Tel Aviv University, Fluid Mech. & Heat Transfer Dept.

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Advanced Spectral Methods, Nonlinear Dynamics, and the Nile River

Michael Ghil

Ecole Normale Supérieure, Paris, and University of California, Los Angeles

Motivation

- Climatic time series have typically broad peaks on top of a continuous, "warm-colored" background → Method
- 2. Connections to nonlinear dynamics \rightarrow Theory
- 3. Need for stringent statistical significance tests \rightarrow *Toolkit*
- 4. Applications to analysis and prediction \rightarrow *Examples*

Joint work with: M. R. Allen, M. D. Dettinger, K. Ide, N. Jiang, C. L. Keppene, D. Kondrashov, M. Kimoto, M. E. Mann, J. D. Neelin, M. C. Penland, G. Plaut, A. W. Robertson, A. Saunders, D. Sornette, S. Speich, C. M. Strong, C. Taricco, Y.-d. Tian, Y. S. Unal, R. Vautard, & P. Yiou (on 3 continents).

http://www.atmos.ucla.edu/tcd

Motivation & Outline

- 1. Data sets in the geosciences are often short and contain errors: this is both an obstacle and an incentive.
- 2. Phenomena in the geosciences often have both regular ("cycles") and irregular ("noise") aspects.
- 3. Different spatial and temporal scales: one person's noise is another person's signal.
- 4. Need both deterministic and stochastic modeling.
- Regularities include (quasi-)periodicity → spectral analysis via "classical" methods (see SSA-MTM Toolkit).
- 6. Irregularities include scaling and (multi-)fractality → "spectral analysis" via Hurst exponents, dimensions, etc. (see Multi-Trend Analysis, MTA)
- 7. Does some combination of the two, + deterministic and stochastic modeling, provide a pathway to prediction?

For details and publications, please visit these two Web sites:

- TCD <u>http://www.atmos.ucla.edu/tcd/</u> → key person Dmitri Kondrashov!
- E2-C2 http://www.ipsl.jussieu.fr/~ypsce/py_E2C2.html

Climatic Trends & Variability

• Standard view — Binary thinking, dichotomy:

Trend — Predictable (completely), deterministic, reassuring, good;

Variability — Unpredictable (totally), stochastic, disconcerting, bad.

- In fact, these two are but extremes of a spectrum of, more or less predictable, types of climatic behavior, between the totally boring & the utterly surprising.
- (Linear) Trend = Stationary >

Set Ballie & Tory wash Some

Periodic > Quasi-periodic >

Deterministically aperiodic >

Random Noise

• Here ">" means "better, more predictable", &

Variability = Periodic + Quasi-periodic +

Aperiodic + Random

Time Series in Nonlinear Dynamics

The 1980s — decade of greed & fast results

(LBOs, junk bonds, fractal dimension).

Packard et al. (1980), Roux et al. (1980);

Mañe (1981), Ruelle (1981), Takens (1981);

• Method of delays: $\ddot{x}_i = f_i(x_1, \dots, x_n) \Leftrightarrow x^{(n)} = F(x^{(n-1)}, \dots, x)$ $\ddot{x} = F(x, \dot{x}) \Rightarrow \begin{cases} \dot{x} = y, \\ \dot{y} = F(x, y) \end{cases}$

Differentiation ill-posed \Rightarrow use differences instead!

1st Problem — smoothness:

Whitney embedding lemma doesn't apply to most attractors (e.g.,Lorenz) 2nd Problem — noise;

3rd Problem — sampling: long recurrence times.

• Some rigorous results on convergence:

Smith (1988, Phys. Lett. A), Hunt (1990, SIAM J. Appl. Math.)

and the second second

Spectral Density (Math)/Power Spectrum (Science & Engng.)



Power Law for Spectrum

 $S(f) \sim f^{-p} + poles$

i.e. linear in log-log coordinates

For a 1st-order Markov process or "red noise" p = 2

"Pink" noise, p = 1 (1/*f*, flicker noise)

"White" noise, p = 0

Low-order dynamical (deterministic) systems

have exponential decay of S(f) (linear in log-linear coordinates)

e.g. for Smale horseshoe $\forall k \exists 2^k$ unstable orbits of period k

N.B. Bhattacharaya, Ghil & Vulis (1982, *J. Atmos. Sci.*) showed a spectrum $S \sim f^{-2}$ for a nonlinear PDE with delay (doubly infinite-dimensional)

Power Law for Spectrum (cont'd)

• Hypothesis: "Poles" correspond to the least unstable periodic orbits

"unstable limit cycles"

"Poincaré section"



• Major clue to the physics

that underlies the dynamics

N.B. Limit cycle not necessarily elliptic, i.e. not

 $(x,y) = (a_f sin(ft), b_f cos(ft))$



Spectrum



Singular Spectrum Analysis (SSA)

Spatial EOFs

SSA

$$\phi(x,t) = \sum a_k(t)e_k(x)$$

$$S - lag$$

$$X(x+s) = \sum a_k(t)e_k(s)$$

$$C_{\phi}(x, y) = E\phi(x, \omega)\phi(y, \omega)$$
$$= \frac{1}{T} \int_{o}^{T} \phi(x, t)\phi(y, t)d$$
$$C_{\phi}e_{k}(x) = \lambda_{k}e_{k}(x)$$

Colebrook (1978); Weare & Nasstrom (1982); Broomhead & King (1986: BK); Fraedrich (1986)

BK+VG: Analogy between Mañe-Takens embedding and the Wiener-Khinchin theorem

$$C_X(s) = EX(t+s, \omega)\phi(s, \omega)$$
$$= \frac{1}{T} \int_o^T X(t)X(t+s)dt$$

$$C_{X}e_{k}(s) = \lambda_{k}e_{k}(s)$$



Power Spectra & Reconstruction

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• A. Transform pair:

$$X(t+s) = \sum_{k=1}^{M} a_k(t)e_k(s), e_k(s) - EOF$$

The e_{k} 's are adaptive filters,

$$a_k(t) = \sum_{s=1}^{M} X(t+s)e_k(s), a_k(t) - PC$$

the a_k 's are filtered time series.

B. Power spectra

$$S_X(f) = \sum_{k=1}^M S_k(f); \quad S_k(f) = R_k(s); \quad R_k(s) \approx \frac{1}{T} \int_0^T a_k(t) a_k(t+s) dt$$

C. Partial reconstruction

$$X^{K}(t) = \frac{1}{M} \sum_{k \in K} \sum_{s=1}^{M} a_{k}(t-s)e_{k}(s);$$

in particular: $K = \{1, 2, ..., S\}$ or $K = \{k\}$ or $K = \{l, l+1; \lambda_l \approx \lambda_{l+1}\}$

Singular Spectrum Analysis (SSA)

Time series





T-EOFs



RCs

Selected parts of the series can be reconstructed, via *Reconstructed Components* (RCs)



- SSA is good at isolating oscillatory behavior via paired eigenelements.
- · SSA tends to lump signals that are longer-term than the window into
 - one or two trend components.

Selected References:

Vautard & Ghil (1989, *Physica* D); Ghil *et al.* (2002, *Rev. Geophys.*) 12/28

Singular Spectrum Analysis (SSA) and M-SSA (cont'd)



- Break in slope of SSA spectrum distinguishes "significant" from "noise" EOFs
- Formal Monte-Carlo test (Allen and Smith, 1994) identifies 4-yr and 2-yr ENSO oscillatory modes. A window size of M = 60 is enough to "resolve" these modes in a monthly SOI time series

SSA (prefilter) + (low-order) MEM



In good agreement with MTM peaks of **Ghil & Vautard (1991,** *Nature***)** for the Jones *et al.* (1986) temperatures & stack spectra of Vautard *et al.* (1992, *Physica D*) for the IPCC "consensus" record (both global), to wit 26.3, 14.5, 9.6, 7.5 and 5.2 years.

Peaks at 27 & 14 years also in Koch sea-ice index off Iceland (Stocker & Mysak, 1992), etc. Plaut, Ghil & Vautard (1995, Science)



- •Ported to Sun, Dec, SGI, PC Linux, and Mac OS X
- •Graphics support for IDL and Grace
- •Precompiled binaries are available at <u>www.atmos.ucla.edu/ tcd/ssa</u>
- Includes Blackman-Tukey FFT, Maximum Entropy Method, Multi-Taper Method (MTM), SSA and M-SSA.
- •Spectral estimation, decomposition, reconstruction & prediction.
- •Significance tests of "oscillatory modes" vs. "noise."

Test Options Plot Options Reconstruction Log file Help				
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- Free!!!
- Data management with *named vectors & matrices.*
- Default values button.

kSpectra Toolkit for Mac OS X

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- **\$\$** ... but: *Project files*, *Automator WorkFlows*, *Spotlight* and more!
- <u>www.spectraworks.com</u>

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The Nile River Records Revisited: How good were Joseph's predictions?

Michael Ghil, ENS & UCLA Yizhak Feliks, IIBR & UCLA, Dmitri Kondrashov, UCLA

17/28

Why are there data missing?



 Byzantine-period mosaic from Zippori, the capital of Galilee (1st century B.C. to 4th century A.D.); photo by Yigal Feliks, with permission from the Israel Nature and Parks Protection Authority)

What to do about gaps?

 Most of the advanced *filling-in* methods are different flavors of *Optimal Interpolation* (OI: Reynolds & Smith, 1994; Kaplan 1998).

Drawbacks: they either (i) require error statistics to be specified *a priori*; or (ii) derive it **only** from the interval of dense data coverage.

EOF Reconstruction (Beckers & Rixen, 2003): (i) iteratively compute **spatial-covariance** matrix using **all the data**; (ii) determine via cross-validation "**signal**" EOFs and use them to fill in the missing data; accuracy is similar to or better than **OI** (Alvera-Azcarate *et al.* 2004).

Drawbacks: uses **only** spatial correlations => cannot be applied to very **gappy** data.

We propose *filling in* gaps by applying iterative SSA (or M-SSA):

Utilize both spatial and temporal correlations of data => can be used for highly **gappy** data sets; simple and easy to implement!

Historical records are full of "gaps"....



Annual maxima and minima of the water level at the nilometer on Rodah Island, Cairo.

SSA (M-SSA) Gap Filling

Main idea: utilize both spatial and temporal correlations to iteratively compute selfconsistent lag-covariance matrix; M-SSA with M = 1 is the same as the EOF reconstruction method of Beckers & Rixen (2003)

Goal: keep "signal" and truncate "noise" — usually a few leading EOFs correspond to the dominant oscillatory modes, while the rest is noise.

(1) for a given window width *M*: center the original data by computing the unbiased value of the mean and set the missing-data values to zero.

(2) start iteration with the first EOF, and replace the missing points with the reconstructed component (RC) of that EOF; repeat the SSA algorithm on the new time series, until convergence is achieved.

(3) repeat steps (1) and (2) with two leading EOFs, and so on.

(4) apply cross-validation to optimize the value of *M* and the number of dominant SSA (M-SSA) modes *K* to fill the gaps: a portion of available data (selected at random) is flagged as missing and the RMS error in the reconstruction is computed.

Synthetic I: Gaps in Oscillatory Signal



Very good gap filling for smooth modulation; OK for sudden modulation.

Synthetic II: Gaps in Oscillatory Signal + Noise



Nile River Records



- High level
- Low level

MC-SSA of Filled-in Records



SSA results for the extended Nile River records;

arrows mark highly significant peaks (at 95%), in both SSA and MTM.

25/28

Periods	Low	High	High-Low
40–100yr	64 (9.3%)	64 (6.9%)	64 (6.6%)
20–40yr		[32]	
10–20yr	12.2 (5.1%), 18.0 (6.7%)		12.2 (4.7%), 18.3 (5.0%)
5–10yr	6.2 (4.3%)	7.2 (4.4%)	7.3 (4.4%)
0–5yr	3.0 (2.9%), 2.2 (2.3%)	3.6 (3.6%), 2.9 (3.4%), 2.3 (3.1%)	2.9 (4.2%),

Table 1a: Significant oscillatory modes in short records (A.D. 622–1470)

Table 1b: Significant oscillatory modes in extended records (A.D. 622–1922)

Periods	Low	High	High-Low
40–100yr	64 (13%)	85 (8.6%)	64 (8.2%)
20–40yr		23.2 (4.3%)	
10–20yr	[12], 19.7 (5.9%)		12.2 (4.3%), 18.3 (4.2%)
5–10yr	[6.2]	7.3 (4.0%)	7.3 (4.1%)
0–5yr	3.0 (4%), 2.2 (3.3%)	4.2 (3.3%), 2.9 (3.3%), 2.2 (2.9%)	[4.2], 2.9 (3.6%), 2.2 (2.6%)

Significant Oscillatory Modes



SSA reconstruction of the 7.2-yr mode in the extended Nile River records:(a) high-water, and (b) difference.Normalized amplitude; reconstruction in the large gaps in red.

Instantaneous frequencies of the oscillatory pairs in the low-frequency range (40–100 yr). The plots are based on multi-scale SSA [Yiou *et al.*, 2000]; local SSA performed in each window of width W = 3M, with M = 85 yr.

How good were Joseph's predictions?



The Nile River Basin initiative will greatly modify the flow along the longest & bestdocumented river system in the world ...

Going with the Flow

Mediterranean Sea

Idina, Zita and Delta Barrages Cairo & El Saff E GYPT

Asyut Barrage Nag Hammadi Barrage Esna Barrage Aswan High Dam Lake Nasser Toshka Valley The Nile is already dotted with dams. New projects could help spread the river's wealth of water, but environmentalists fear ecological disaster

> Existing dam or barrage Dam project propose or under construction Cavity-fed power station ____Proposed canal project

SUDAN

White N

DEMOCRATIC

REPUBLIC

OF CONGO

Khartoum + bel Aulia Dam Khashm El Girba Dam Tekeze Dam

onglei Canal

UGANDA

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ala +

RWANDA

Lake

Victoria

Dek Island Dek Island Tis Abay

Blue Nile * Addis Ababa ETHIOPIA

ETHIOPIA POPULATION: 77 MILLION GDP: \$8 BILLION

Tis Abay, a new 73-MW hydro plant just below the Blue Nile Falls, took Ethiopia's paltry national power capacity to 770 MW three years ago; a \$224 million, 185-m-tall dam at fekeze on a tributary of the Atbara River being built by the Chinese firm responsible for much of China's Three Gorges Dam will soon add 300 MW more. But it's what comes next that could change Ethiopia and the Nile forever: the Blue Nile alone has the potential to generate some 30,000 MW of power for the nation, and fficials have identified more than 100 sites for large-scale hydropower development schemes along the Nile and the country's other rivers. Development will help power the country, but it will also cut the flow of water that reaches Sudan and Egypt, block sediment transfer, and require the relocation of thousands of people.

UGANDA POPULATION: 29 MILLION GDP: \$6.8 BILLION Built by the British in the 1950s and extended in

EGYPT POPULATION: 74 MILLION GDP: \$75.1 BILLION

Unlike other Nile states, Egypt has almost fully

Opened in 1971, the 2,100-MW Aswan High Dam

irrigation scheme to water some 220,000 hectares of land in the Toshka Irrigation Scheme is

scheduled for completion by 2017. Further north, a

series of barrages, most of them originally built by

the British but many since updated, help provide

the Nile to the northern Sinai to make it habitable.

Many of the dams along the White and Blue Niles and the Atbara tributary in Sudan, built between the 1950s and 70s, are now silting up. The 13-MW Khashmi El Grhab Dam has lost almost half of its capacity to sittation, as has the 15-MW Sennar Dam. Like the Sennar, the 280-MW Roseires Dam, currently Sudan's higgest, was built for

Imgation but converted to hydropower production. These dams will soon be dwarfed by the \$1.8 billion Merowe Dam, which will produce 1,250 MW but will also put acres

of agricultural land underwater, displace some 50,000 people, flood a trove of ancient Nublan artifacts and,

environmentalists fear, change the local ecosystem forever A 300-MW dam at Kajbar is also being built.

Soon after it crosses the border into Sudan, the White Nile dis-

appears into the **Sudd**, a 130,000-sq-km swamp—the largest in the world. More than half the White Nile's water is lost

thousands of acres of farmland, was begun in 1978 but stalled

with the outbreak of civil war In Sudan in 1983. A peace deal signed in late 2004 could eventually restart the ambitious.

through evaporation or by being absorbed into thick aquatic vegetation and marshy soil. The **Jonglei Canal**, a joint Sudan-Egypt project to bypass the Sudd and use the water to irrigate

much-needed power to Egypt. When it is completed, the Salam Canal will divert water from

SUDAN POPULATION: 36 MILLION GDP: \$19.6 BILLION

NORTH

SOUTH

schomo

is currently the biggest on the river. An ambitious

tapped the hydropower potential of the Nile.

2000, Ower Falls Dam has a 380-WM capacity, but generates much less due to hydraulic bottlenecks that occur when insufficient water gets through to turn all the turbines. Owen Falls will be joined in the next decade by a dam at **Bujagali Falls**, a few kilometers down river. Costing around \$300 million, Bujagali will provide 200 MW of power, but will also force the relocation of villagers and flood the Bujagali Falls, a popular tourist site. Ugandan officials also have plans for a 180-MW dam at **Karuma**, as well as other sites along the Nile.

400 miles 400 kilometers GDP SOURCE: WORLD BANK