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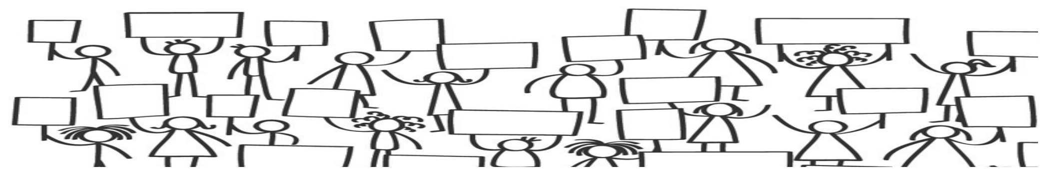
Program

18/1	Elementary statistics – 1
25/1	Elementary statistics - 2
8/2	Exercises – Computer room
15/2	Fourier Analysis -1
22/2	Fourier Analysis -2, stochastic processes
1/3	Exercises – Computer room
8/3	Exercises – Computer room
15/3	
22/3	Principal component analysis -1
29/3	Principal component analysis -2
5/4	Exercises – Computer room
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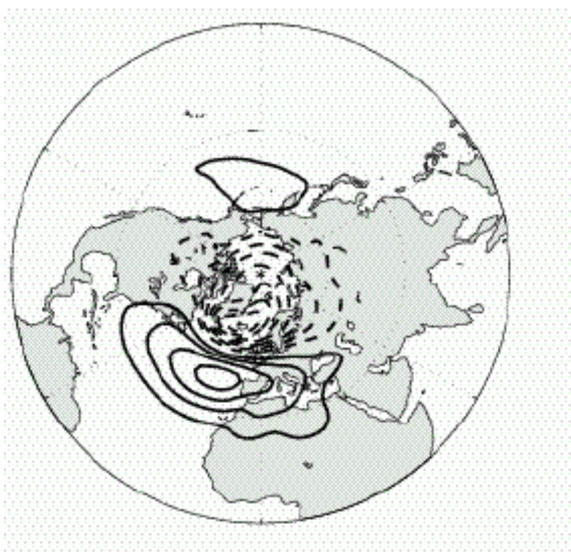
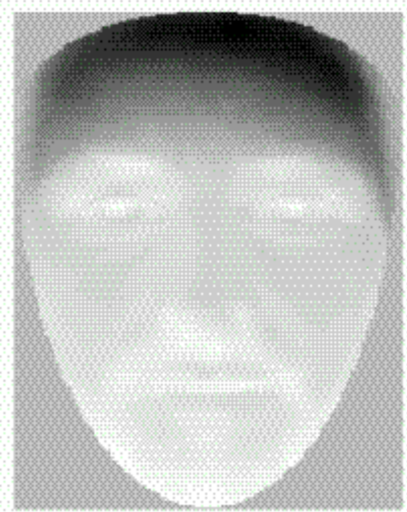
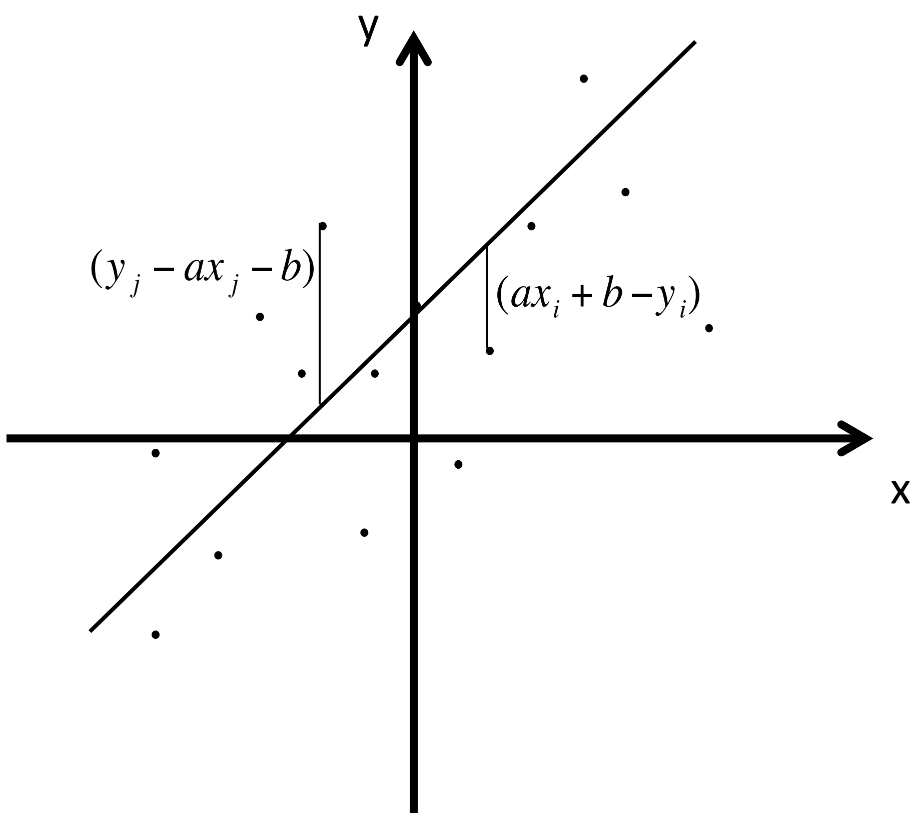
10/5	Principal component analysis: Complements
17/5	catch-up, we will see
7/6	Exam

.Lesson 4.
Principal Component Analysis





Reminders: regression and correlation



We want to minimize:

$$\sum_{i=1}^{N}\left(y_i - ax_i - b\right)^2$$

We take the derivative with respect to *a* and *b* and we obtain the two conditions:

$$a)\quad \sum x_i\left(y_i - ax_i - b\right) = 0$$

$$b)\quad \sum\left(y_i - ax_i - b\right) = 0$$

Condition b) gives:

$$b)\quad \sum_{i=1}^{N} y_i - a\sum_{i=1}^{N} x_i - Nb = 0 \Rightarrow \boxed{b = \bar{y} - a\bar{x}}$$

Substituting b) into a) gives:

$$a)\quad \sum_{i=1}^{N}\left(y_i x_i - ax_i^2 - \bar{y}x_i - a\bar{x}x_i\right) = \sum_{i=1}^{N}\left(y_i' x_i' - ax_i'^2\right)$$

Where we have introduced the definitions :

$$x_i = \bar{x} + x', y_i = \bar{y} + y'.$$

Hence

$$a = \frac{\sum_{i=1}^{N} x_i' y_i'}{\sum_{i=1}^{N} x_i'^2}$$

$$b = \bar{y} - a\bar{x}$$

Regression

How good is the regression?

The regression is not perfect: $\hat{y}_i = ax_i + b \neq y_i$

Introducing the error $y_i^* = y_i - \hat{y}_i$ We can write $y_i = ax_i + b + y_i^*$

And the variance of y becomes:

$$\overline{y'^2} = \overline{a^2 x'^2} + \overline{y^{*2}} \quad \Rightarrow \quad \frac{\overline{a^2 x'^2} + \overline{y^{*2}}}{\overline{y'^2}} = 1$$

Explained variance + unexplained variance = 1

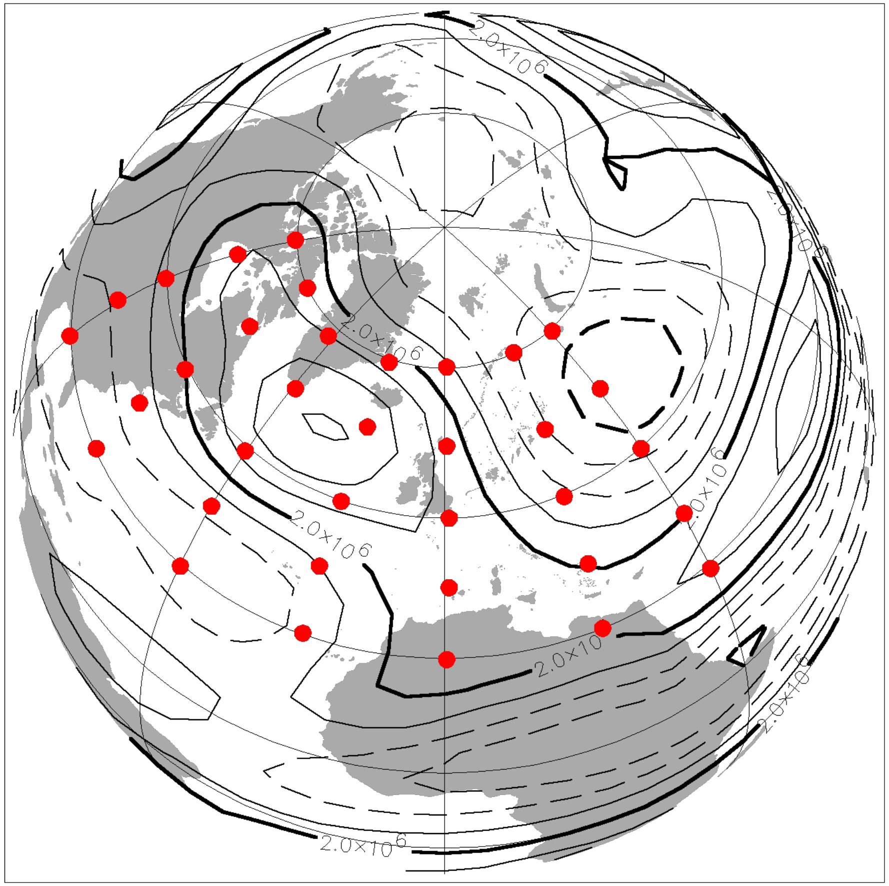
Substituting the value of *a* found above we find:

$$\frac{\overline{a^2 x_i'^2}}{\overline{y_i'^2}} = \frac{\left(\overline{x'y'}\right)^2}{\overline{x'^2}\,\overline{y'^2}} = r^2, \qquad r = \frac{\overline{x'y'}}{\sigma_x \sigma_y}$$

Is the correlation coefficient

$$r^2 = \frac{\text{Explained Variance}}{\text{Total Variance}}; \quad 1 - r^2 = \frac{\text{Unexplained Variance}}{\text{Total Variance}}$$

A geophysical map is a vector belonging to $\Re^N$



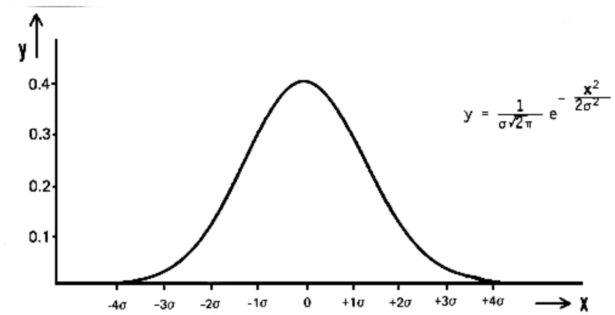
How big is N??

Atmosphere: 10^{45}

Weather / Climate models: 10^{9-10}

Analysing $\mathbf{x}(t)$

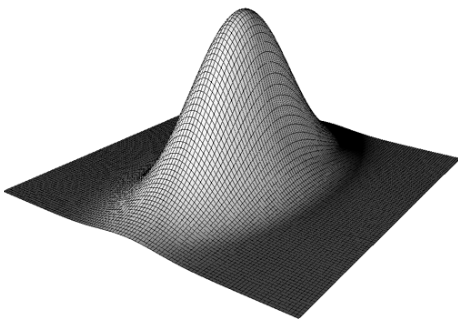
1D



You are now familiar with scalar time series statistics. Mean, variance, correlation, spectra, etc.

What happens with vector time series?

2D

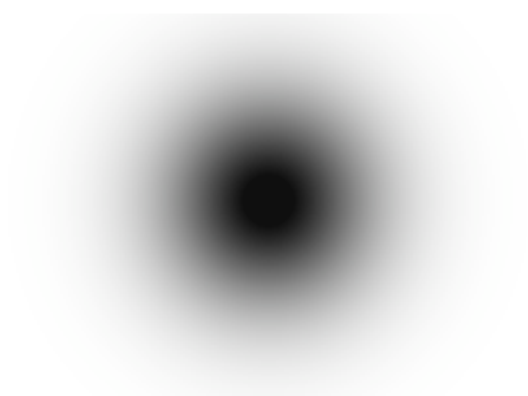


The mean is easy. Let's suppose $\overline{\mathbf{x}} = 0$

But what takes the place of variance?

The covariance matrix: $\overline{\mathbf{x}\mathbf{x}^T}$

3D

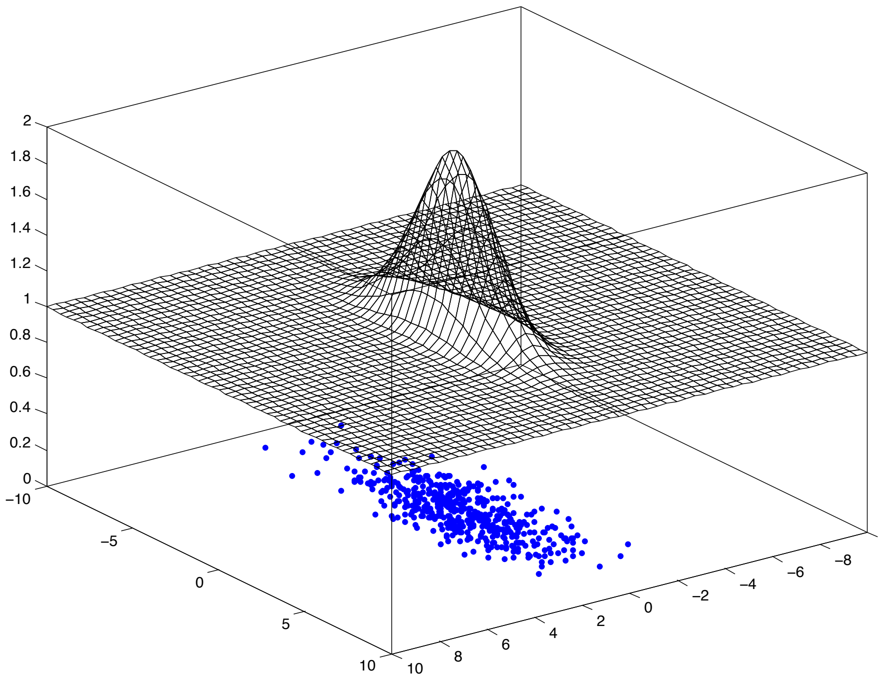


$$C = \begin{pmatrix} \overline{x_1 x_1} & \overline{x_1 x_2} & \ldots\ldots & \ldots\ldots & \overline{x_1 x_N} \\ \overline{x_2 x_1} & \overline{x_2 x_2} & \ldots\ldots & \ldots\ldots & \overline{x_2 x_N} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \overline{x_N x_1} & \overline{x_N x_2} & \ldots\ldots & \ldots\ldots & \overline{x_N x_N} \end{pmatrix}$$

C gives the variance of the sample in any given direction in phase space. So if $\mathbf{e}$ is a unitary vector,

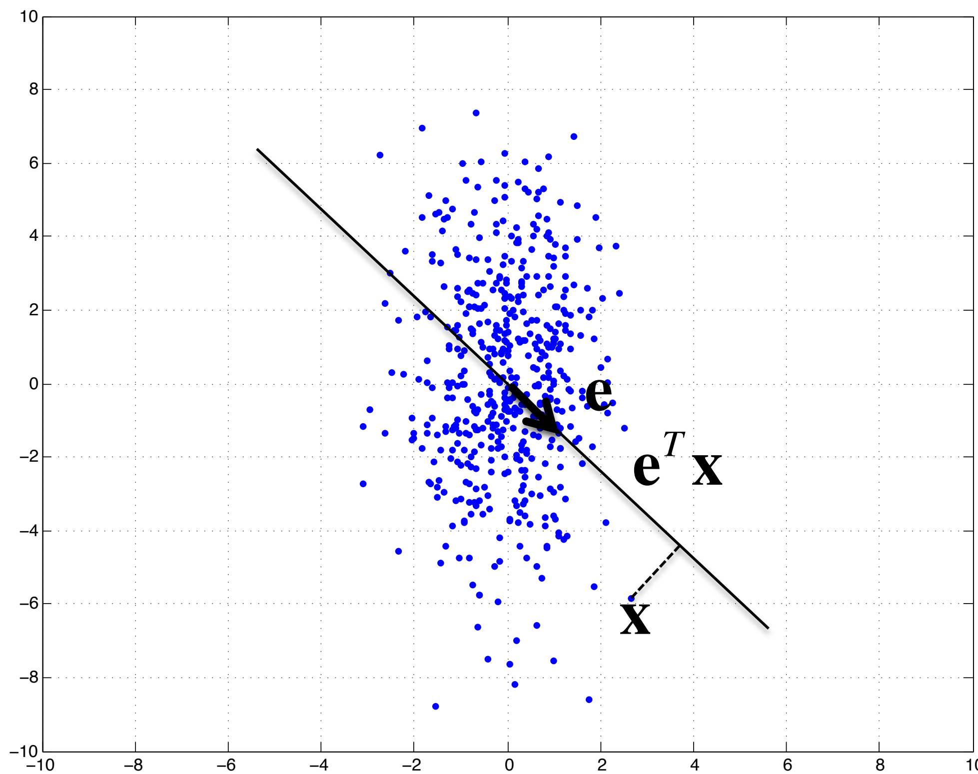
$$\mathbf{e}^T C \mathbf{e}$$

is the variance in the direction $\mathbf{e}$.



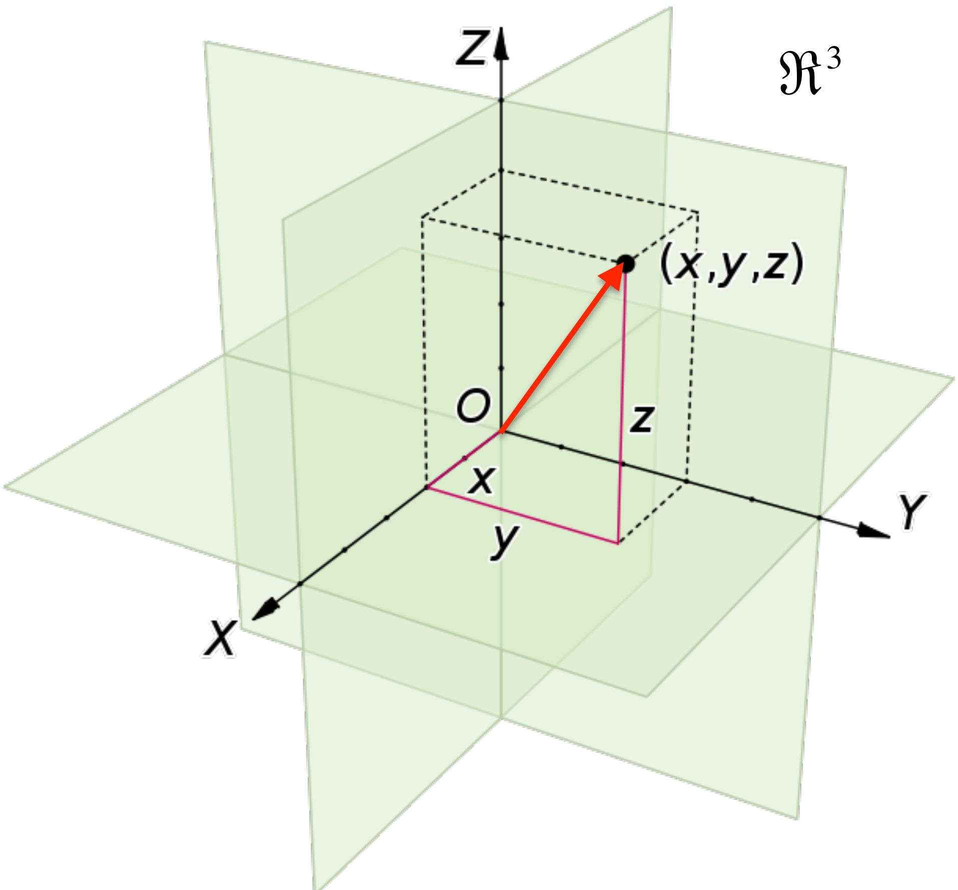
$$\sigma^2(\mathbf{e}) = \mathbf{e}^T C \mathbf{e}$$

$$\overline{(\mathbf{e}^T \mathbf{x})^2} = \mathbf{e}^T \, \overline{\mathbf{x}\mathbf{x}^T} \, \mathbf{e} = \mathbf{e}^T C \mathbf{e}$$



Reminder: Euclidean spaces

Norms, scalar product, distance, bases….



You are familiar with the cartesian coordinate system in 3d.
A vector $\mathbf{x} \in \Re^3$ can be represented by its components:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \sum_{i=1}^{3} x_i \mathbf{e}_i$$

where:

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

is the euclidean basis, or canonical basis (or standard, or natural)

You also know what is the length of a vector:

$$|\mathbf{x}| = \sqrt{x_1^{\,2} + x_2^{\,2} + x_3^{\,2}}$$

Generalising to N dimensions

$$\mathbf{x} = \sum_{i=1}^{N} x_i \mathbf{e}_i$$

where:

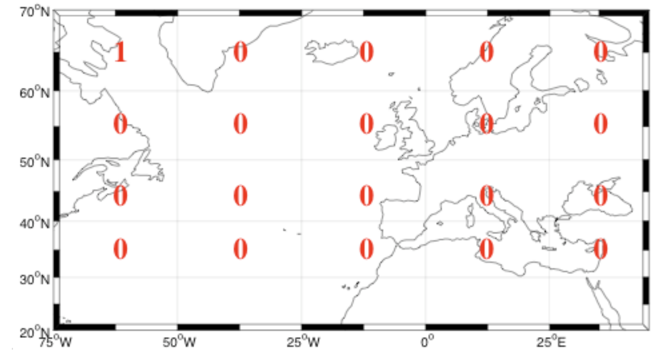
$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \mathbf{e}_N = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

EXAMPLE

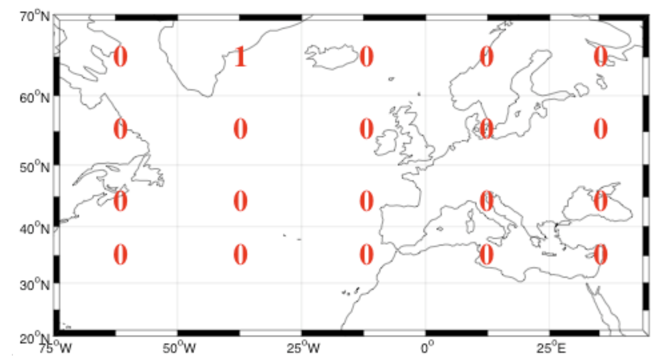
A grid can be seen as a linear basis of the vector space $\Re^N$

$$\mathbf{x}(t) = \sum_{i=1}^{N} c_i(t)\ \mathbf{e}_i$$

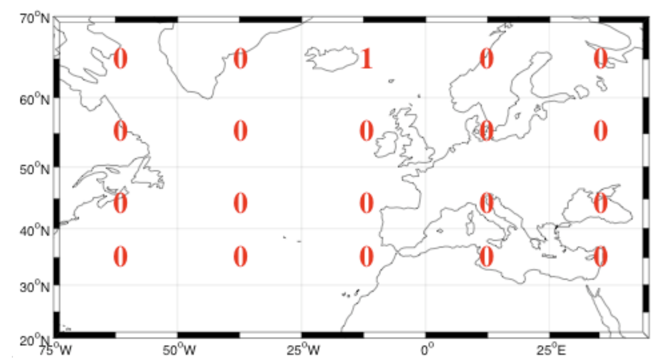
$\mathbf{e}_1$



$\mathbf{e}_2$



$\mathbf{e}_3$



$$\mathbf{x} \in \Re^N,\ N = 20$$

$$\mathbf{x}(t) = \sum_{i=1}^{N} c_i(t)\mathbf{e}_i$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

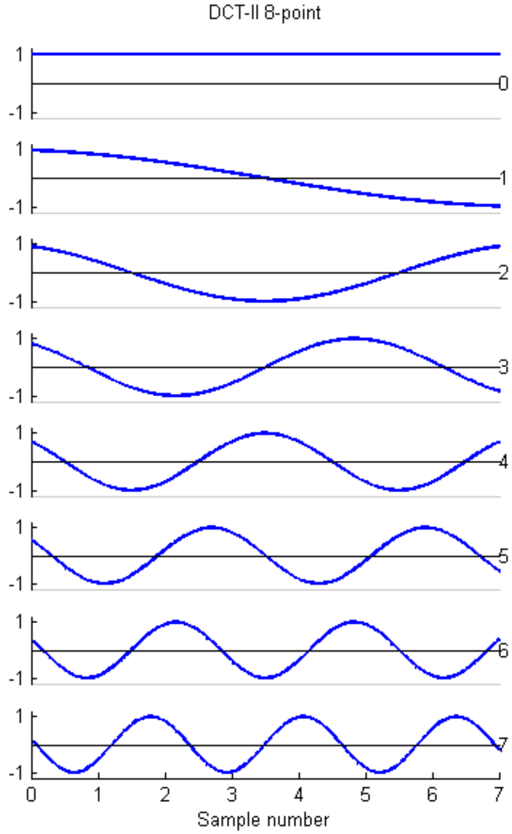
Etc…

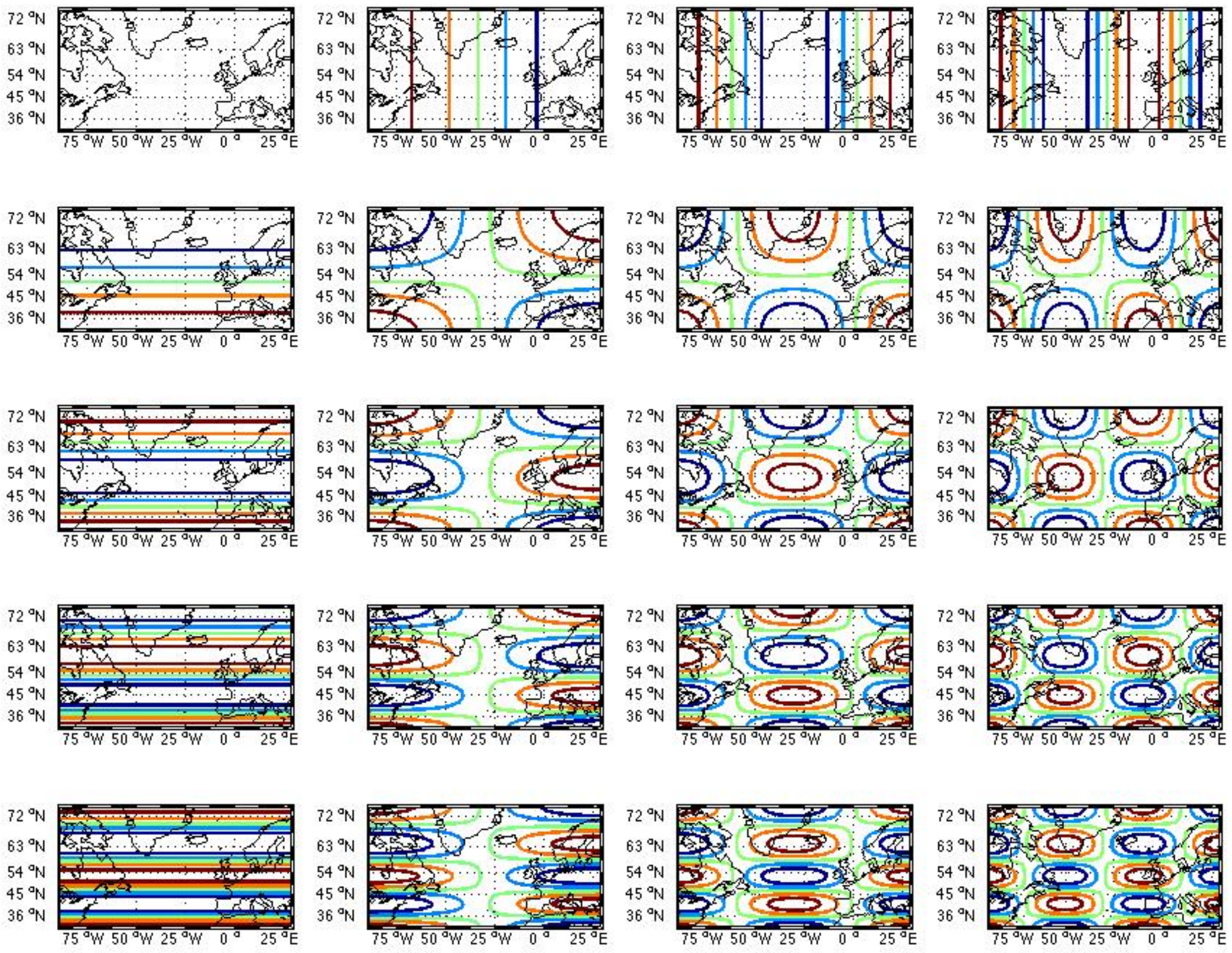
Do you know any other basis?

Yes you do: The Fourier basis.

2D

1D





In order to introduce a geometry, we need to have a concept of angles and distances.

This is done by introducing a scalar product. The standard scalar product is

$$\mathbf{x} \bullet \mathbf{y} = \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{N} x_i y_i = 2xy\cos\vartheta$$

or, using a matrix notation:

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y} = \begin{pmatrix} x_1 & x_2 & \cdots & x_{N-1} & x_N \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N-1} \\ y_N \end{pmatrix} = \sum_{n=1}^{N} x_n y_n$$

A scalar product induces a norm in a standard way, which generalizes the idea of length:

$$\|\mathbf{x}\|^2 = \langle \mathbf{x}, \mathbf{x} \rangle = \sum_{i=1}^{N} {x_i}^2$$

This in turn induces a definition of distance:

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{\sum_{i=1}^{N} (x_i - y_i)^2}$$

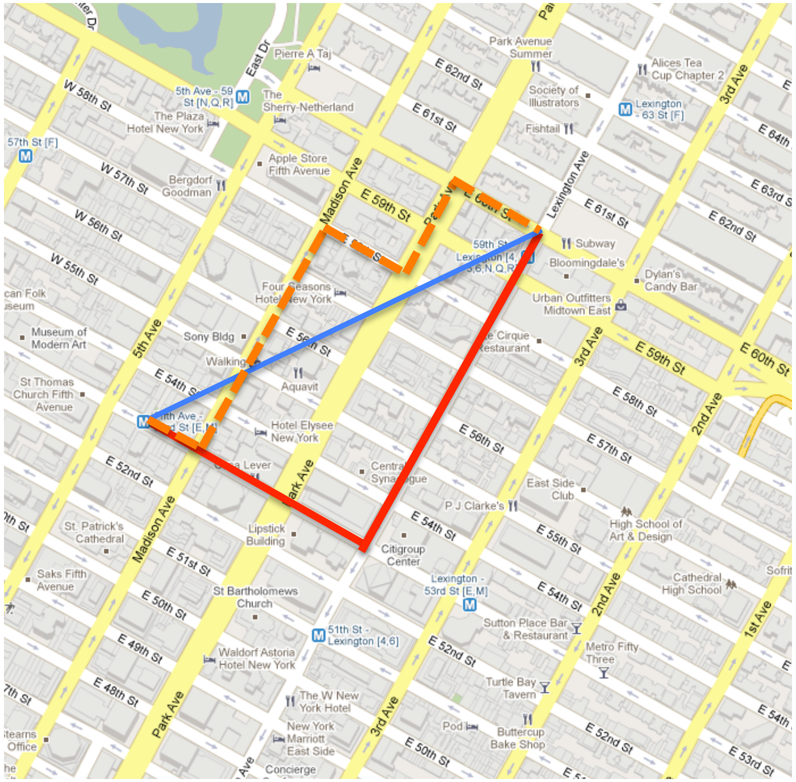
Scalar products, norms and distances are in no way unique. There are several possible choices, given that they satisfy the definition (ask your friends at the math department, or see a geometry handbook for that….)

Exemples of other norms:

The “Manhattan” norm, which induces the “taxi” distance:

$$\|\mathbf{x}\| = \sum_{i=1}^{N} |x_i|$$

$$d(\mathbf{x},\mathbf{y}) = \sqrt{\sum_{i=1}^{N} |x_i - y_i|}$$

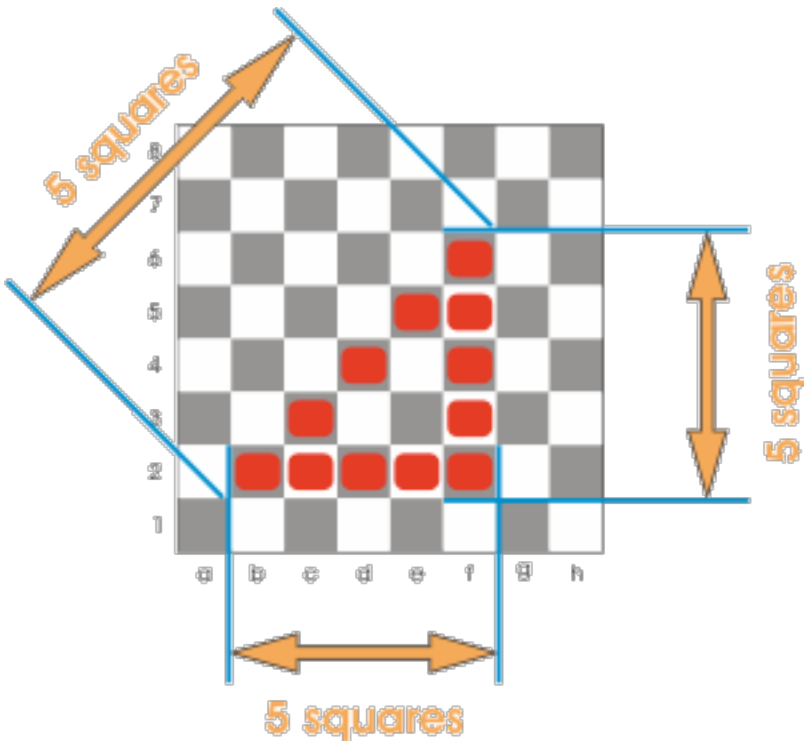


The general case: the p norm

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{N} |x_i|^p \right)^{\frac{1}{p}}$$

The $p = \infty$ norm, or the *sup* or Chebyshev norm, which induces the chessboard distance:

$$\|\mathbf{x}\|_\infty = \sup |x_i|$$



A “statistically interesting” norm (Mahalanobis norm):

$$\|\mathbf{x}\|^2 = \left\langle \mathbf{x}, C^{-1}\mathbf{x} \right\rangle$$

Prashanta Mahalanobis
(1893 –1972)

If *C* is diagonal, the distance becomes:

$$d(\mathbf{x},\mathbf{y}) = \sqrt{\sum_{i=1}^{N} \frac{(x_i - y_i)^2}{\sigma_i^{\,2}}}$$

Which means that each component is normalized by its own variance. It is useful in case of vectors of heterogeneous observations.

Given two norms, there is always a matrix, called Metric Matrix, that transforms one norm into the other.

$$\|\mathbf{x}\|_a^{\;2} = \langle \mathbf{x}, \mathbf{x} \rangle_a$$

$$\|\mathbf{x}\|_b^{\;2} = \langle \mathbf{x}, \mathbf{x} \rangle_b$$

$$\langle \mathbf{x}, \mathbf{x} \rangle_b = \langle \mathbf{x}, M_b \mathbf{x} \rangle_a$$

Spectral theorem

Ask your mathematician friends for all the nice hypotheses and symbols. Here, just a special simple result is given.

All the matrices for which this is true:

$$\langle \mathbf{y}, L\mathbf{x} \rangle = \langle L\mathbf{y}, \mathbf{x} \rangle$$

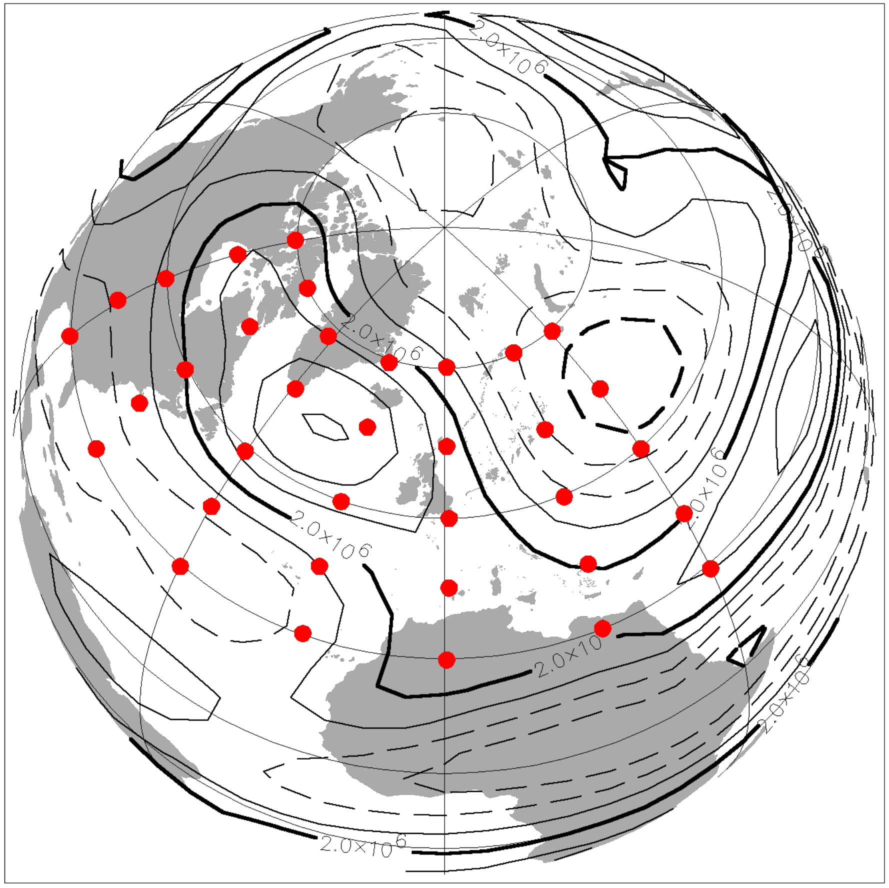
have eigenvectors that define an orthonormal basis for the vector space. In other words, all symmetric (self-adjoint) matrices ($L = L^T$) have an ortonormal complete set of eigenvectors.

In yet other words, for any symmetric matrix L *(any self-adjoint operator)*, there exist two orthogonal matrices and a diagonal matrix for which:

$$D = M^{-1} L M = M^T L M$$

We' ll encounter this later on…

A geophysical map is a vector belonging to $\Re^N$

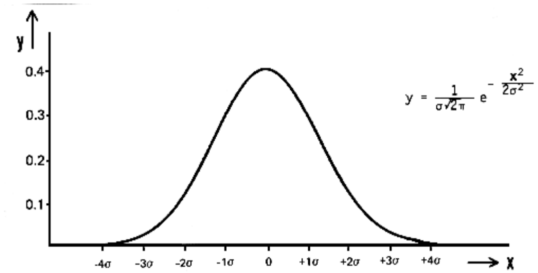


How big is N??

Atmosphere: 10^{45}

Weather / Climate models: $10^{9\text{-}10}$

Analysing $\mathbf{x}(t)$



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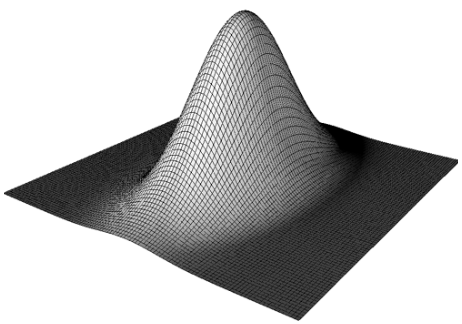
What happens with vector time series?



The mean is easy. Let's suppose $\overline{\mathbf{x}} = 0$

But what takes the place of variance?

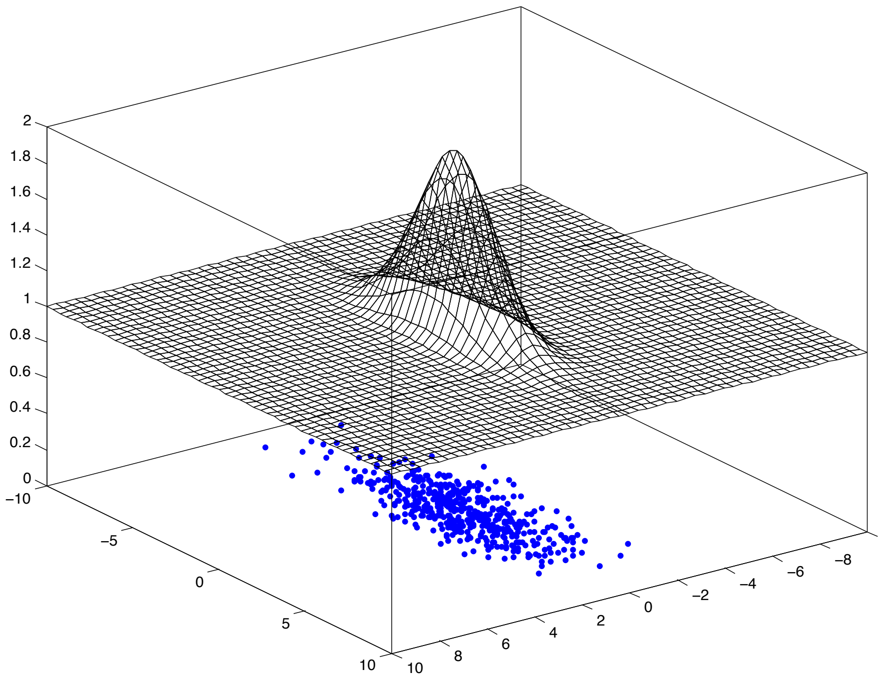
The covariance matrix: $\overline{\mathbf{x}\mathbf{x}^T}$

$$C = \begin{pmatrix} \overline{x_1x_1} & \overline{x_1x_2} & \ldots\ldots & \ldots\ldots & \overline{x_1x_N} \\ \overline{x_2x_1} & \overline{x_2x_2} & \ldots\ldots & \ldots\ldots & \overline{x_2x_N} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \overline{x_Nx_1} & \overline{x_Nx_2} & \ldots\ldots & \ldots\ldots & \overline{x_Nx_N} \end{pmatrix}$$

C gives the variance of the sample in any given direction in phase space. So if $\mathbf{e}$ is a unitary vector,

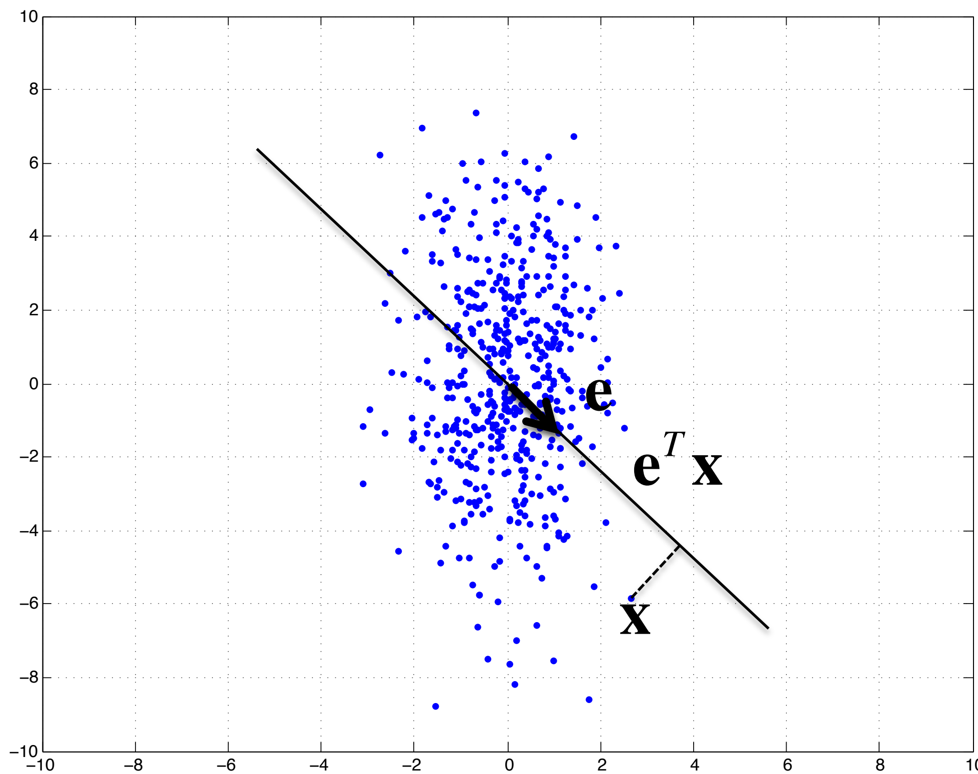
$$\mathbf{e}^T C \mathbf{e}$$

is the variance in the direction $\mathbf{e}$.



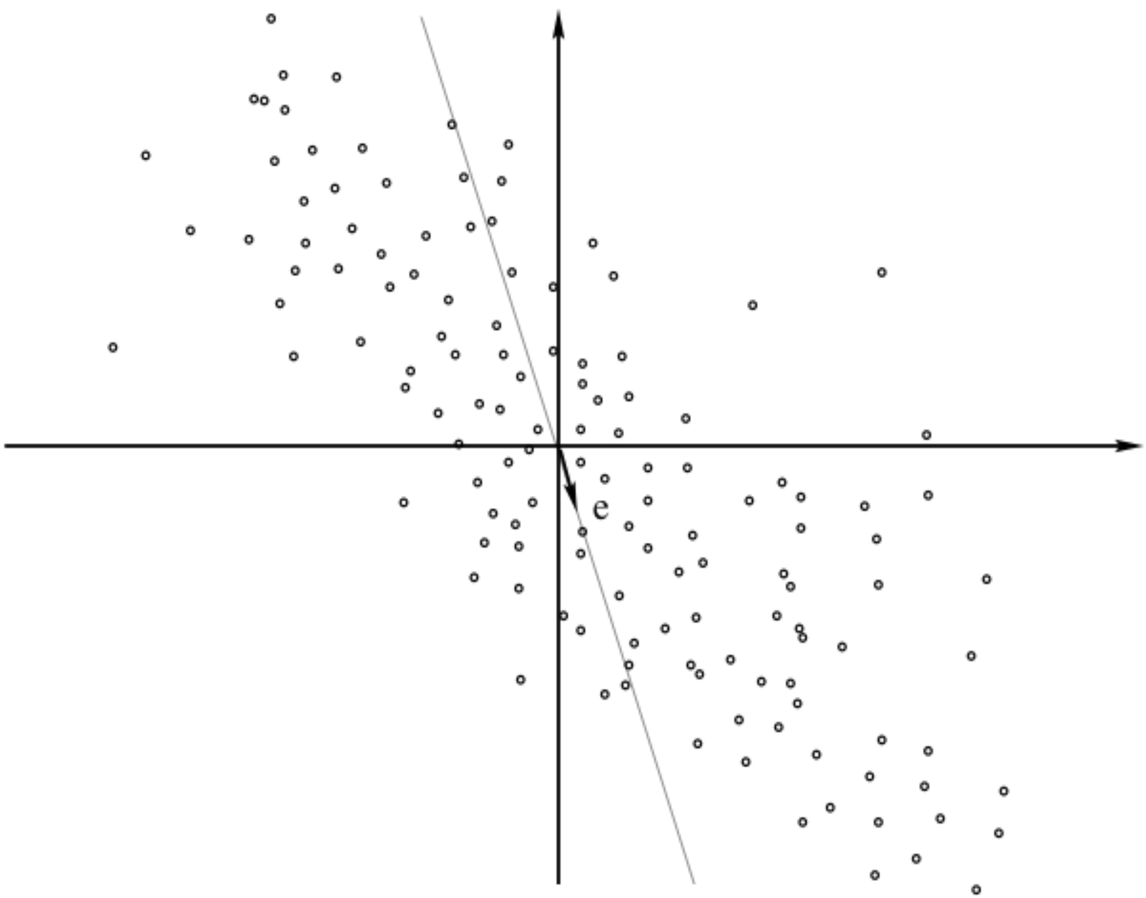
$$\sigma^2(\mathbf{e}) = \mathbf{e}^T C \mathbf{e}$$

$$\overline{(\mathbf{e}^T \mathbf{x})^2} = \mathbf{e}^T \, \overline{\mathbf{x}\mathbf{x}^T} \, \mathbf{e} = \mathbf{e}^T C \mathbf{e}$$



Principal Component Analysis

<u>Problem</u>: find the direction e that maximises the variance of a sample of vectors.



It is a constrained Maximization problem. We want to find the maximum:

$$Max(\mathbf{e}^T C\mathbf{e})$$

Submitted to the constraint:

$$\|\mathbf{e}\|^2 = \mathbf{e}^T\mathbf{e} = 1$$

The problem can be solved via the Lagrange multiplyer *λ.*
The maximum to be found is:

$$Max\left[\mathbf{e}^T C\mathbf{e} - \lambda(\mathbf{e}^T\mathbf{e} - 1)\right]$$

Differentiating with respect to $\mathbf{e}$

$$\frac{\partial}{\partial \mathbf{e}}\left[\mathbf{e}^T C\mathbf{e} - \lambda(\mathbf{e}^T\mathbf{e} - 1)\right] = 2C\mathbf{e} - 2\lambda\mathbf{e} = 0$$

Hence: $\boxed{C\mathbf{e} = \lambda\mathbf{e}}$

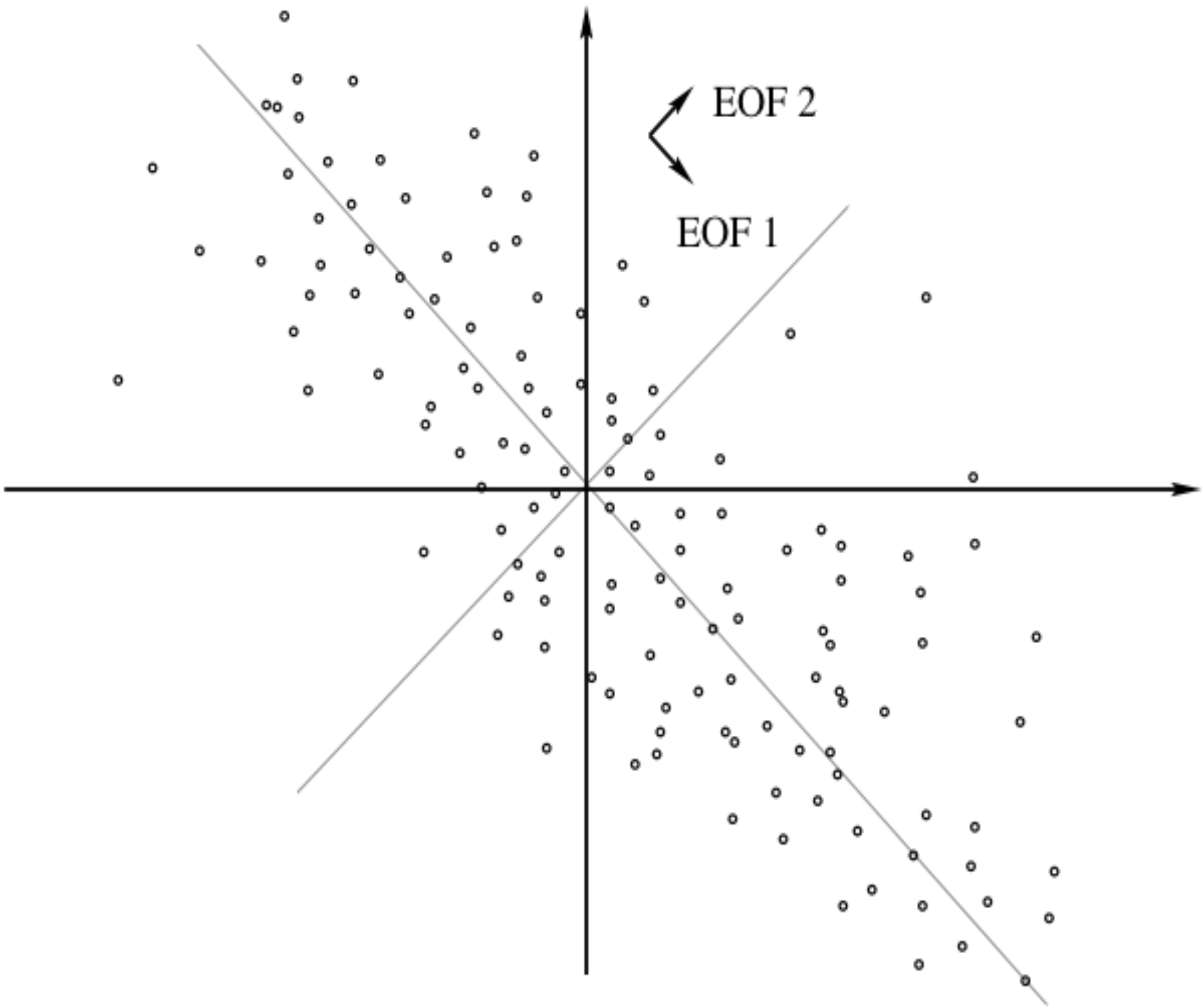
The maximization problem is simply the eigenvalue problem for C.

In geophysical applications, these eigenvectors are usually called EOFs (Empirical Orthogonal functions).

C is a symmetric matrix, consequently – by the spectral theorem - it has a complete orthonormal set of eigenvectors. The EOFs are an orthonormal basis for $\Re^N$:

$$\mathbf{x}(t) = \sum_{n=1}^{N} c_n(t)\mathbf{e}_n,$$

$$\text{where } c_n(t) = \langle \mathbf{x}(t), \mathbf{e}_n \rangle$$



The first EOF is the direction along wich the variance of the sample is maximum.
The second EOF is the direction along wich the variance is maximum, under constraint of orthogonality with the first, and so forth.

PROPERTIES

an important property is that for any given truncation $T<N$:

$$\mathbf{x}(t) = \sum_{n=1}^{T} c_n(t)\mathbf{e}_n + R$$

the EOFs are the linear basis that minimises the residual *R,* given the chosen norm.

This is very efficient for data compression.

What about the eigenvalues λ_n?

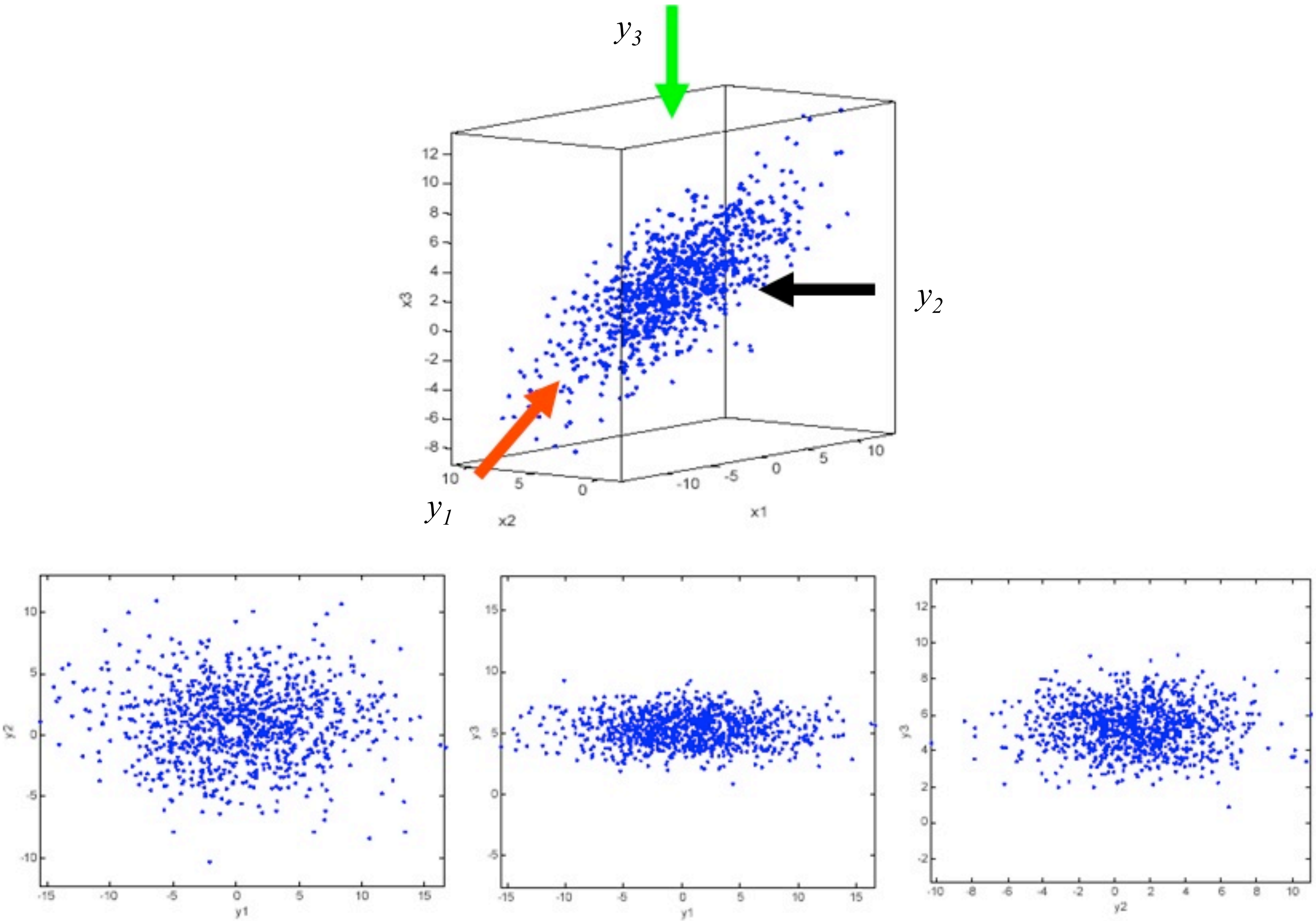
$$\mathbf{e}_n{}^T C \mathbf{e}_n = \mathbf{e}_n{}^T \lambda_n \mathbf{e}_n = \lambda_n$$

So the n-th eigenvalue is the variance explained in the direction of the n-th EOF.

Since the total variance of $\mathbf{x}(t)$ is :

$$Var(\mathbf{x}(t)) = \sum_{n=1}^{N} \lambda_n$$

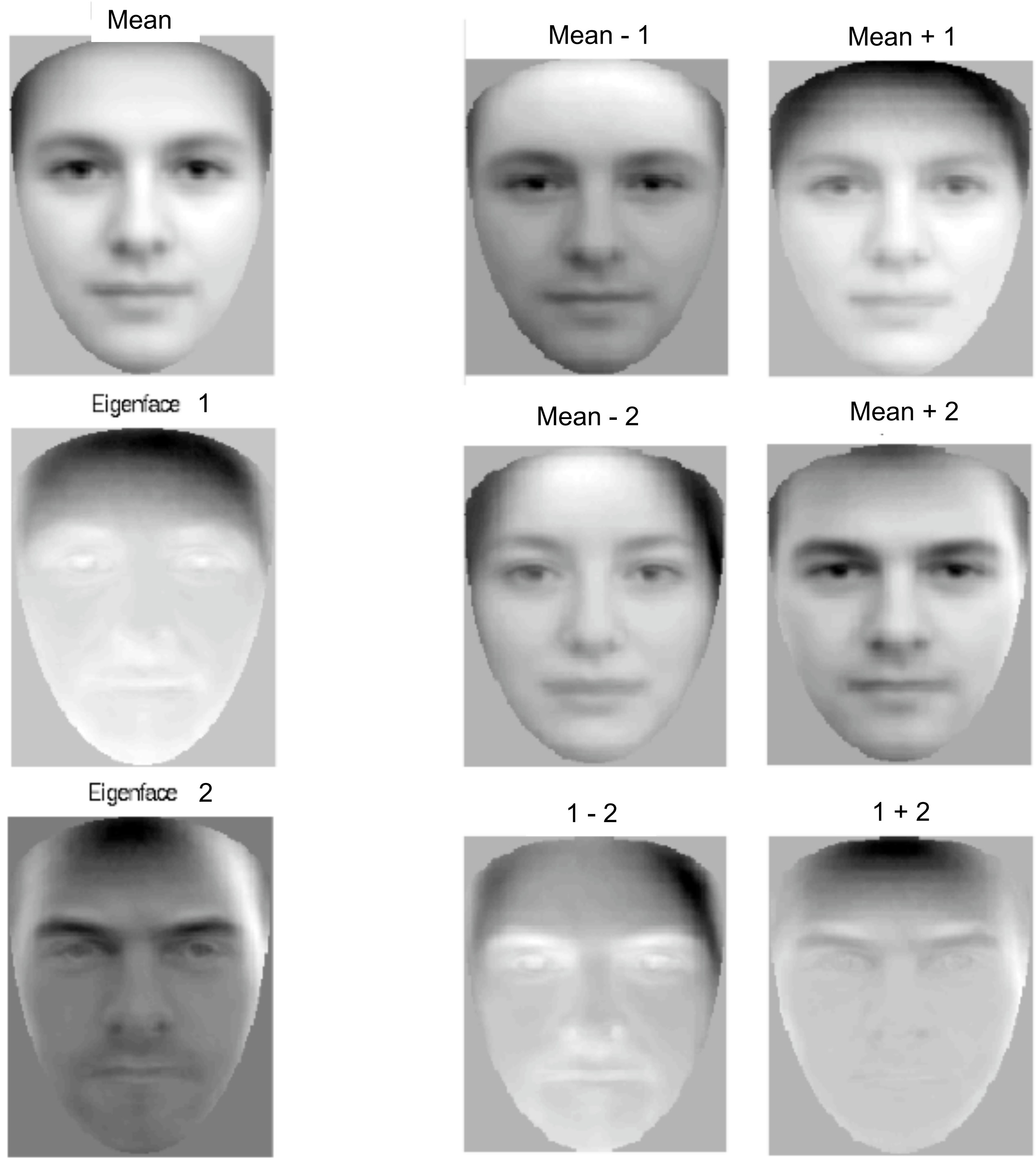
One can express the percent of variance explained by an EOF as $\dfrac{\lambda_i}{\sum_{n=1}^{N} \lambda_n}$



y3
y2
y1
x3
x2
x1
y2
y1
y3
y1
y3
y2

Example 1:
Principal component analysis of human faces

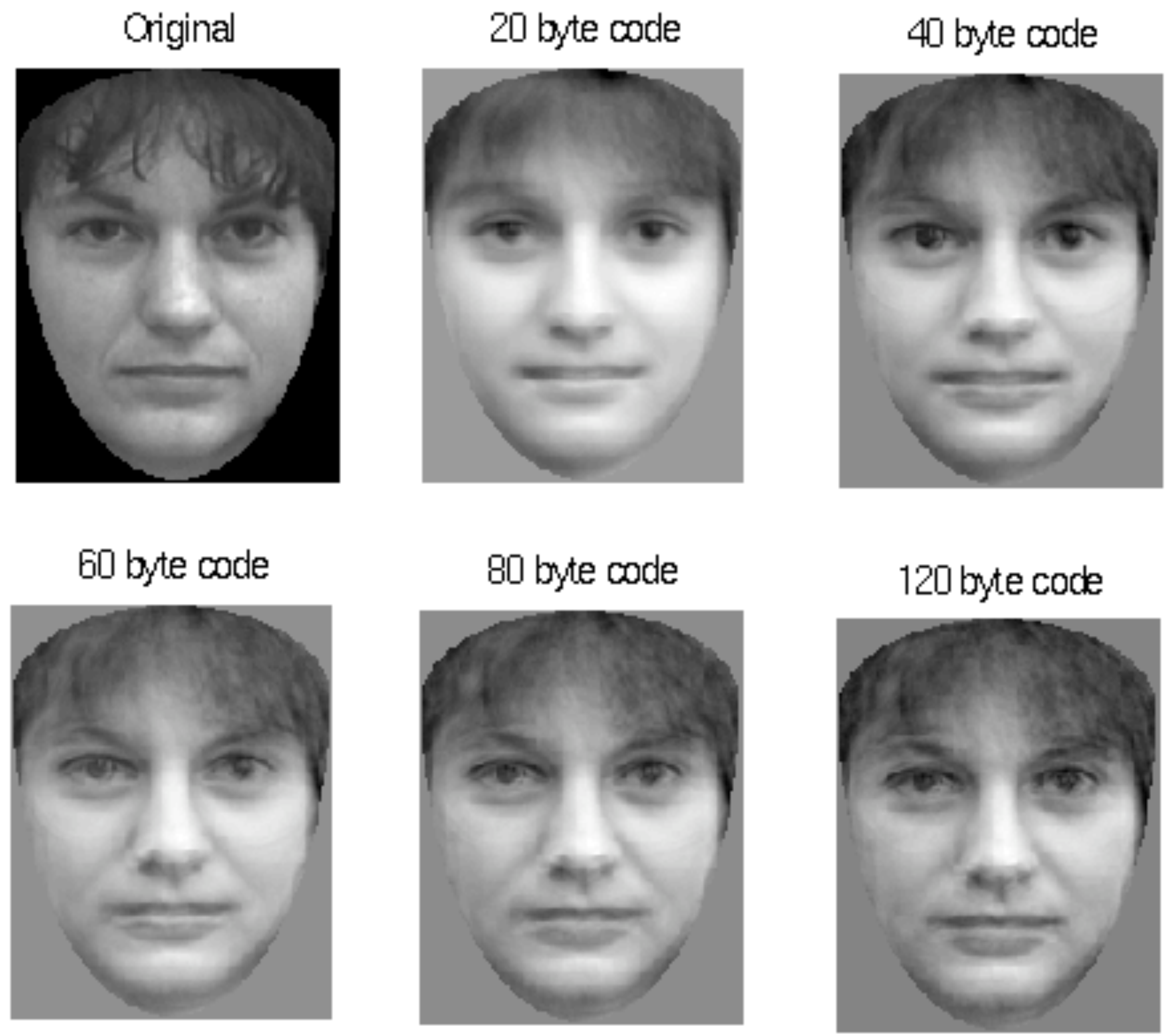
Digitalized photos of the faces of students of the university of Kent.
It allows to compute the « mean face » and the first EOFs of faces.



Credit: C.J. Solomon and J.P. Brooker, Univ. of Kent: /www.ukc.ac.uk/physical -sciences/aog/facereco

« Truncating a face »

Reconstruction of a vector by projection on the EOF basis. The effect of the truncation.



Google:

“faces principal component analysis”

- A lot of fun stuff-

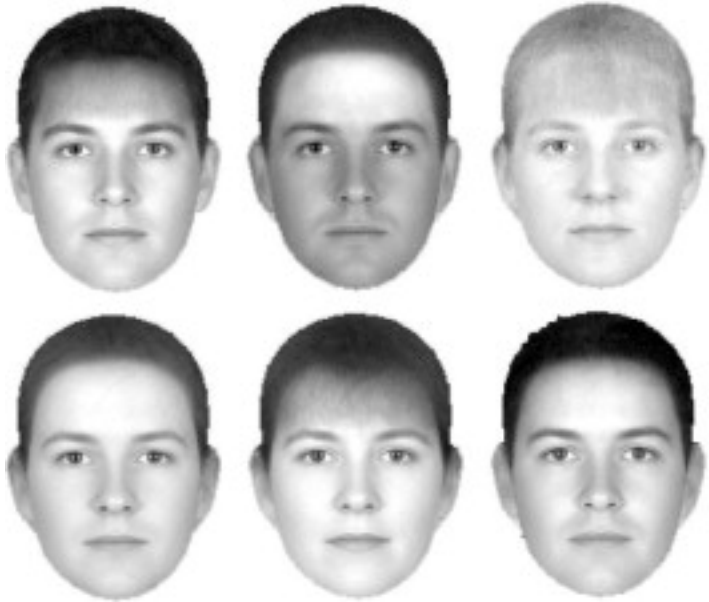


Figure I. Analysis of face components. From left to right, the first three image components resulting from a principal components analysis, illustrated by subtracting (top row) and adding (bottom row) the component to the average face (although note that the sign of the change is arbitrary). These early components are largely dominated by lighting and hair effects, but note that the latter strongly codes face gender.

Figure II. Analysis of face shape. From left to right, the first two and the ninth shape components, illustrated by subtracting (top row) and adding (bottom row) the component to the shape of the average shape-free face. The first codes head size, along with an element of face gender (women in this set have smaller heads, even after normalizing for pupil centres). The ninth is included because it clearly captures another aspect of sex differences.

Peter J.B. Hancock, Vicki Bruce, A.Mike Burton. Recognition of unfamiliar faces. Trends in Cognitive Sciences, Volume 4, Issue 9, 1 September 2000, Pages 330–337

The problem of maximizing the variance is equivalent to the problem of finding the direction e having the largest projection on the data sample. Or alternatively, of finding the straight line of smallest distance (given a definition of distance) from all the data sample.

In fact, we can write the mean square projection:

$$\overline{(\mathbf{e}^T M\mathbf{x})^2} = \mathbf{e}^T M \overline{\mathbf{x}\mathbf{x}^T} M\mathbf{e} = \mathbf{e}^T MCM\mathbf{e}$$

Which is equivalent to the variance definition of before for the canonic metric, i.e. for M equal to the identity.

We can reformulate for a general metric the EOF formula. The maximization problem becomes:

$$\frac{\partial}{\partial \mathbf{e}}\left[\mathbf{e}^T MCM\,\mathbf{e} - \lambda(\mathbf{e}^T M\,\mathbf{e} - 1)\right] = 2MCM\,\mathbf{e} - 2\lambda M\,\mathbf{e} = 0,$$

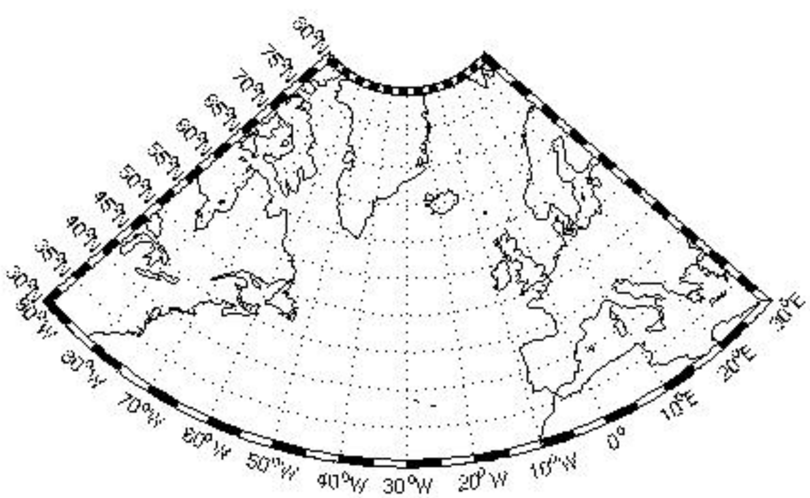
Hence: $$CM\mathbf{e} = \lambda\mathbf{e}.$$ Generalized eigenvalue problem

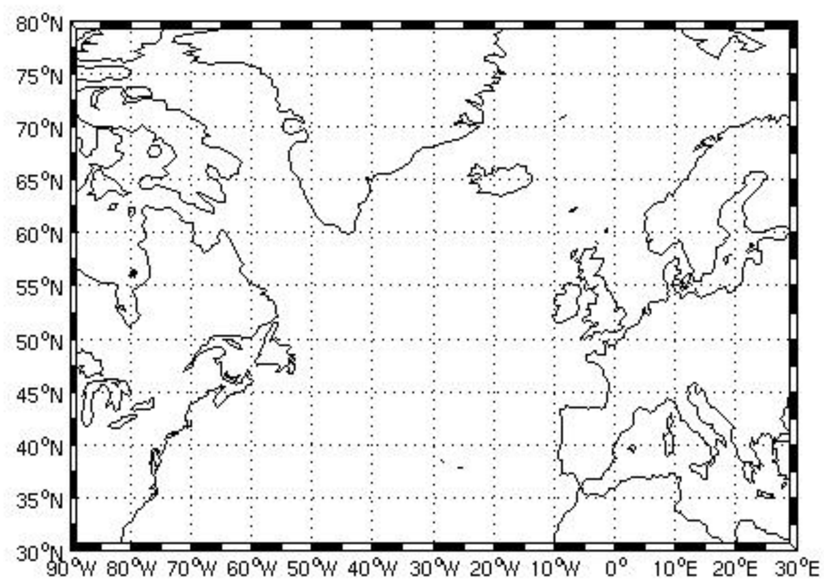
M can be the metric matrix of the canonical norm, or any other norm.

The EOFs do depend on the choice of a norm.

Area-weighting norm

A useful norm is the area-weighting norm, used when the data analysed are represented on a lat-lon regular grid.





In this case, the scalar product is the the area integral of the product of two fields. In the discrete approximation it becomes the following sum. It measures « how much two fields are similar ».

$$\langle x,y\rangle = \int_0^{2\pi}\int_{-\pi}^{\pi} xy\, R^2 \cos\theta \, d\theta d\lambda = R^2 \sum_{n=1}^{N} x_n y_n \cos\theta_n =$$

$$= x^T M y.$$

Where M is the metric defined as follows:

$$M = R^2 \begin{pmatrix} \cos\theta_1 & 0 & 0 & \cdots & 0 \\ 0 & \cos\theta_2 & & & \vdots \\ 0 & & \cos\theta_3 & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \cos\theta_N \end{pmatrix}$$

Just a trick

There is a trick to solve the eigenvalue problem in the case of this norm:

We can make the variable change $\mathbf{x}' = \mathbf{m}\mathbf{x}$, where $\boldsymbol{M} = \mathbf{m}\mathbf{m}$. This way, the eigenvector of $\mathbf{C}' = \overline{\mathbf{x}'\mathbf{x}'^T}$ are the eigenvector of $\mathbf{C}\boldsymbol{M}$, multiplied by $\boldsymbol{m}$.

Proof:

$$\mathbf{C}' = \overline{\mathbf{x}'\mathbf{x}'^T} = \overline{\mathbf{m}\mathbf{x}(\mathbf{m}\mathbf{x})^{\mathbf{T}}} = \mathbf{m}\overline{\mathbf{x}\mathbf{x}^{\mathbf{T}}}\mathbf{m}^{\mathbf{T}} = \mathbf{m}\mathbf{C}\mathbf{m}$$

Hence the eigenvalue problem to be solved is

$$\mathbf{m}\mathbf{C}\mathbf{m}\ \mathbf{e} = \lambda\mathbf{e}$$

$$\mathbf{m}\mathbf{C}\mathbf{m}\ \mathbf{m}\mathbf{m}^{-1}\mathbf{e} = \lambda\mathbf{e}$$

$$\mathbf{C}M\ \ \mathbf{m}^{-1}\mathbf{e} = \lambda\ \ \mathbf{m}^{-1}\mathbf{e}$$

Conclusion: first mutliply all your data by the square root of the cosine of latitude, then compute the EOFs. After, divide them by the square root of the cosine of latitude.

Statistical significance of EOFs.

It is complicated, but the standard error of the eigenvectors and of the eigenvalues can be computed. See:

North et al, 1982: “Sampling Errors in the Estimation of Empirical Orthogonal Functions“. Mon Wea Rev, 110, 699–706.

There is a « rule of thumb »:

$$\Delta\lambda_i \approx \sqrt{\frac{2}{N}}\lambda_i$$

$$\Delta\mathbf{e}_i \approx \frac{\Delta\lambda_i}{\lambda_i - \lambda_j}\mathbf{e}_j$$

Where λ_j is the eigenvalue closest in value to λ_i

Example 2

Principal component analysis of 500 mb geopotential height maps

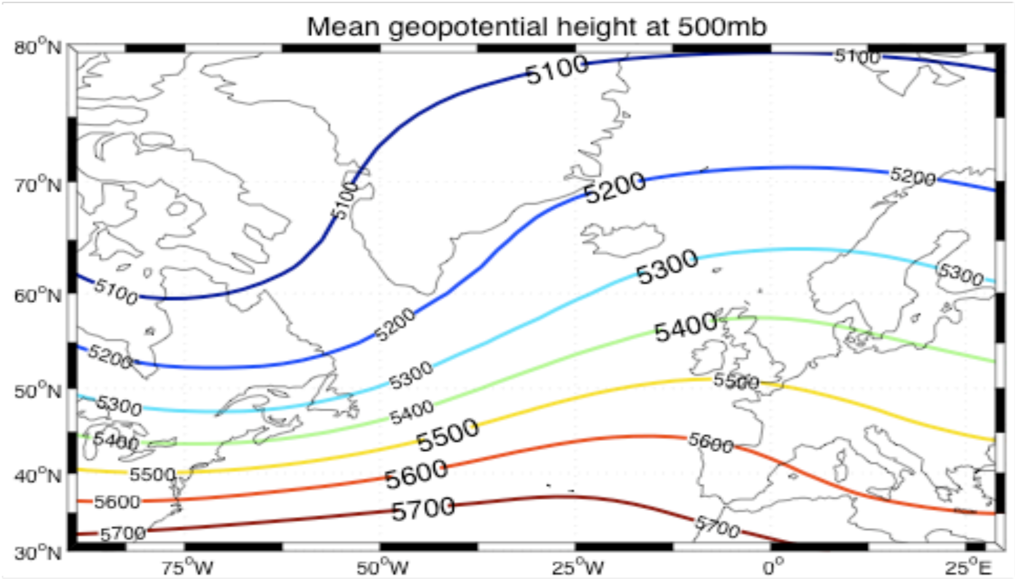
Geopotential height is defined by $$Z(p) = \frac{R}{g}\int_{p_0}^{p} T \, d\ln p$$

Intuitively, it can be seen at the height from the ground at which a pressure p is found – more or less.

Remember that by geostrophy it is:

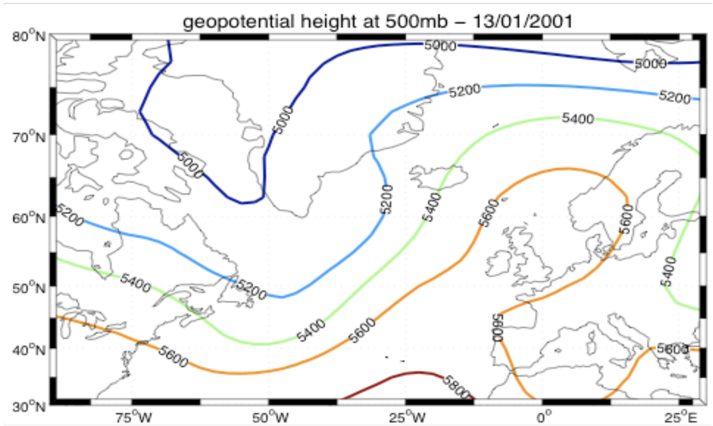
$$fu = g\frac{\partial Z}{\partial y}$$

$$fv = -g\frac{\partial Z}{\partial x}$$

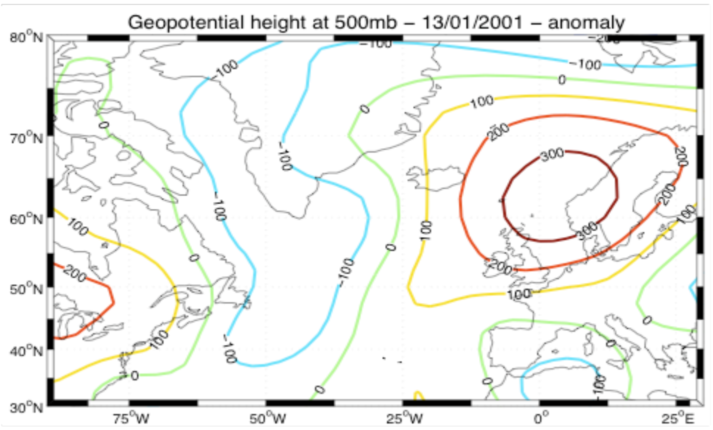


In this case the time series $\mathrm{x}(\mathrm{t})$ is a series of daily meteorological maps projected on a lat-lon grid of 25x49 points. Hence one can say that $\mathbf{x} \in \Re^N$ and N=1225.

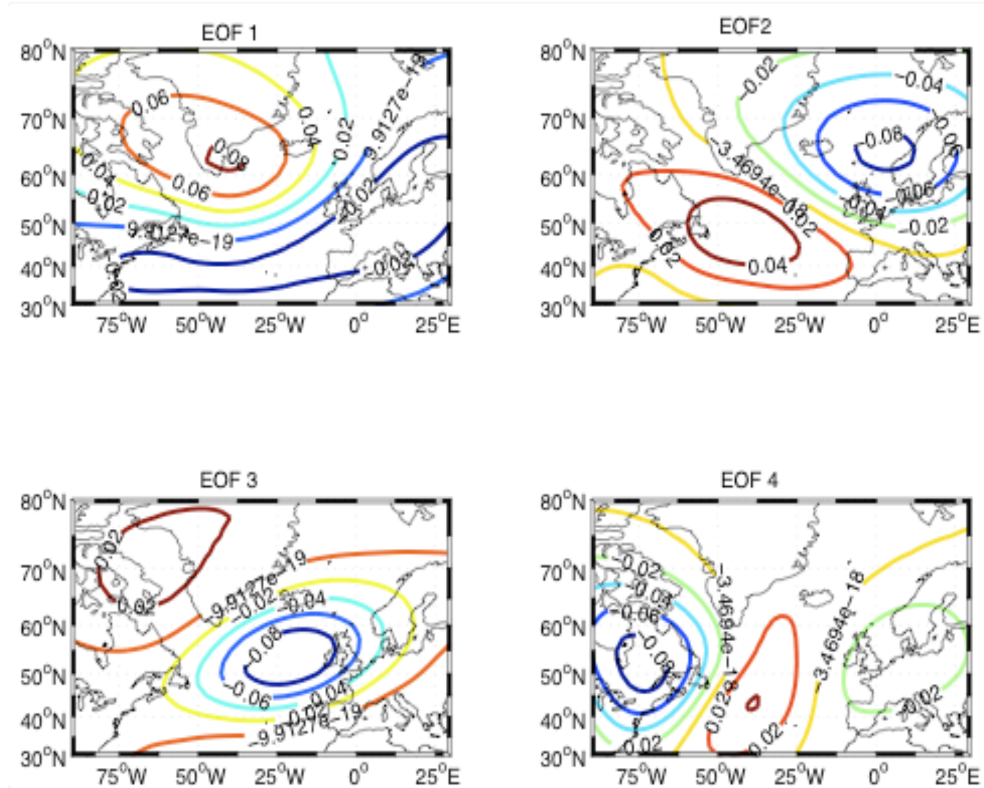
Example of $\mathrm{x}(t)$ for a given day

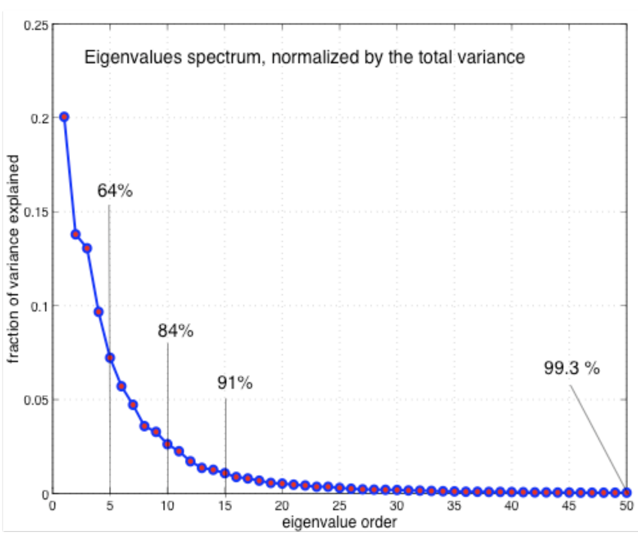


And subtracting the mean $\mathbf{x}(t) - \overline{\mathbf{x}}$



And here are the EOFs





And the eigenvalues spectrum

Normalization of EOFs

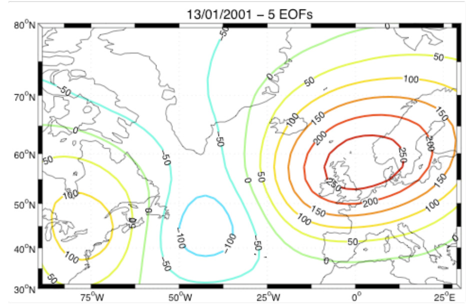
EOFs are normalized, but sometimes one can visualize the amount of variance explained in the direction of an EOF by multiplying it by the correspondent eignevalue: $\mathbf{e}_n' = \lambda_n \mathbf{e}_n$

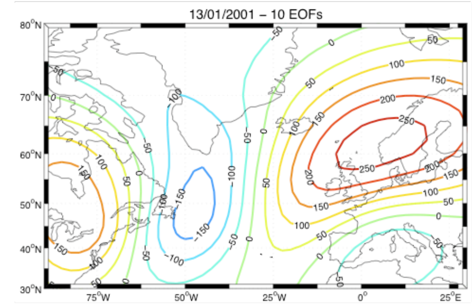
In this case, the Principal Components have variance 1

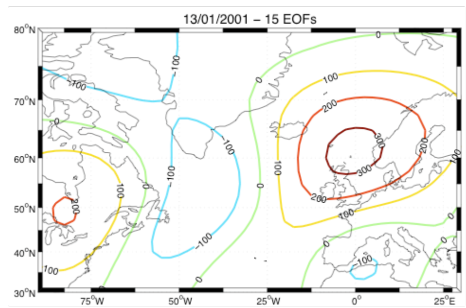
$$\mathbf{x}(t) = \sum_{n=1}^{N} c_n'(t)\mathbf{e}_n'$$

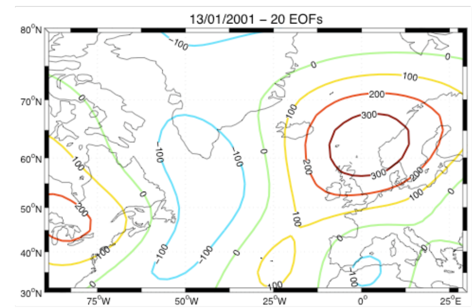
Truncating a map

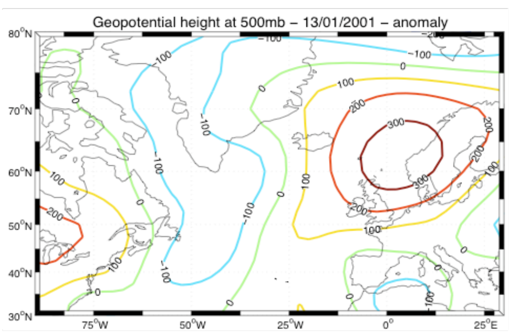
Reconstruction of a given vector of the time series, or also of a given map, on a truncated series of EOFs

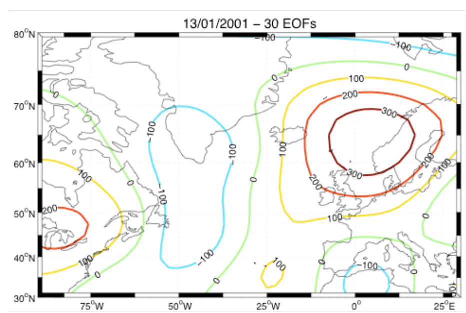


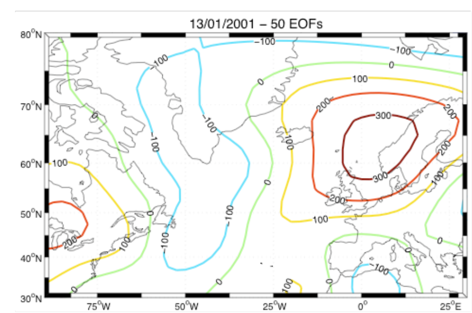










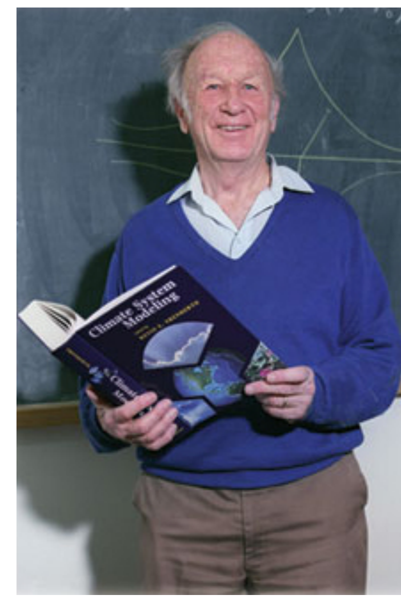


PCA has many names, according to application.

Depending on the field of application, it is also named the discrete **Karhunen–Loève transform** (KLT), the **Hotelling transform** in multivariate quality control, **proper orthogonal decomposition** (POD) in turbulence, **singular value decomposition** (SVD) of X, **eigenvalue decomposition** (EVD) of $X^T X$ in linear algebra, **Factor analysis** in social sciences, **Eckart-Young Theorem** in psychometrics, **Schmidt-Mirsky theorem, Empirical Orthogonal Functions** (EOF) in meteorological science, **Empirical Eigenfunction Decomposition**, **Empirical Component Analysis**, Quasiharmonic Modes, **Spectral Decomposition** in noise and vibration, and **Empirical Modal Analysis** in structural dynamics.

[*Wikipedia « Principal component analysis »*]

Karl Pearson
(1857 – 1936)



Ed Lorenz
(1917 – 2008)

Physical Interpretation

The EOFs are a statistical construct, so they cannot *a priori* be linked to a given physical mechanism. They are the signature of the dynamics of a given physical system. The physical interpretation is done *a posteriori* by the user. Sometimes it is evident, sometimes not.

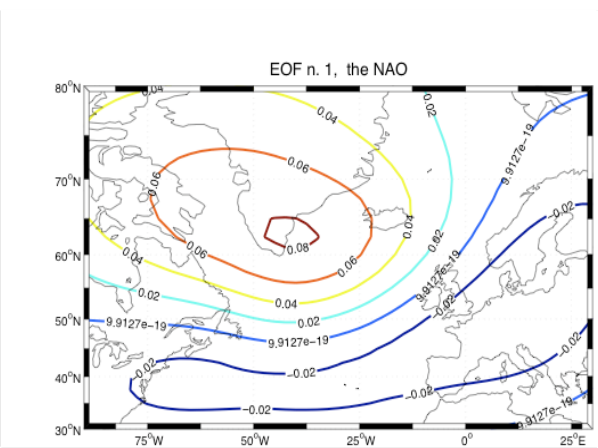
In the following we will see two examples of physical phenomena, or better of the EOF signatures of two physical phenomena:

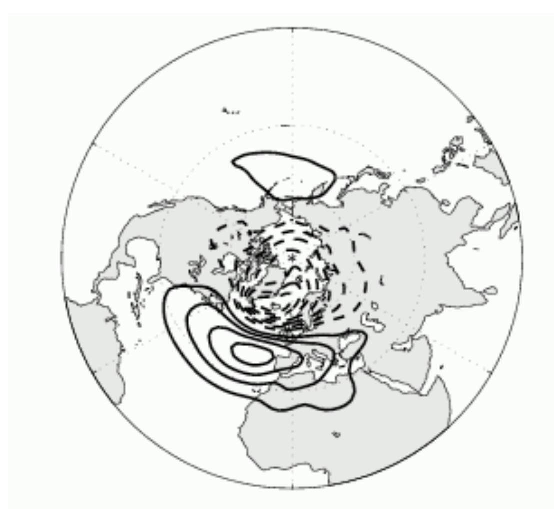
1) The North Atlantic Oscillation (NAO)
2) The El Niño Southern Oscillation (ENSO)

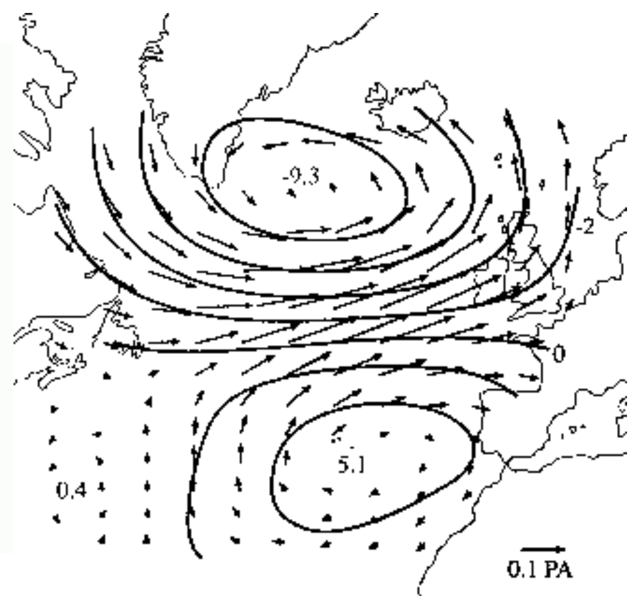
Example 1:
The North Atlantic Oscillation

The first EOF of geopotential height is very well known to meteorologists. It is the sign of a phenomenon so important in the North Atlantic region, that it was given a name: **NAO.**

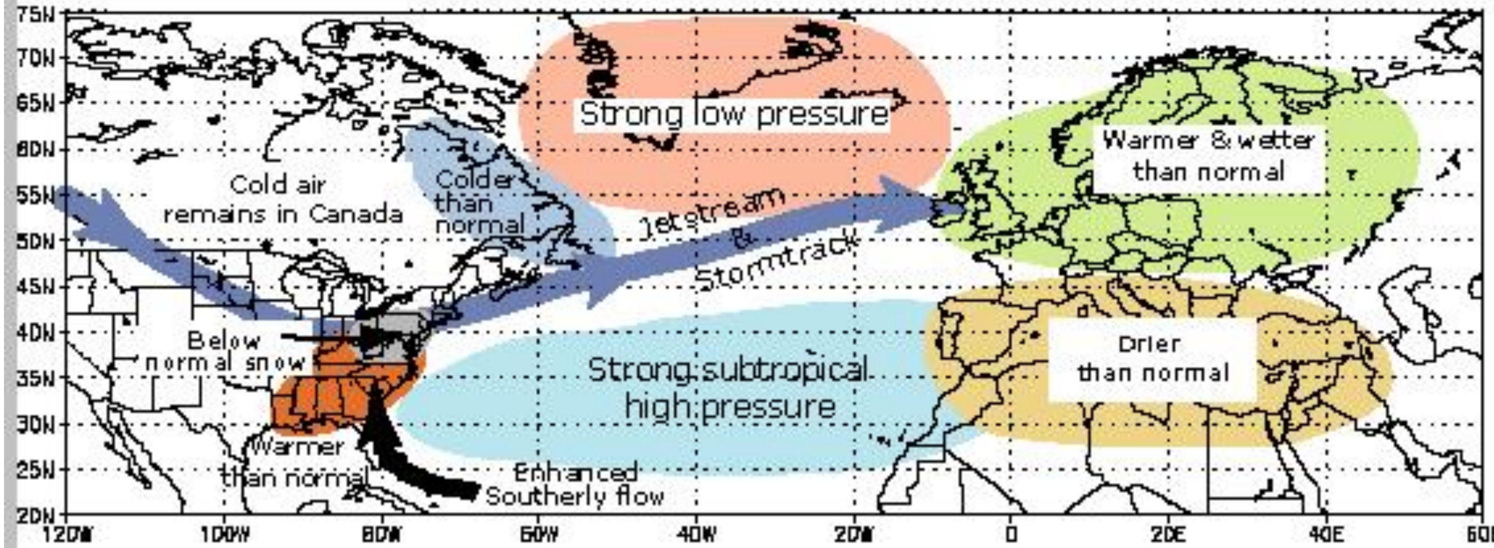
It has a dipolar structure and represents an anticorrelation between the Greenland ridge and the Açores anticyclon.



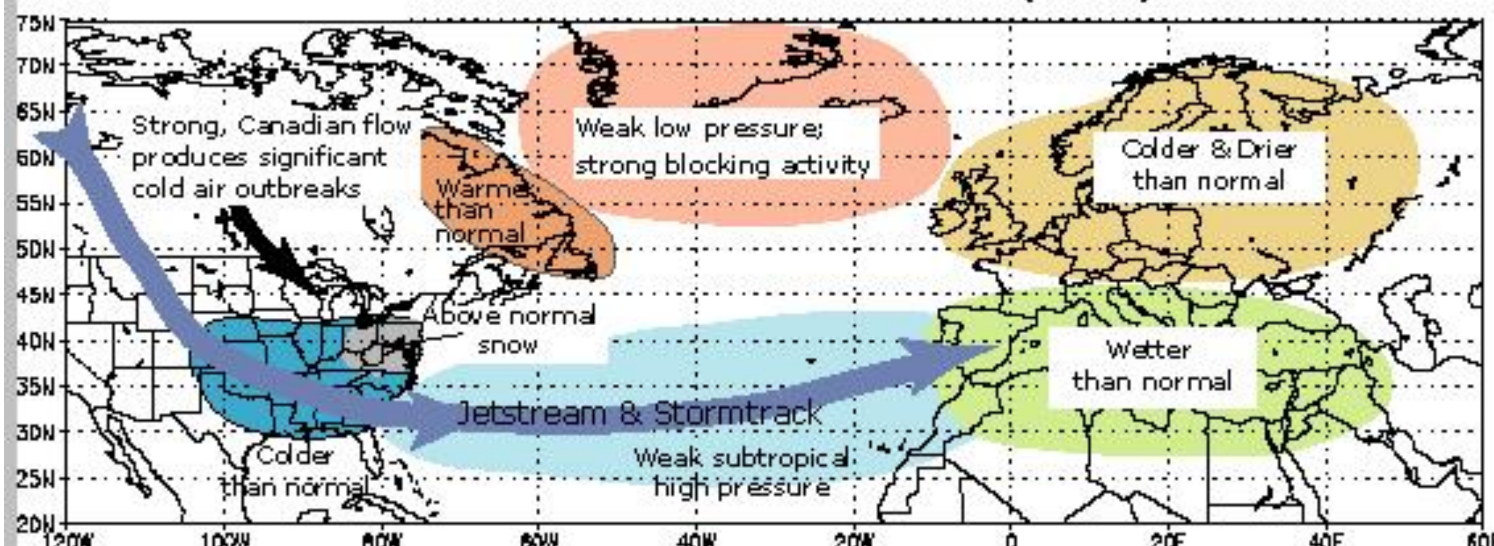


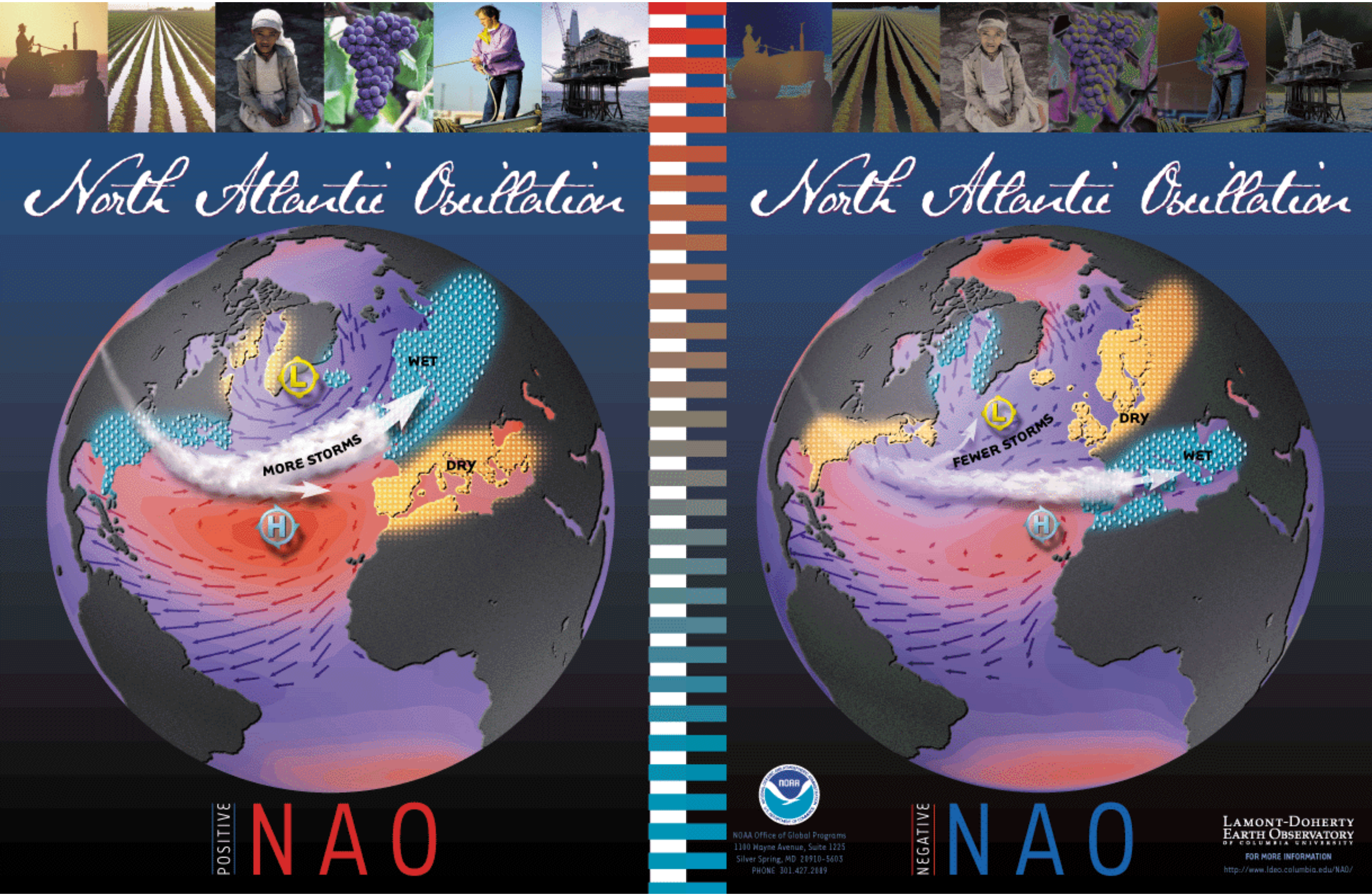


Positive Phase of the Wintertime North Atlantic Oscillation (NAO)



Negative Phase of the Wintertime North Atlantic Oscillation (NAO)




North Atlantic Oscillation
WET
MORE STORMS
DRY
L
H
POSITIVE
NAO
North Atlantic Oscillation
DRY
FEWER STORMS
WET
L
H
NOAA Office of Global Programs
1100 Wayne Avenue, Suite 1225
Silver Spring, MD 20910-5603
PHONE 301.427.2089
NEGATIVE
NAO
Lamont-Doherty
Earth Observatory
of Columbia University
FOR MORE INFORMATION
http://www.ldeo.columbia.edu/NAO/

IMPACTS ASSOCIATED WITH A POSITIVE NAO YEAR.

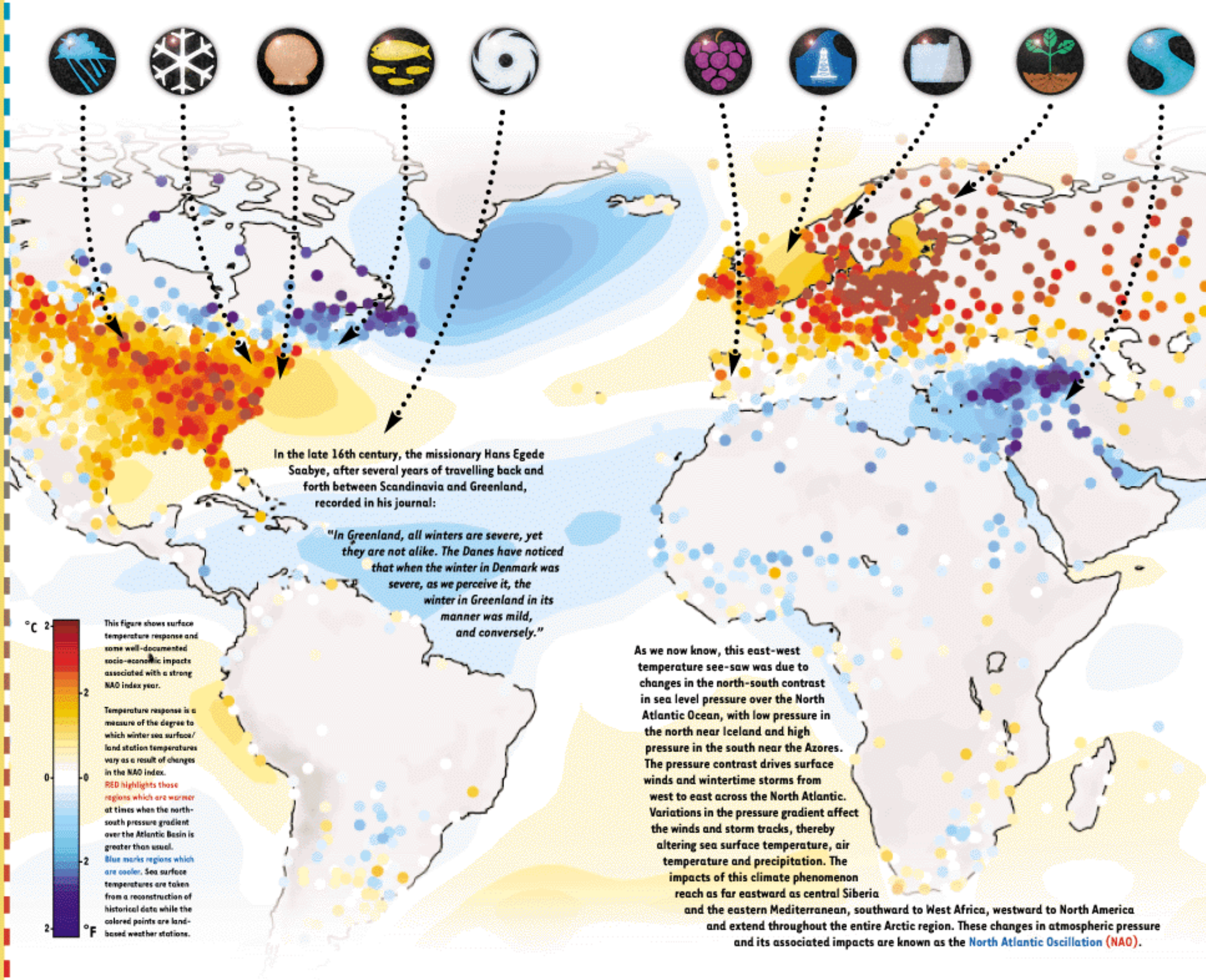
NORTHEASTERN US
Increased temperature results in decreased number of snow days

NORTH SEA
Increased wave height affects safety of oil rigs and their operators

NORWAY
Surplus water in hydroelectric reservoirs provides potential for selling surplus electricity

SCANDINAVIA
Length of the plant growth season is lengthened by 20 days

CENTRAL US
Increased precipitation and river runoff



This figure shows surface temperature response and some well-documented socio-economic impacts associated with a strong NAO index year.

Temperature response is a measure of the degree to which winter sea surface/land station temperatures vary as a result of changes in the NAO index. RED highlights those regions which are warmer at times when the north-south pressure gradient over the Atlantic Basin is greater than usual. Blue marks regions which are cooler. Sea surface temperatures are taken from a reconstruction of historical data while the colored points are land-based weather stations.

In the late 16th century, the missionary Hans Egede Saabye, after several years of travelling back and forth between Scandinavia and Greenland, recorded in his journal:

"In Greenland, all winters are severe, yet they are not alike. The Danes have noticed that when the winter in Denmark was severe, as we perceive it, the winter in Greenland in its manner was mild, and conversely."

As we now know, this east-west temperature see-saw was due to changes in the north-south contrast in sea level pressure over the North Atlantic Ocean, with low pressure in the north near Iceland and high pressure in the south near the Azores. The pressure contrast drives surface winds and wintertime storms from west to east across the North Atlantic. Variations in the pressure gradient affect the winds and storm tracks, thereby altering sea surface temperature, air temperature and precipitation. The impacts of this climate phenomenon reach as far eastward as central Siberia and the eastern Mediterranean, southward to West Africa, westward to North America and extend throughout the entire Arctic region. These changes in atmospheric pressure and its associated impacts are known as the North Atlantic Oscillation (NAO).

IMPACTS ASSOCIATED WITH A NEGATIVE NAO YEAR.

TROPICAL ATLANTIC/ GULF COAST
Warmer sea surface temperatures cause increases in number and strength of hurricanes

ATLANTIC
Increased growth and recruitment of Northern Cod

EASTERN LONG ISLAND
Decreased "brown tide" events increase scallop harvests

PORTUGAL & SPAIN
Increased grape and olive harvests

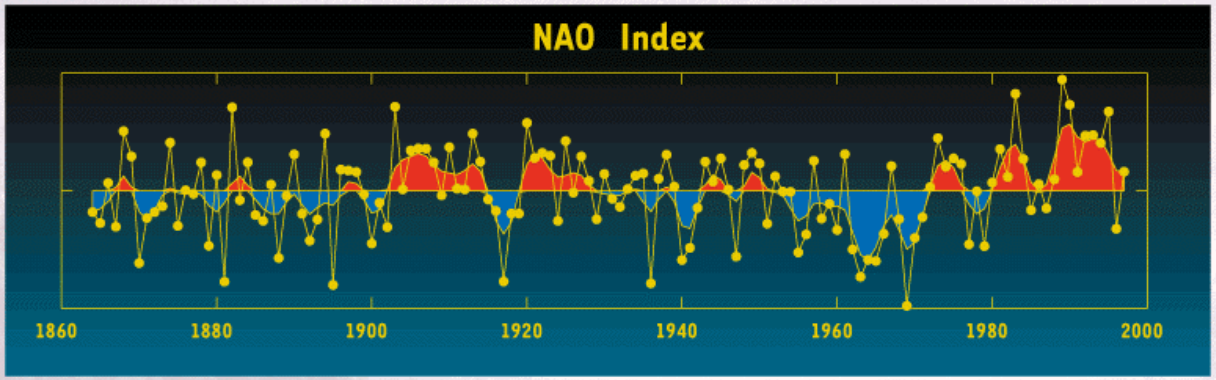
TURKEY
Increased precipitation and streamflow in the Tigris-Euphrates River Basin

THE NORTH ATLANTIC OSCILLATION NAO

The NAO is a large-scale see-saw in atmospheric mass between the subtropical high located near the Azores and the sub-polar low near Iceland. An index can be derived that tracks the behavior of the NAO through time. The index shows both high frequency and low frequency variability. In the later portion of the record there is a positive trend, steadily increasing with time. The high and low frequency variability of the NAO is believed to be related to natural variations in the climate system, while the trend witnessed over the last 30 years may be caused by anthropogenic impacts such as ozone depletion and increased CO_2 emissions. One of the fundamental questions driving NAO-related research is:

How do these two influences, natural climate variability and global warming, interact?

The NAO index is defined as the anomalous pressure difference between the Icelandic Low and the Azores High. The figure at right shows the measured sea level pressure difference between Stykkishomur, Iceland and Lisbon, Portugal over the period 1865-1998 during the winter season (December through March).




POSITIVE NAO


NEGATIVE NAO

NATURAL VARIABILITY

NAO & the Atlantic Ocean

The NAO is the dominant mode of winter climate variability in the North Atlantic region. The corresponding index varies from year to year, but also exhibits a tendency to remain in a positive or negative phase for intervals lasting several years (see red and blue sections of the NAO index above).

The characteristic time scale of atmospheric circulation anomalies are only on the order of weeks. However, the ocean, with its large capacity to absorb heat, has significant long-term memory, and may set the pace for decadal variations in the NAO. Ocean currents have the ability to propagate temperature anomalies across the Atlantic, which may influence the dynamics of the overlying atmosphere. As a result, some scientists believe that decadal variations in the NAO are due to 'two-way' communication between the ocean and atmosphere. Other scientists have suggested that the oceanic variability is merely the integrated response of the ocean to high frequency variability in the atmosphere. Another hypothesis is that the NAO might be influenced by variability in the tropical Atlantic Ocean. Once the interactions between the ocean, atmosphere, and land are more clearly understood, it may be possible to forecast year-to-year changes in the NAO.

ANTHROPOGENIC CHANGE

NAO & Global Warming

Over the past thirty years, the NAO has steadily strengthened, rising from its low index state in the 1960s to a historic maximum in the early 1990s. This trend accounts for a significant portion of Northern Hemisphere wintertime temperature increase over Eurasia, a major component of the recent warming. Consequently, the NAO has made its way into the global warming debate.

More recently, scientists became aware of a connection between variations in temperature at the earth's surface and the strength of the stratospheric winter vortex, located about 60 km above the earth's surface. Changes in stratospheric circulation can be forced by several different mechanisms including ozone depletion, volcanic dust, and CO. Rising CO_2 concentrations cool and strengthen the stratospheric winter vortex which translates into stronger surface winds. Enhanced surface westerly winds are consistent with a positive NAO index. These changes, which modulate the temperature over northern Eurasia and America, are sometimes referred to as the Arctic Oscillation.

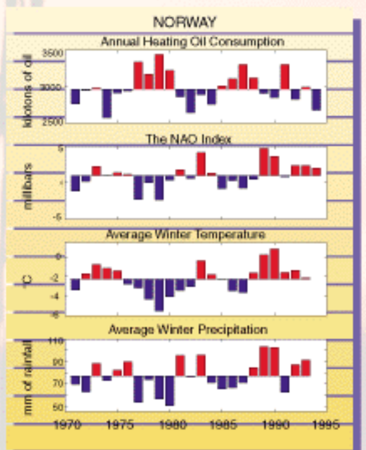
Human Impacts

ENERGY PRODUCTION & CONSUMPTION

US HYDROPOWER PRODUCTION

In the United States hydropower supplies 12% of the nation's electricity. Hydropower produces more than 90,000 megawatts of electricity, which is enough to meet the needs of 28.3 million consumers. Hydropower accounts for over 90% of all electricity that comes from renewable resources (such as solar, geothermal, wind and biomass).

A primary goal of reservoir operators at hydropower facilities is optimizing flood protection vs. energy generation. If reservoir operators under-estimate flood volume, the reservoir system will be unable to fully regulate flow. As a result, water must be spilled over into spillways. Environmental damage due to flooding and financial loss due to decreased generating capacity result. The link between a positive NAO and increased East Coast precipitation suggests that reservoir operators in this region could gain from knowing more about the NAO.



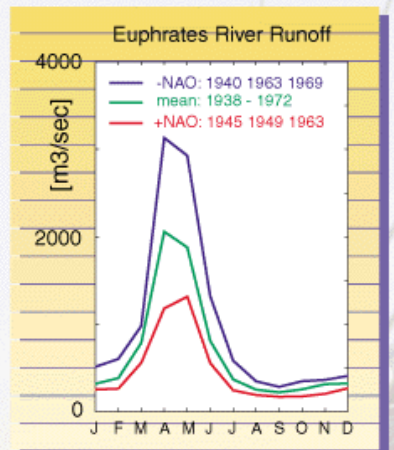
ENERGY CONSUMPTION AND PRODUCTION IN NORWAY AND THE NAO

The demand for heating oil in Norway clearly shows human sensitivity to changes in the NAO. Cooler winters and a generally negative NAO prevailed during the late 1970's resulting in a greater demand for heating oil. Things changed in the early 1980's as the NAO index switched to a positive phase and Norway became warmer, resulting in decreased demand for heating oil. These changes in demand vary by 10-15% of the average demand between 1970 - 1995.

Norway is the world's sixth largest hydropower producer, and the largest producer of hydropower in Europe. Annual winter precipitation in Norway can be thought of as a surrogate for streamflow and hence hydropower generation. Between 1980 and 1993, a period of increasingly positive NAO years, precipitation was higher than normal, resulting in increased water inflow for power generation.

HYDROLOGY & WATER RESOURCE MANAGMENT

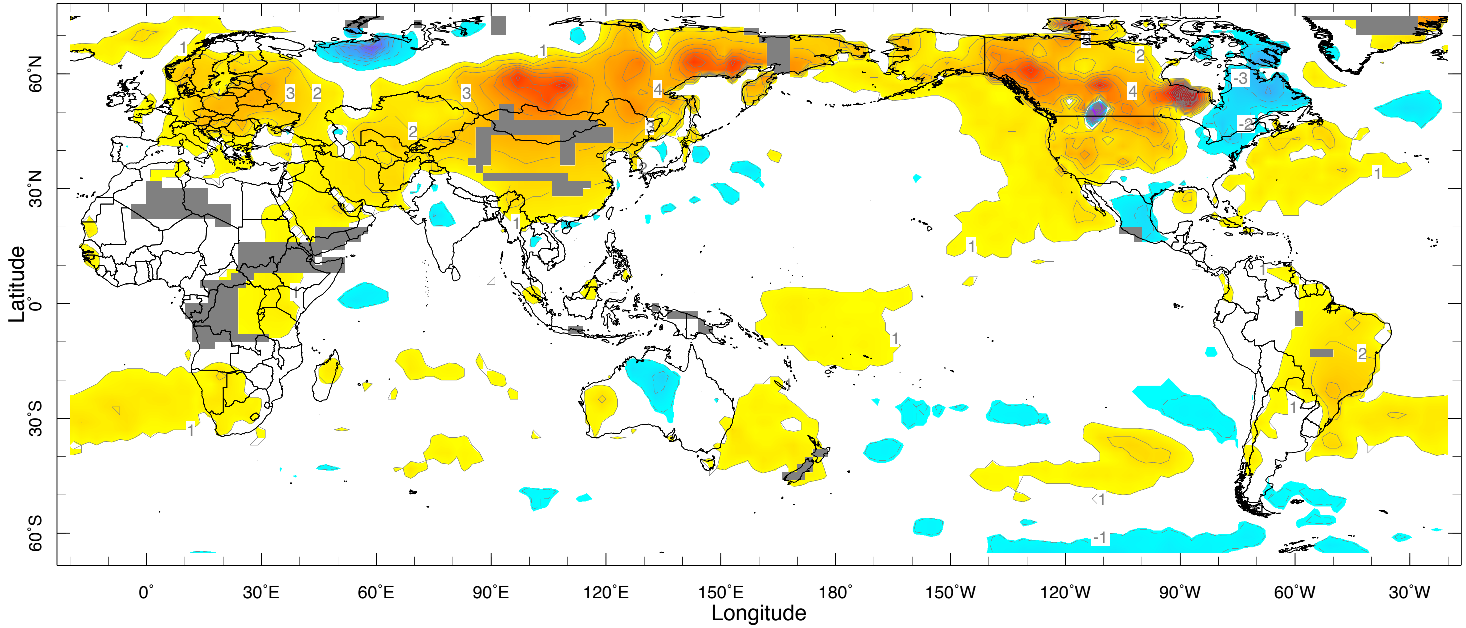
Freshwater constitutes only ~2.5% of the total volume of water on earth, and two-thirds of it is trapped in glacial ice. Only 0.77% of freshwater is held in places more accessible to humans such as aquifers, lakes, rivers, and the atmosphere. River runoff is the most accessible source and accounts for much of the water used for irrigation agriculture, industry, and hydropower generation. New dam construction has the potential to increase accessible runoff by ~10% over the next 30 years, however population is projected to increase by more than 45% during that period. As a result humans will become increasingly sensitive to natural variations in precipitation and river runoff.



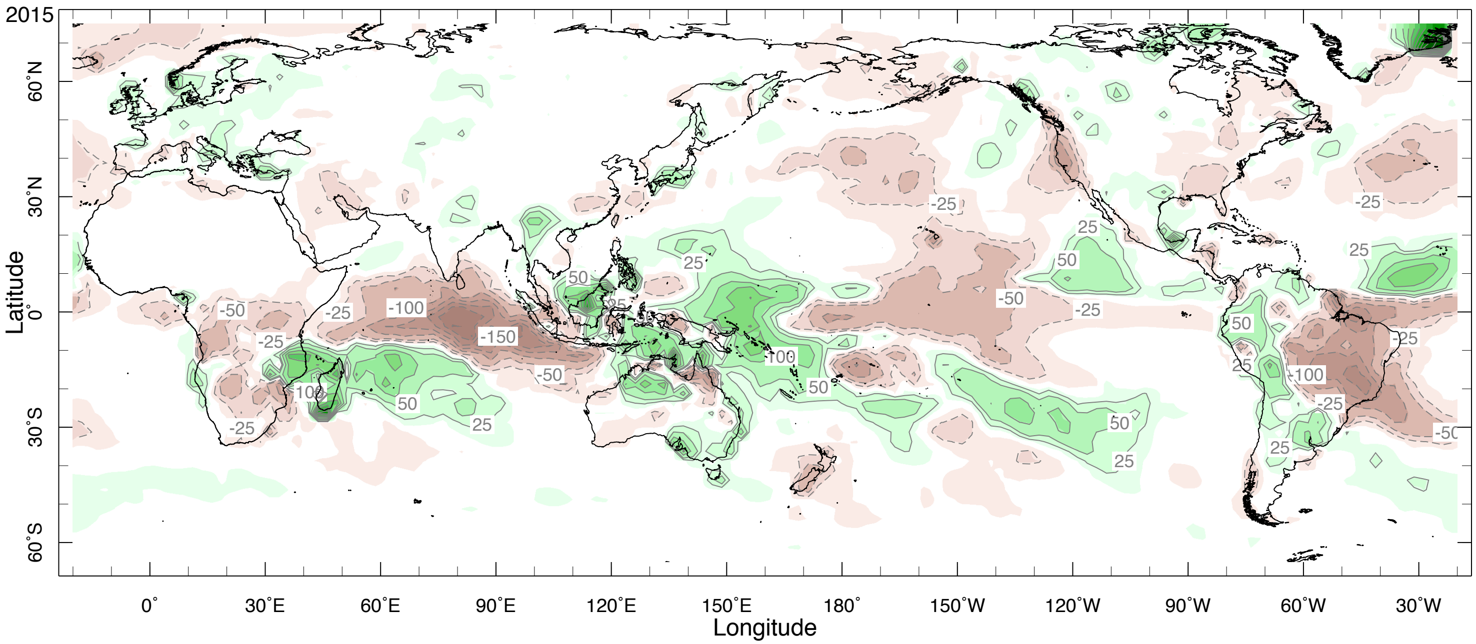
Perhaps the most sensitive of all regions is the Middle East, where usable freshwater is already scarce. With population increasing by 3.2% each year and irrigation practices consuming upwards of 80% of available water supply, water is a key variable affecting regional public health and political stability. Much of the current focus in Middle Eastern water policy has been the environmental and socio-economic impacts associated with increased damming along the Tigris-Euphrates River system.

Turkey, because it has the good fortune of being situated at the headwaters of the Tigris-Euphrates River system, can literally turn off the supply of water to its downstream neighbors and has threatened to do so on occasion. For example, when the Ataturk Dam was completed in 1990, Turkey stopped the flow of the Euphrates entirely for one month, leaving Iraq and Syria in considerable distress. However natural climate variability, which has no political alliances, can be attributed to variations in Turkish precipitation and Euphrates River runoff and is linked to changes in the NAO. Even the recent trend in the NAO index can be seen in historical precipitation data; with droughts occurring in Turkey during the 1980s and the early 1990s and wet conditions generally occurring during the 1960s and the late 1970s.

Human Impacts

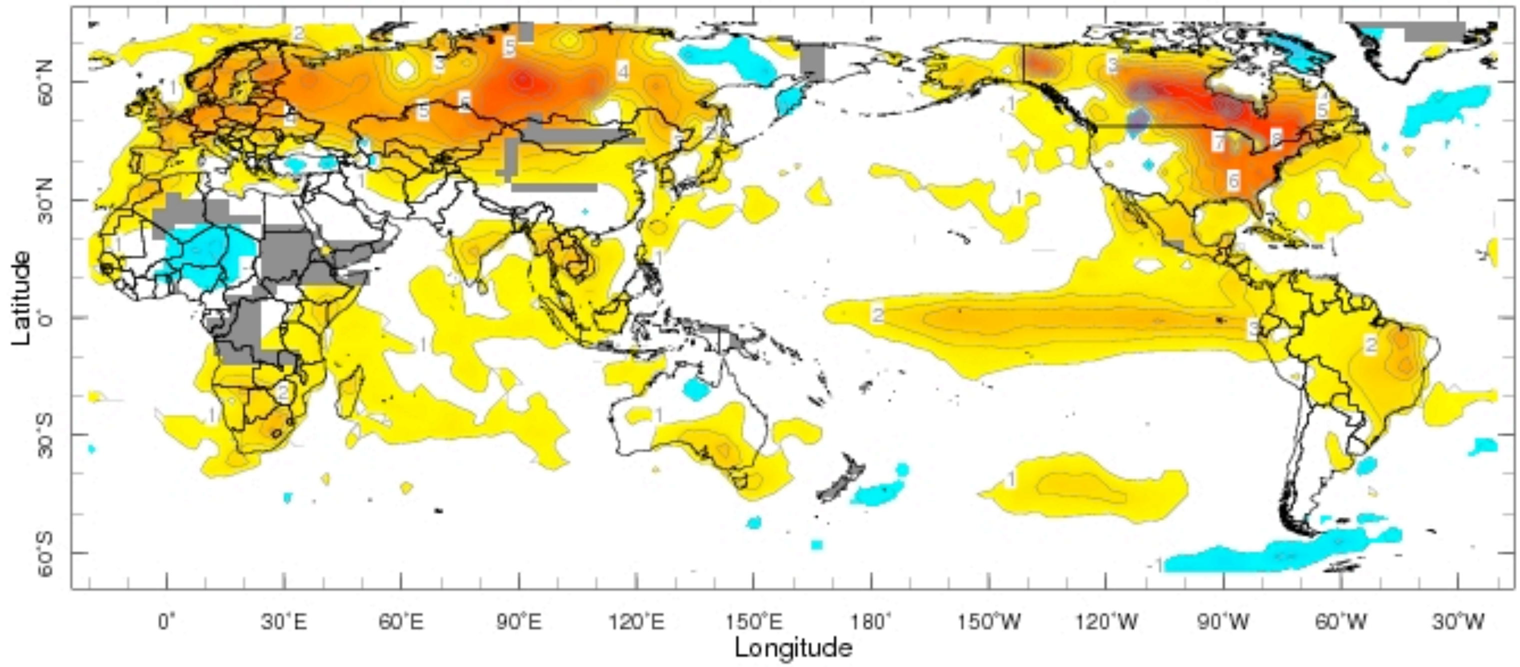


Jan 2015



Jan 2015

Monthly averages for January 2015 strong and persistent Positive NAO
(credit: http://iridl.ldeo.columbia.edu/)

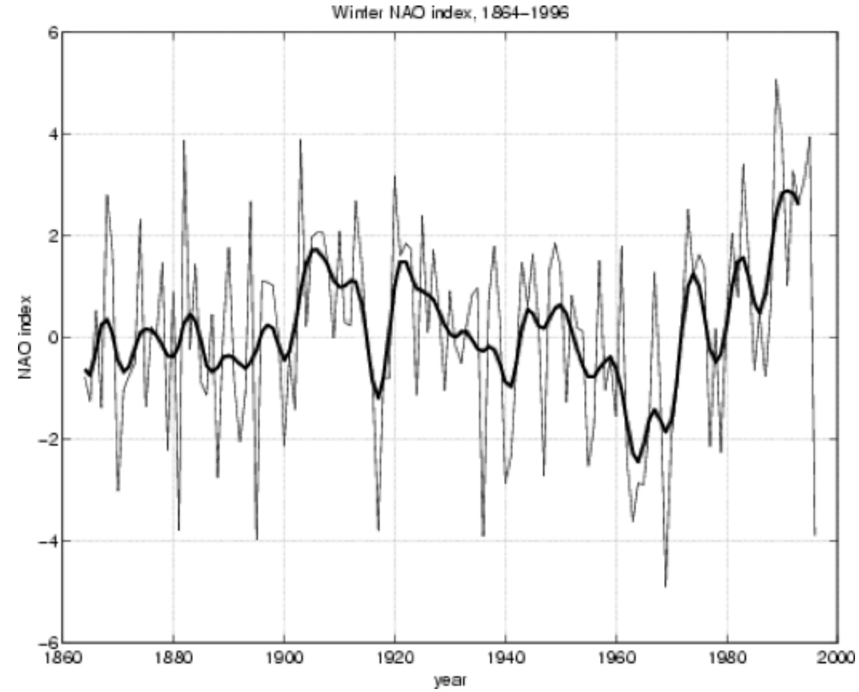


60°N
30°N
0°
30°S
60°S
Latitude
0°
30°E
60°E
90°E
120°E
150°E
180°
150°W
120°W
90°W
60°W
30°W
Longitude

Dec 2015

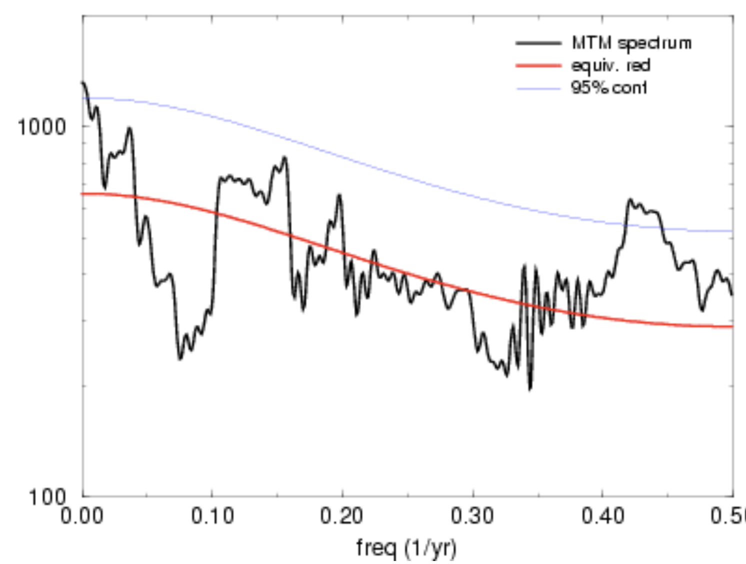
NAO variability

Analysing the time series $\mathbf{x}(t)$ we can see that the NAO has many time scales of variability.

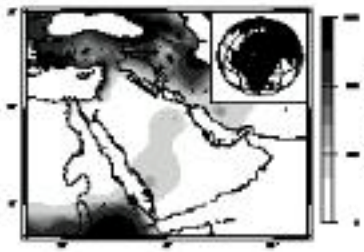


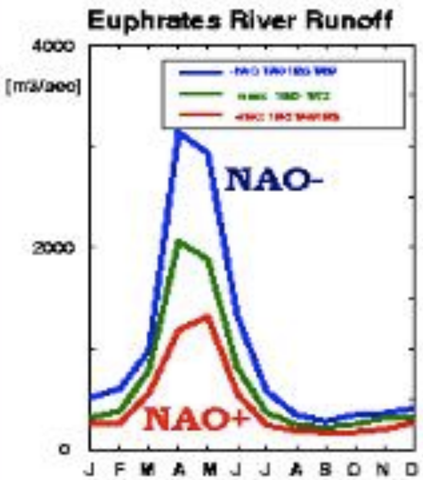
NAO index (Dec–Mar) 1864–1996

Its spectrum shows power at all frequencies and is compatible with a red noise process. It doesn't really show any signficant peaks (still debated).



NAO and Water Resources in Turkey and the Middle East





Cullen 1998

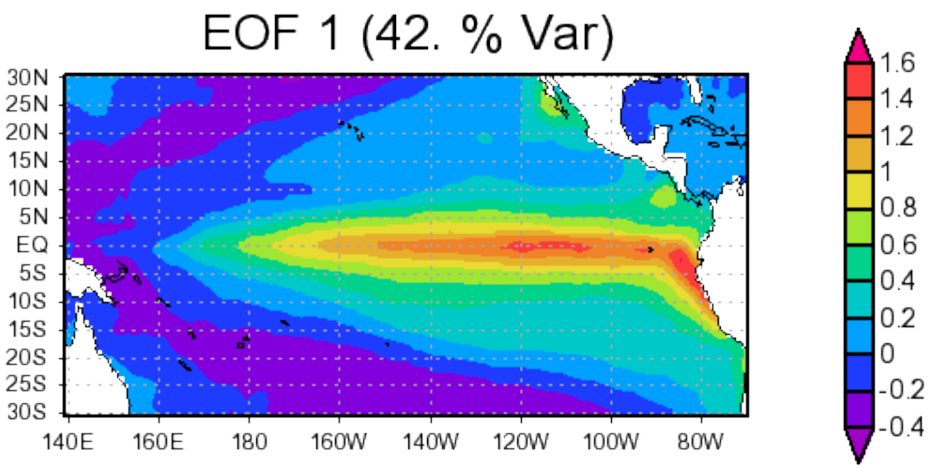
Precipitation in Turkey is well correlated with the NAO.

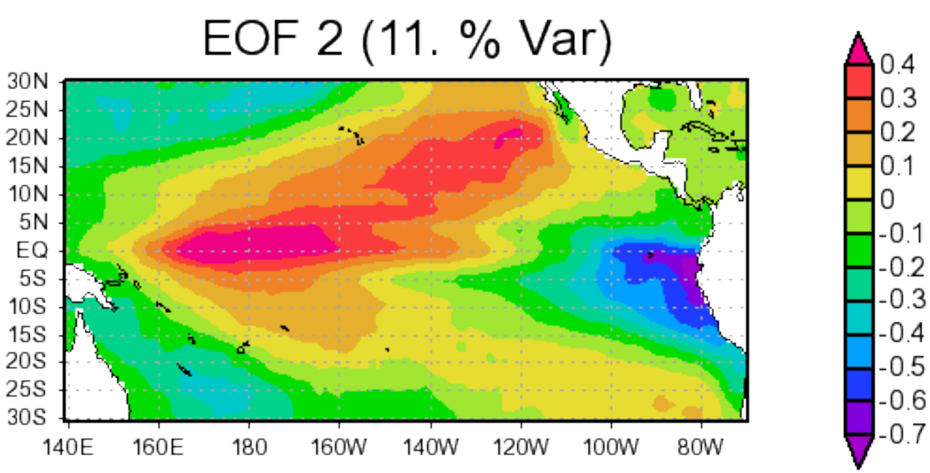
As a result spring stream flow in the Euphrates River varies by about 50% with the NAO.

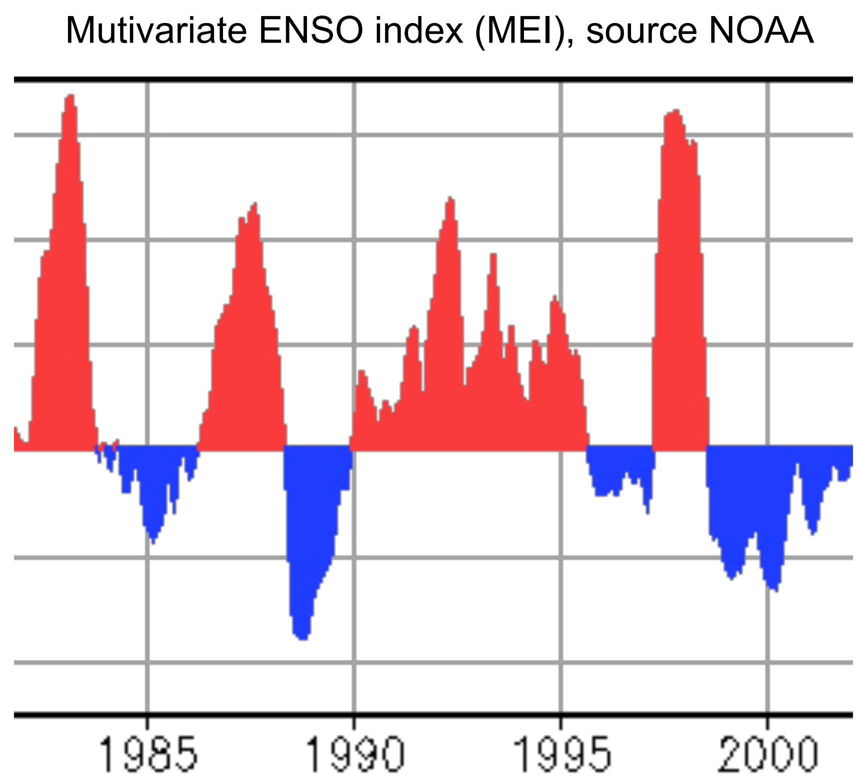
An upward trend in the NAO will lead to drought conditions in the Middle East.

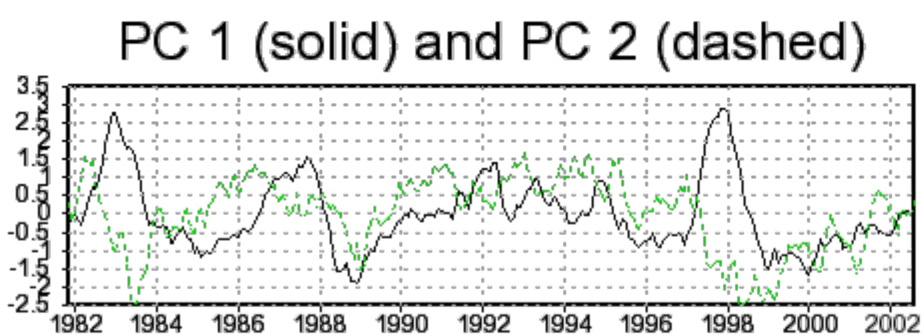
Example 2:

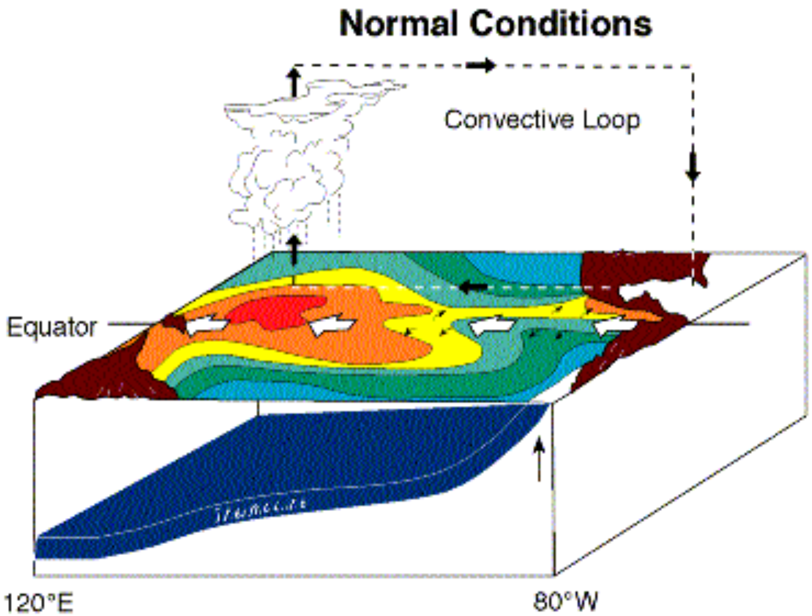
EOFs of Sea Surface Temperatures. Equatorial Pacific ocean.

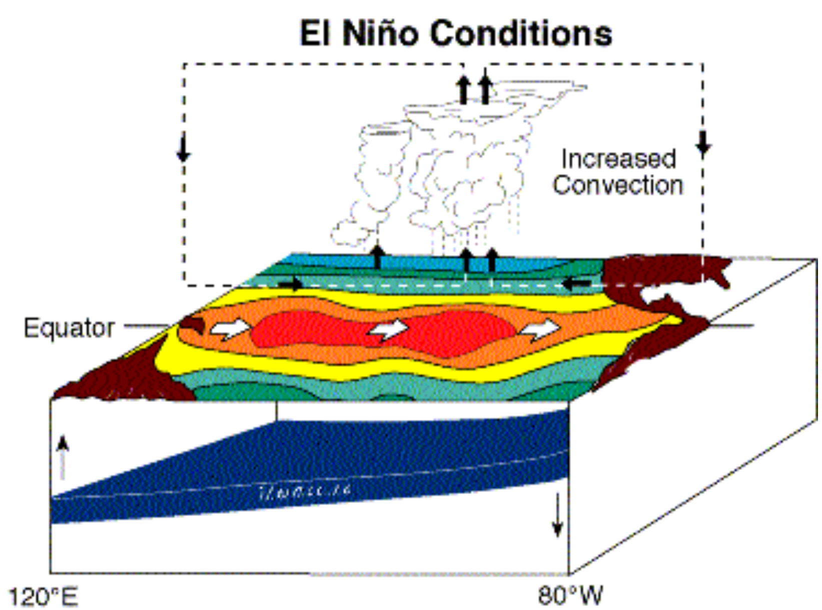










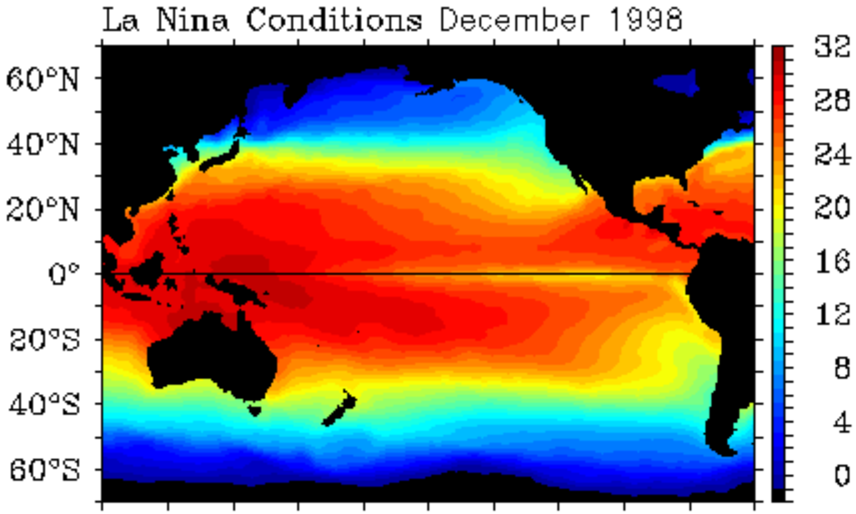


NOAA/PMEL/TAO

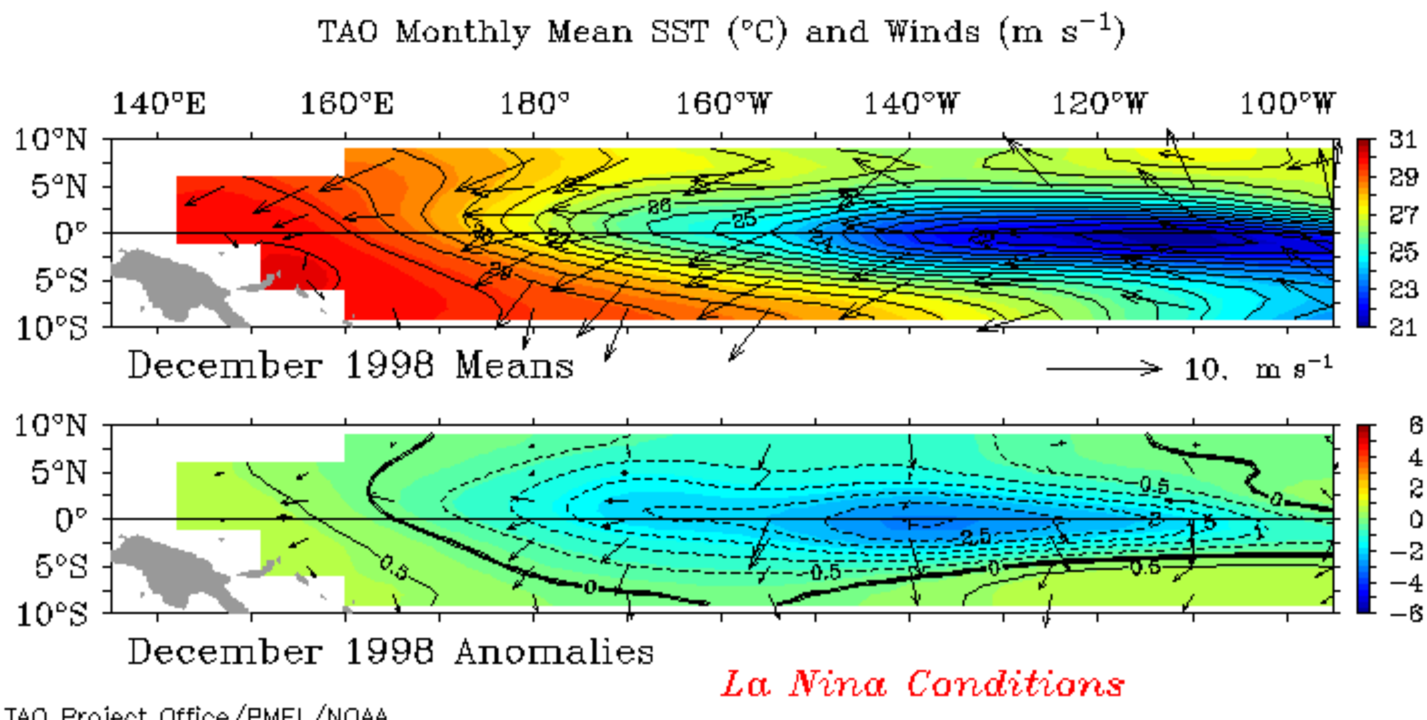
Dynamical mechanisms of ENSO (El Niño Southern Oscillation).

Te EOFs are often the sign of a physical mechanisms creating the variability, but it is not necessarily so.

NORMAL CONDITIONS

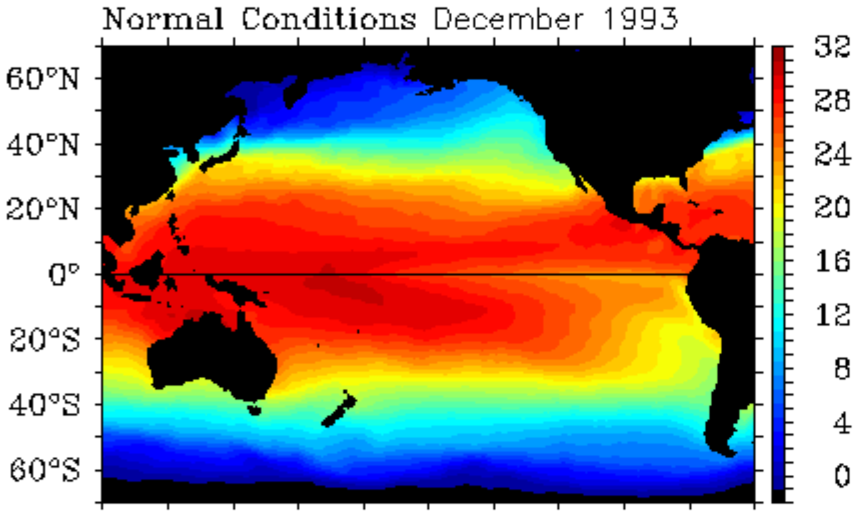


Reynolds SSTdata



TAO data

LA NIÑA CONDITIONS



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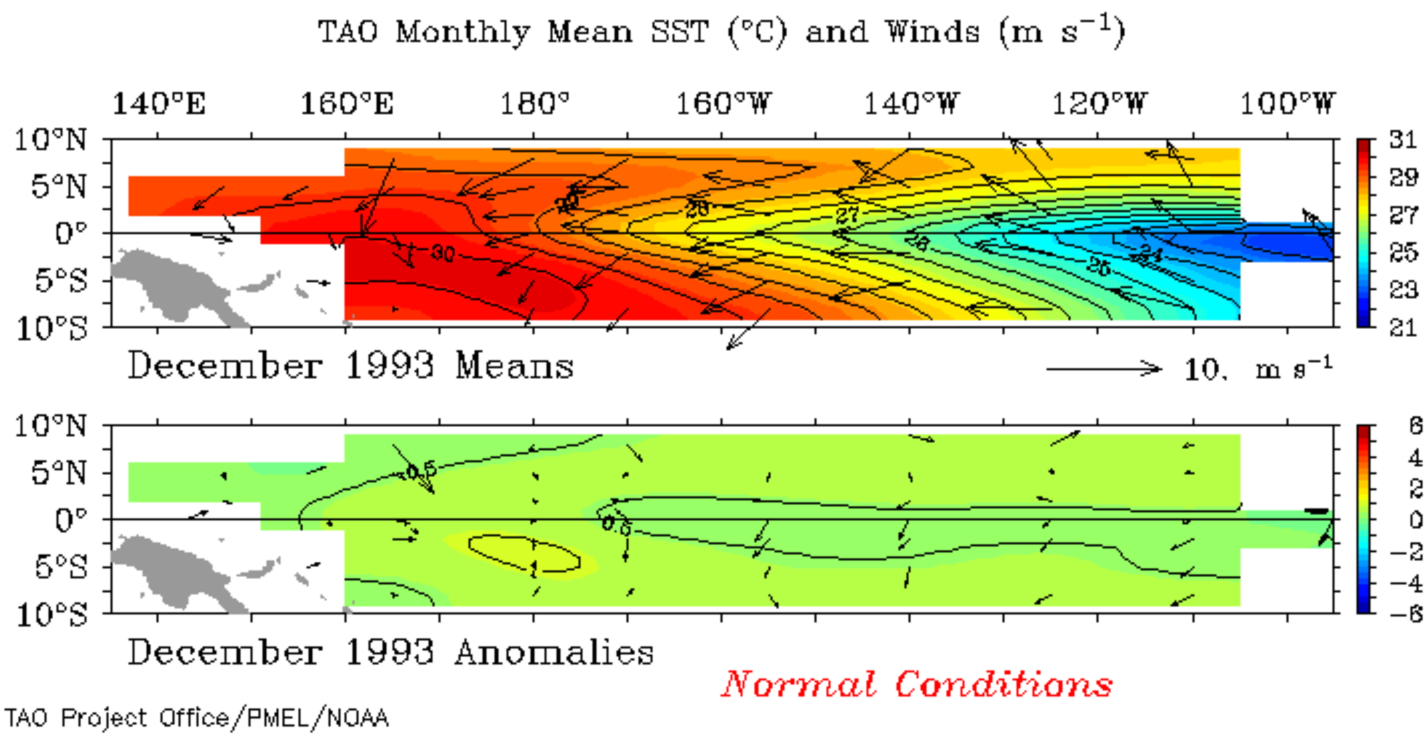

TAO data

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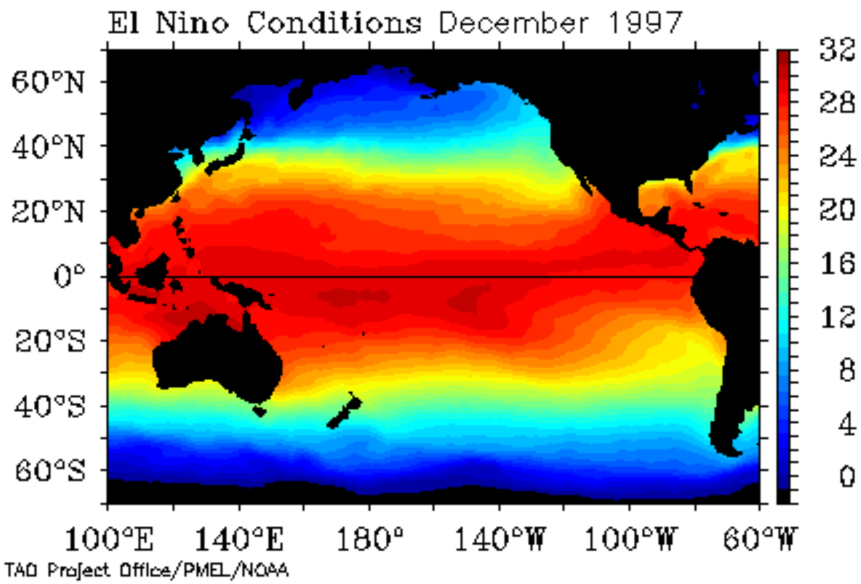

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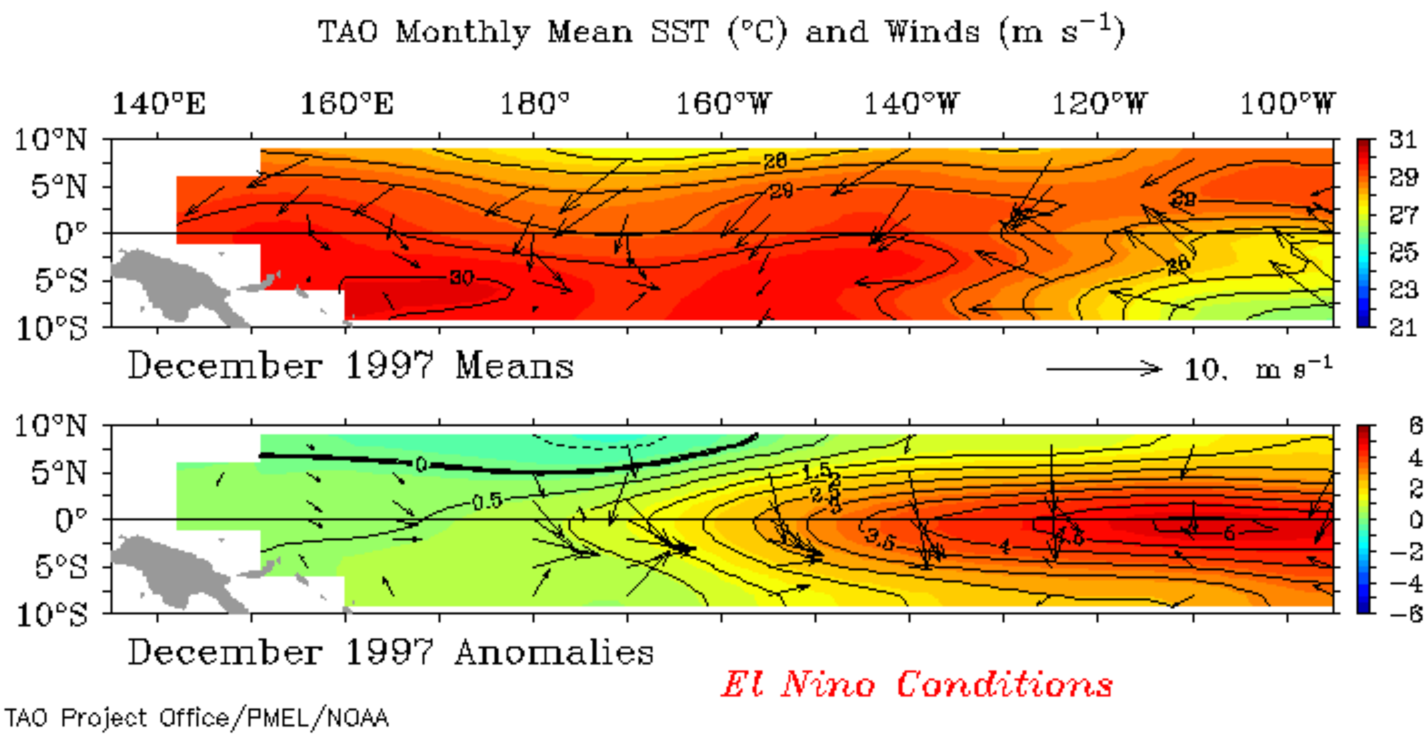
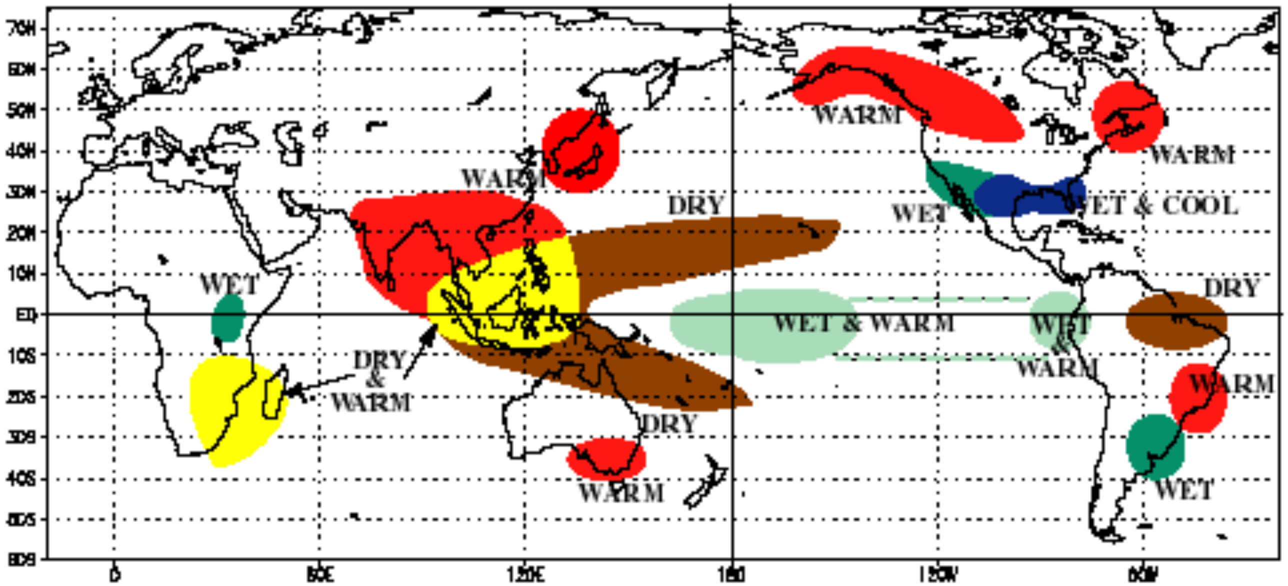
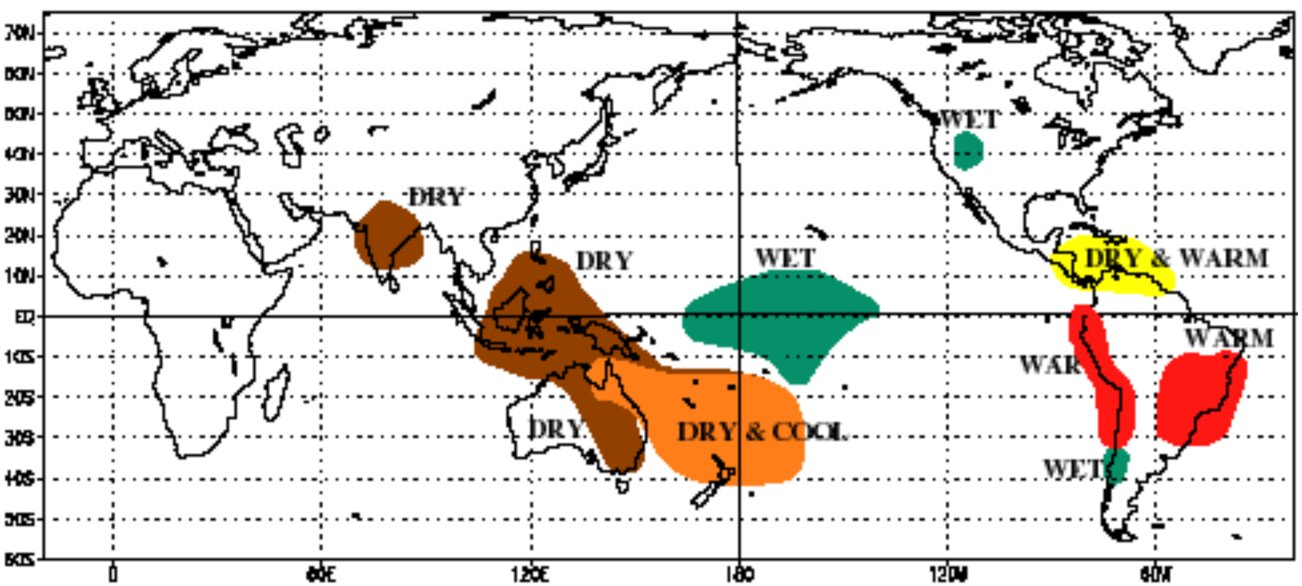


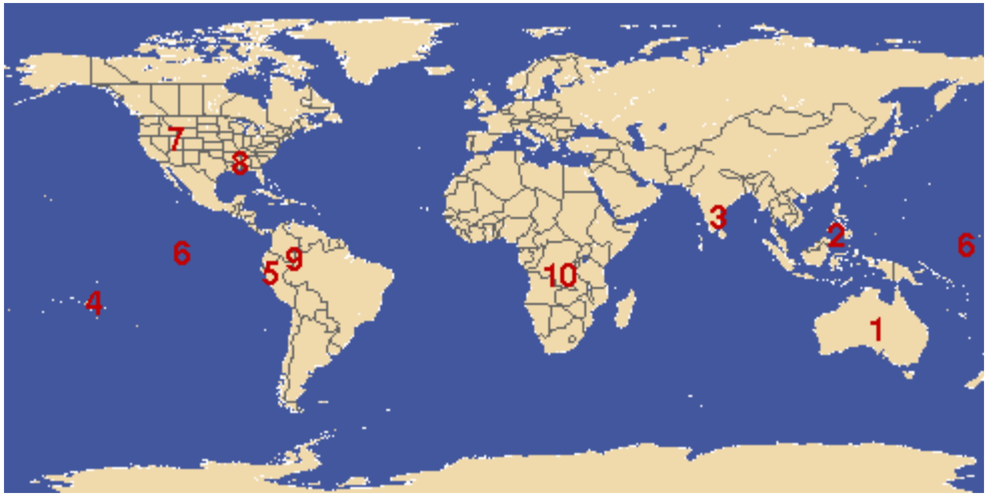
TAO data

WARM EPISODE RELATIONSHIPS DECEMBER - FEBRUARY



WARM EPISODE RELATIONSHIPS JUNE - AUGUST





Impacts of 1983 El Niño

1 Australia-Drought and bush fires
2 Indonesia, Philippines-Crops fail, starvation follows
3 India, Sri Lanka-Drought,fresh water shortages
4 Tahiti- tropical cyclones
5 South America-Fish industry devastated
6 Across the Pacific-Coral reefs die
7 Colorado River basin-Flooding, mud slides
8 Gulf states-Downpours cause death, property damage
9 Peru, Ecuador-Floods, landslides
10 Southern Africa-Drought, disease, malnutrition

EXERCISE

Get the daily 500hPa geopotential height data for the Euro-Atlantic region in winter (December-January-February) from the course webpage and compute the EOFs with area-weighting norm.

Roadmap:

1)Read data in (command *Dataset of module netCDF4*) (Test1: Plot the map of a given date)

2)Compute the time mean of the data (Test2: plot the time mean on a map)

3)Subtract the mean to each data field (Test3: plot the anomaly map of the same day as above)

4)Change the shape of data from grid to column vector (command *reshape*)

5)Define weight as cosine of latitude

6)Multiply data by weight

7)Compute covariance matrix

8) Diagonalize covariance matrix

9) Postprocess data (divide by weight, reshape)

10) Plot eigenvalue spectrum

11) Plot EOFs , save EOFs

Use the python function (cylmap.py) for tracing geophysical maps. You should put it in the correct directory.