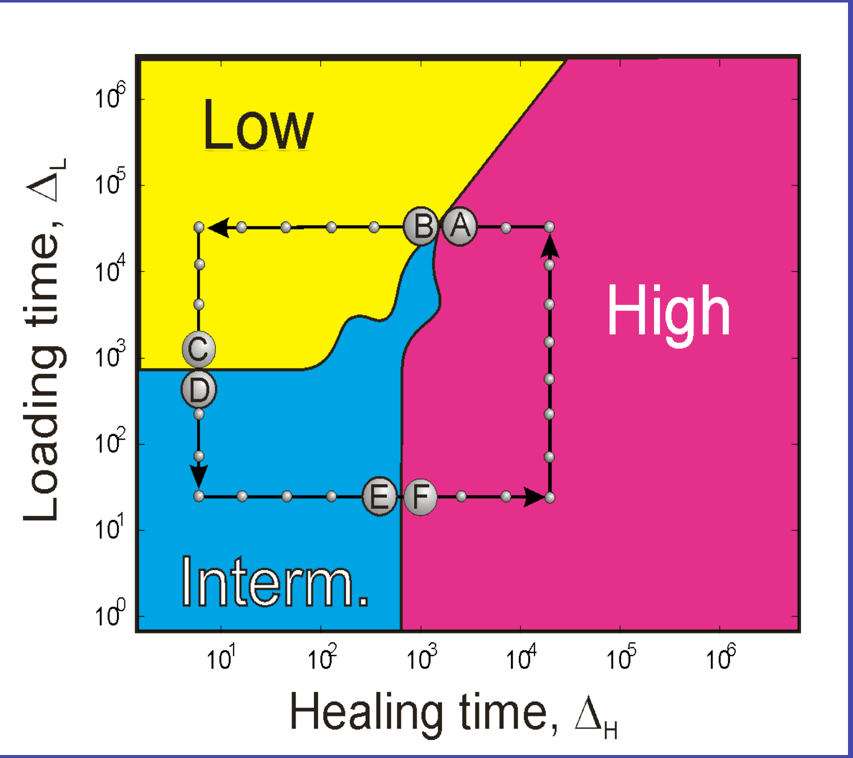
I. Zaliapin, V. Keilis-Borok & M. Ghil (2003a, *J. Stat. Phys.*)

BDE model of colliding cascades

Regime diagram:

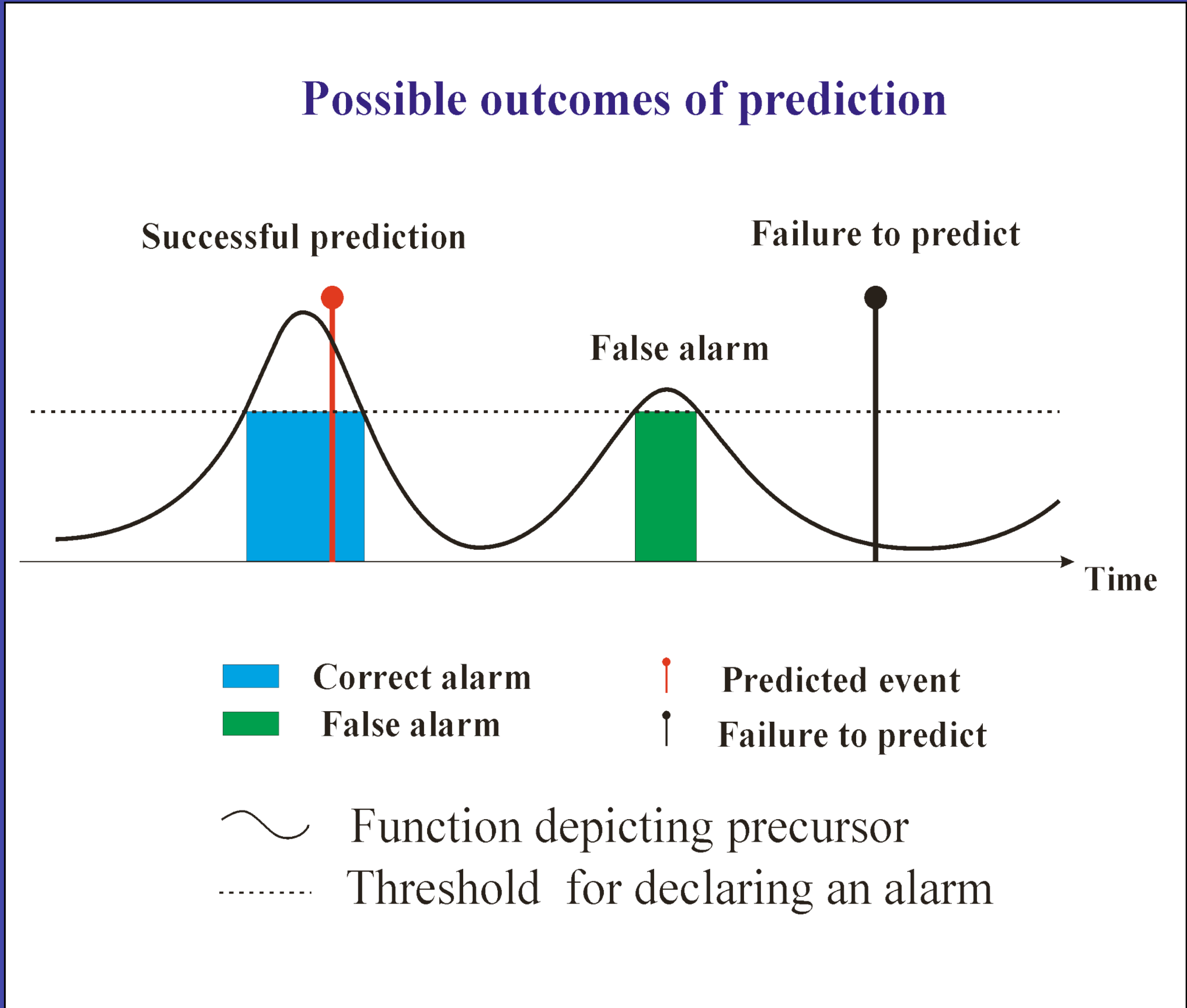
Transition between regimes





I. Zaliapin, V. Keilis-Borok & M. Ghil
(2003a, *J. Stat. Phys.*)

Forecasting algorithms for natural and social systems: Can we beat statistics-based approach?



Ghil and Robertson (2002, *PNAS*)
Keilis-Borok (2002, *Annu. Rev. Earth Planet. Sci.*)

Minimax prediction strategy

P – set of parameters for precursor Π
(e.g. magnitude threshold, time window, *etc.*)

$\Pi_t(P)$ – Boolean alarm process

$\tau(P)$ – fractional time covered by alarms

$n(P)$ – fractional number of unpredicted target events

$f(P)$ – fractional number of false alarms

Minimax prediction strategy 1:

$$P = \arg\min\left[f(P)\right]$$

$$A_{\text{collective}} = \Pi_1 \vee \Pi_2 \vee \ldots \vee \Pi_n$$

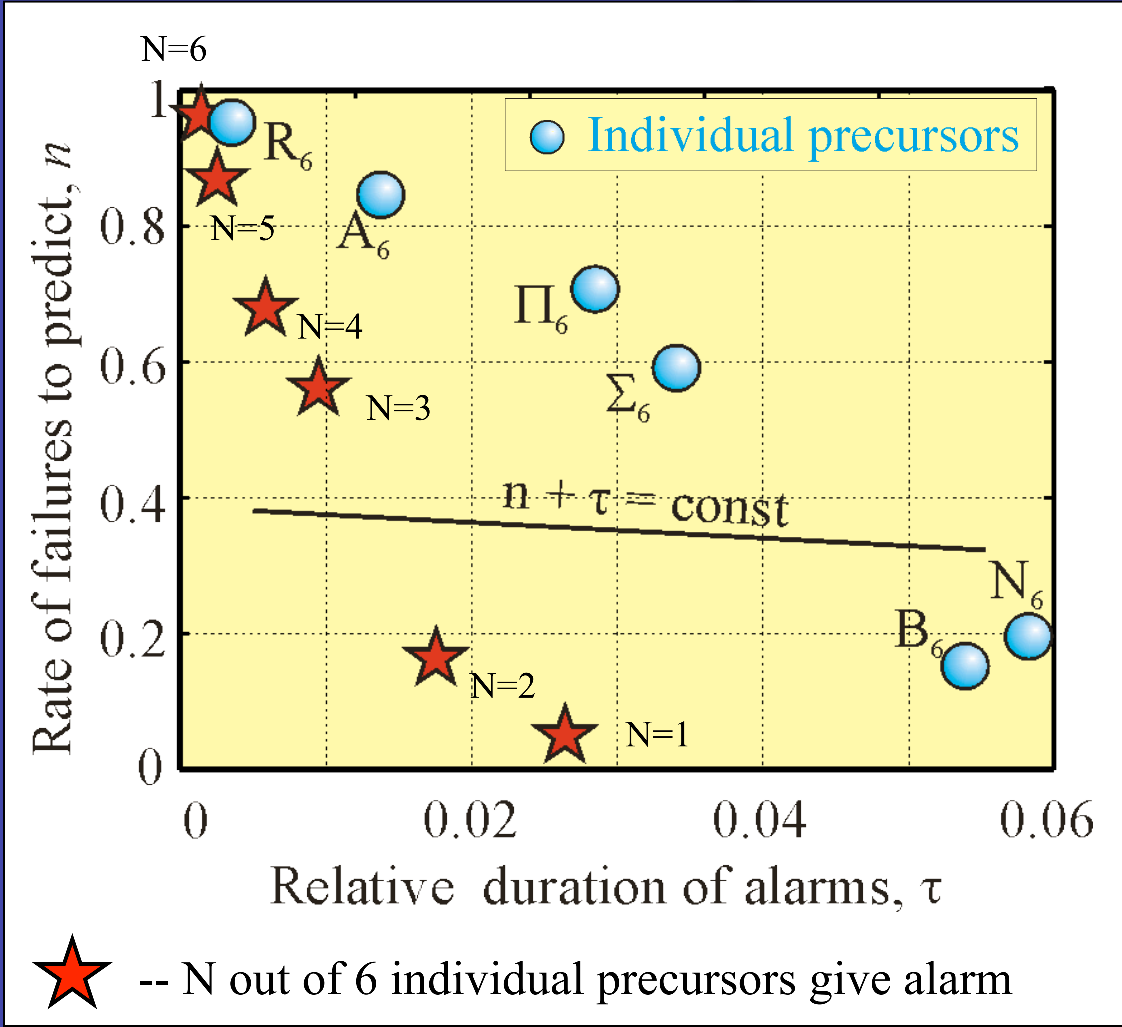
Minimax prediction strategy 2:

$$P = \arg\min\left[n(P)\right]$$

$$A_{\text{collective}} = \Pi_1 \wedge \Pi_2 \wedge \ldots \wedge \Pi_n$$

BDE model

Minimax prediction strategy 1

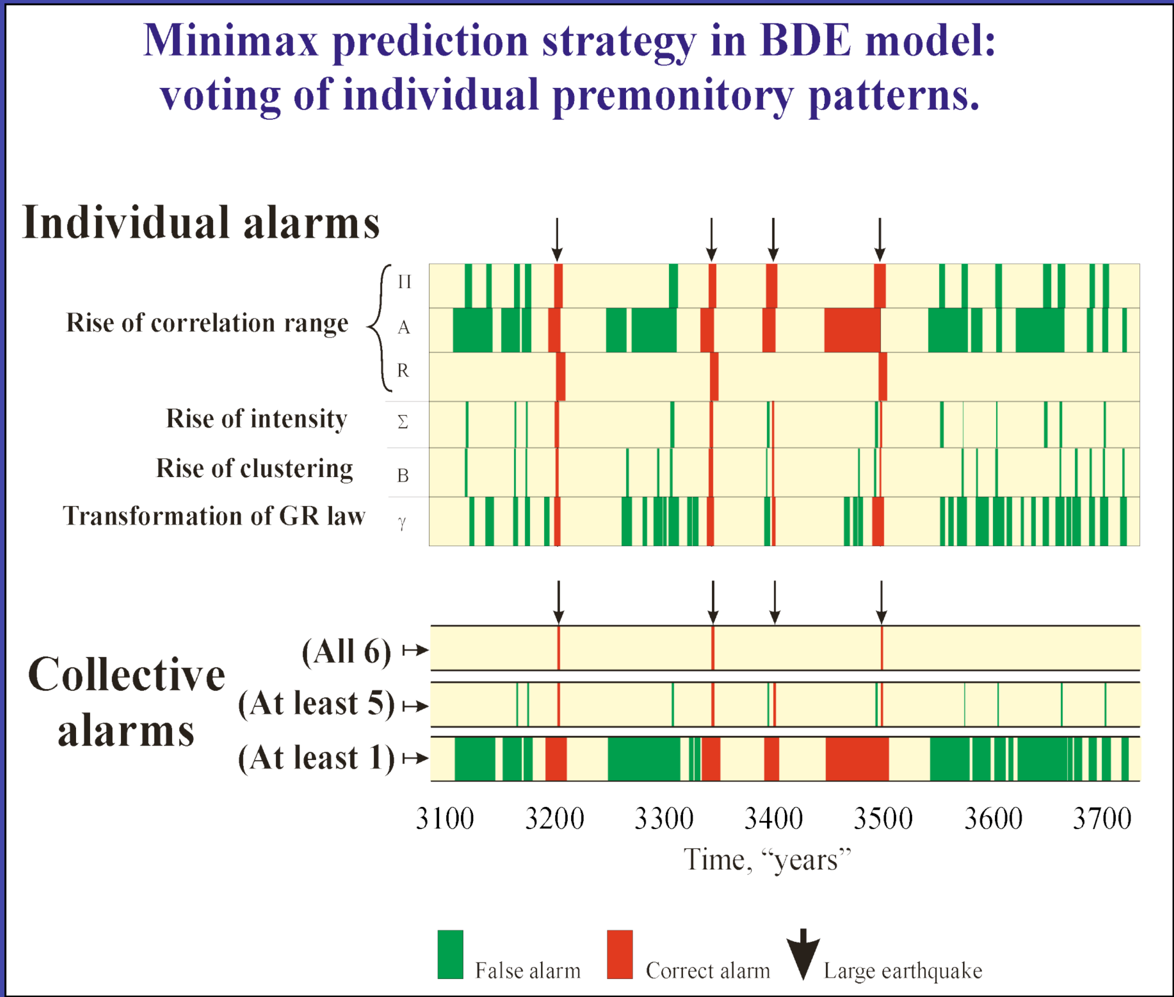


Individual patterns are tuned to eliminate false alarms at the cost of having more failures to predict. Collectively, errors of both kinds are drastically reduced.

After Zaliapin, Keilis-Borok, & Ghil (2003b, *J. Stat. Phys.*)

BDE model

Minimax prediction strategy 2



Individual patterns are tuned to eliminate failures to predict at the cost of having more false alarms.
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After Zaliapin, Keilis-Borok, & Ghil (2003b, *J. Stat. Phys.*)

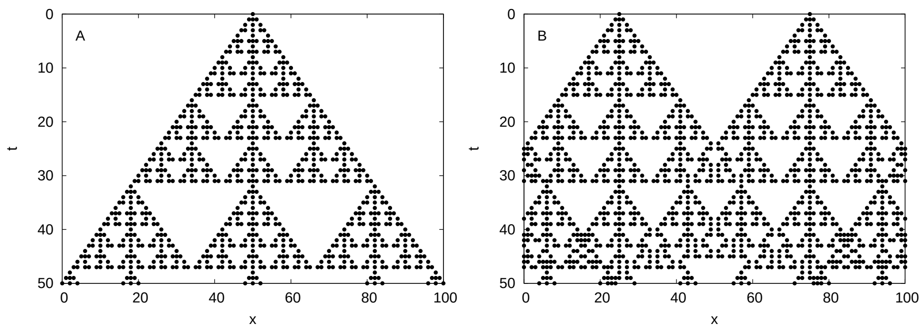

Fig. 13. Solutions of the "partial BDE" (34): (a) for a single nonzero site at $t = 0$; and (b) the collision of two "waves," each originating from such a site. For the space and time steps $\theta_t = \theta_z = 1$, this BDE is equivalent to the elementary cellular automaton (ECA) with rule 150; empty sites ($u_i(j) = 0$) in white and occupied sites ($u_i(j) = 1$) in black.

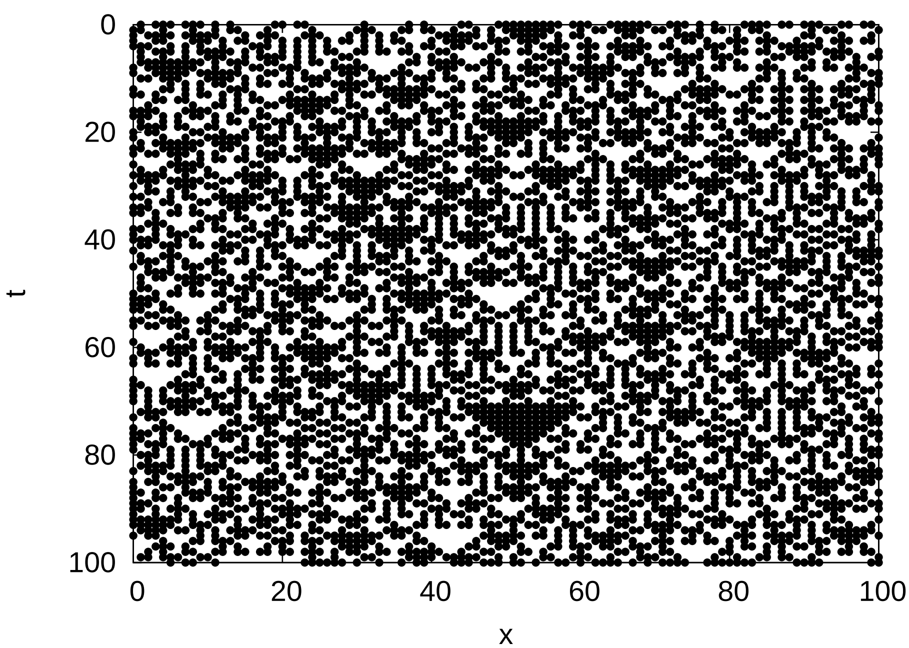

Fig. 14. The solution of the BDE (34) starting from a random initial state of length $N = 100$. The qualitative behavior is characterized by "triangles" of empty or occupied sites but without any recurrent pattern; this behavior does not depend on the particular random initial state.

Table 2. Results on partial BDEs

PDE $\partial_t v =$	Approx.	PBDE $u_i(t+\theta_t) =$	ECA rule	ECA class	Behavior
$\partial_z v$	$\bigtriangledown$ (I order)	$u_{i-1}(t)$	170	—	conservative
$\partial_{zz} v$	$\vee$	$[u_{i-1}(t) \vee u_{i+1}(t)] \bigtriangledown u_i(t)$	54	I	dissipative
$\partial_{zz} v$	$\wedge$	$[u_{i-1}(t) \wedge u_{i+1}(t)] \bigtriangledown u_i(t)$	108	II	dissipative
$\partial_{zz} v$ $\partial_z v$	$\bigtriangledown$ (I order) $\bigtriangledown$ (II order)	$[u_{i-1}(t) \bigtriangledown u_{i+1}(t)] \bigtriangledown u_i(t)$	150	III	dissipative

Summary of results on the partial BDEs obtained from the different considered approximations for the spatial derivative in the parabolic and hyperbolic PDEs. Temporal derivative is always approximated by the $\bigtriangledown$ operator. From top to bottom, these are the equations (29), (35), (36) and (33) discussed in the text, respectively. Notice that, though all but the first considered PBDEs are dissipative, only the last one, *i.e.* Eq. (33), displays chaotic behavior in the limit of infinite lattice size.

Short BDE bibliography

Theory

Dee & Ghil (1984, *SIAM J. Appl. Math.*)
Ghil & Mullhaupt (1985, *J. Stat. Phys.*)

Applications to climate

Ghil et al. (1987, *Climate Dyn.*)
Mysak et al. (1990, *Climate Dyn.*),
Darby & Mysak (1993, *Climate Dyn.*),
Saunders & Ghil (2001, *Physica D*)

Applications to solid-earth problems

Zaliapin, Keilis-Borok & Ghil (2003a, b, *J. Stat. Phys.*)

Applications to genetics

Oktem, Pearson & Egiazarian (2003, *Chaos*)
Gagneur & Casari (2005, *FEBS Letters*)

Applications to the socio-economic and computer sciences?

Review paper

Ghil & Zaliapin (2005) A novel fractal way: Boolean delay equations and their applications to the Geosciences, *Invited for book honoring B.Mandelbrot 80th birthday*

Concluding remarks

1. BDEs have rich behavior:
periodic, quasi-periodic, aperiodic, increasing complexity

2. BDEs are relatively easy to study

3. BDEs are natural in a digital world

4. Two types of applications

- strictly discrete (genes, computers)
- saturated, threshold behavior (nonlinear oscillations, climate dynamics, population biology, earthquakes)

5. Can provide insight on a very qualitative level
(~ symbolic dynamics)

6. Generalizations possible
(spatial dependence – "partial" BDEs;
stochastic delays &/or connectives)

Conclusions

Hmmm, this is interesting!

But what does it all mean?

Needs more work!!!