Physics of the atmosphere

Lecture 1 STRATIFIED ATMOSPHERE

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<u>I Introduction and basic principle</u> II Moist convection and clouds

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composition

The atmosphere is stratified:

- Exponential decay of pressure and density from ground to 100 km
- Several layers distinguished from the temperature profile Troposphere from 0 to 12 km (18 km at tropical latitudes)
 - Stratosphere above until 50 km
 - Mesosphere from 50 to 90 km
 - 90% of the mass under 20 km
 - Standard density (à 1013 hPa et 273K): ρ=1,29 kg m⁻³



Troposphere and stratosphere are separated by the tropopause



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Atmospheric composition*

0,7808

0,2095

<0,030

0,0093

416 ppmv

Nitrogen N2 Oxygen O2 Water H2O Argon A CO2

pre-ind: 285 ppmvOzone O310 ppmvMethane CH41,6 ppmvNitrous oxide N2O350 ppbvCO70 ppbvNO, CFC-11, CFC-12< 0,3 ppbv</td>Mean molar mass M=28,96 g

homogeneous homogeneous highly variable homogeneous ~homogeneous (to be updated each year, 2.3 ppmv/y) stratosphere decreases with z decreases with z

*: composition is indicated as volume mixing ratio

Note: Definition of mixing ratios

The proportions of gaseous compounds in the air can be described in terms of mass or volume mixing ratio.

We take as the reference the dry air (without water vapour) with a molar mass of 28.96 g and the corresponding density p_d . Then a minor component with molar mass M and density pexhibits a <u>mass</u> mixing ratio $r_m = p/p_d$. Equivalently, one might consider the <u>volume</u> mixing ratio defined as the ratio between the partial pressure p of the component to that of the dry air p_d , $r_v = p/p_d$. Under the kinetic approximation, the volume mixing ratio is also the proportion of the minor component in number of molecules in the gas. The relation between the two ratios is given by $r_v = r_m M_d/M$. As the mixing ratio is often very small, multiplicative factors are used and the mixing ratio units are ppmv (part per million in volume, factor 10⁶), ppbv (part per billion in volume, factor 10⁹) or pptv (part per trillion in volume, factor 10¹²). Similar notation exists for mass mixing ratio (ppmm, ppbm, ...). Be careful of the notation ppm which is sometimes ambiguous.

Only water, which can reach 3% in volume mixing ratio in warm and moist areas, is able to change the density of air in significant way. This needs to be accounted for motion in the atmospheric boundary layer. The other minor components are in too small proportion to affect the density.

It is more natural to use the mass mixing ratio to describe the thermodynamical properties of the moist air. On the contrary the volume mixing ratio is the most common to describe atmospheric chemistry.

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The mean components (N₂, O₂) composition of the atmosphere is quasi constant up to 100 km.

There are large variations of the minor components (H2O, O3, ...) in this range of altitudes



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I.2 Dry air thermodynamics

Perfect gas law $p = \rho RT$ where $R = R^*/Md = 287 J kg^{-1} K^{-1}$

• Perfect gas enthalpy $H = C_p T$ where $C_p = 1005 J kg^{-1} K^{-1}$, thermal capacity per unit mass at constant pressure. H depends only of the temperature.

• At constant pressure, for a quasi-static transformation: $\delta Q = C_p dT = dH = T dS$ (S: entropy)

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• Plus généralement

\delta Q = T dS = dU + p d(1/p) = dH - 1/p dp

= C_p dT - 1/p dp = C_p dT - RT/p dp = C_p (T/\theta) d\theta

où on introduit la <u>température potentielle</u>

\theta = T(p_0/p)^{\kappa} avec \kappa = R/C_p = 2/7
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I.3 Hydrostatic law and stratification

Along the vertical, the air is mainly under hydrostatic equilibrium: upon averaging over an horizontal box of a few km², the vertical speed is of the order of a few cm/s and the vertical acceleration is negligible with respect to gravity.

• Hydrostatic law $dp/dz + \rho g = 0$

Combining with the perfect gas law obtains dp/p = -g/RT dz and, for a uniform temperature profile (a coarse approximation valid to within 20 % error) To = 255 K , results into

$p = p_0 \exp(-z/H)$

with H = $RT_0/g \approx 7,4$ km, defined as the <u>scale height</u> of the atmosphere. Atmospheric pressure decreases by about one-half every 5 km (since H $ln(2) \approx 5$ km)

The temperature profile cannot be explained with simple laws. It depends on the radiative equilibrium (absorption and emission) of the vertical heat transport by de chaleur by the motion of air parcels.

- **I.4 Temperature distribution** Variations of the vertical temperature profile
- At the ground, variations of the order of 100 K at same location but 50 K on the average between pole and equator.

• At tropopause level, the temperatures vary within a smaller range but reach very small values (190K at tropical topopause). Very small values are also reached in the polar stratosphere during winter.





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Meridional distribution of temperature during boreal winter



The direction of the meridional temperature gradient changes sign by 200 hPa or 12 km.

To be retained

- The atmosphere up to 100 km is composed of nitrogen (N2, 78%), oxygen (O2, 21%) and argon in fixed proportions to which other minor compounds are added in variable proportions. The most important is water vapour which is highly variable in a ratio 1 :10000 from up to 35g/kg at the ground in moist tropical regions to a few mg/kg at the tropical tropopause and in the stratophere.
- Temperature decays by about 6.5 K/km in the troposphere up to the tropopause, above which the temperature rises again in the stratosphere up to the tropopause at about 55 km. The height of the tropopause varies according to the latitude. It is about 6 km at high latitudes, 12 km (300 hPa) at mid-latitudes and 18 km (100 hPa) in the tropical band near the equator. The temperature of the tropical tropopause is very low, near 200 K.
- The atmospheric ozone is for 90 % concentrated in the stratosphere. It filters the solar UV under 290 nm and converts this energy into heating, resulting in the increasing temperaure profile of the stratosphere from the tropopause to the stratopause..
- The atmosphere is stratified. It is basically in hydrostatic equilibrium for all motion as horizontal scales larger than 10 km. This equilibrium is locally broken by convection and turbulent mixing. Pressure and density decrease with altitude as an exponential law in first approximation, decreasing by half every 5 km.
- Air thermodynamics can be effectively described by the perfect gas law.

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- I.5 Dry air thermodynamics (cont'd)
- Perfect gas law $p = \rho RT$ where $R = 287 J kg^{-1} K^{-1}$
- Perfect gas enthalpy $H = C_p T$ where $C_p = 1005 J kg^{-1} K^{-1}$, thermal capacity per unit mass at constant pressure. H depends only of temperature
- At constant pressure, for a quasi-static transform: $\delta Q = C_p dT = dH = T dS$ (S: entropy)

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More generally
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\delta Q = T dS = dU + p d(1/p) = dH - 1/p dp
= C_p dT - 1/p dp = C_p dT - RT/p dp
dS = C_p dT/T - R dp/p = C_p d\theta/\theta
Where we define the <u>potential temperature</u>
\theta = T(p_0/p)^{\kappa} \text{ with } \kappa = R/C_p (= 2/7 \text{ for a di-atomic perfect gaz})
P_0 = 1000 \text{ hPa}
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I.5b Potential temperature

Fast convective motion is essentially adiabatic since the heat exchanges are much slower than the pressure equilibration of the rising parcel and as long as the parcel does not mix with its environment.

Conservation of potential temperature $\theta = T(p_0/p)^{\kappa}$ with $\kappa = R/C_p$



In first approximation, convective motion is adiabatic, that is at constant θ

I.6 Convective instabity of vertical motion as a $\backslash \Gamma$ $\Gamma < \Gamma'$ Stable fucntion of the ambiant temperature profile $\Gamma' = \begin{cases} \Gamma_{d} & \text{Unsaturate} \\ \Gamma_{c} & \text{Saturated} \end{cases}$ Unsaturated Vertical gradient of temperature $\Gamma = - dT/dz = -$ Unstable 6,5 K/km (lapse rate) 0 $\Gamma > \Gamma'$ Adiabatic gradient $\Gamma' = g/Cp = -10 \text{ K/km}$ We use $0 = \delta Q = C_p dT - \frac{1}{\rho} dp$ et $\frac{dp}{dz} + \rho g = 0$ to obtain $\frac{dT}{dz} = \frac{-g}{C_n}$ during an adiabatic displacement The vertical motion of the air (a) Stable $(\Gamma < \Gamma')$ (b) Neutral ($\Gamma = \Gamma'$) (c) Unstable $(\Gamma > \Gamma')$ $T' > T \int f_b > 0$ Τ′ < Τ $f_b = 0$ 0 T' is then local ambient $f_b < 0$ $f_b < 0$ T' < T stable or $f_b = 0$ unstable. Т

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parcel is assumed adiabatic and its new temperature compared to the temperature to determine where the air column is

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Criterion $\Gamma > \Gamma'$ or $d\theta/dz < 0$



Consequence of the dry instability : the mixed boundary layer. To be studied in depth in the 5th lecture





I.7 Vertial motion of an air parcel Notations: **Displaced parcel p** - Thermodynamic properties of the ambient air : \bar{p} , $\bar{\rho}$, \bar{T} \boldsymbol{p}_{p} , $\boldsymbol{\rho}_{p}$, \boldsymbol{T}_{p} - Thermodynamic properties of the moving parcel : p_p , ρ_p , T_p <mark>Ambient air</mark> $ar{\mathbf{p}}$, $ar{
ho}$, $ar{\mathbf{T}}$ displacement Ambient air is in hydrostatic equilibrium: $\frac{\partial \bar{p}}{\partial z} + \bar{\rho} g = 0$ (1) δZ_n The pressure of the moving parcel equilibrates instantly initial position with the ambient pressure : $p_p = \overline{p}$ $\boldsymbol{z}_{p}, \boldsymbol{T}_{s} = \boldsymbol{\overline{T}}(\boldsymbol{z}_{p})$ The equation for the vertical motion of the air parcel is $\rho_p \frac{d^2 \delta z_p}{dt^2} + \rho_p g + \frac{\partial \bar{p}}{\partial z} = 0$ (2) Combining (1) and (2), we obtain $\frac{d^2 \delta z_p}{dt^2} = g \frac{\bar{\rho} - \rho_p}{\rho_p} = g \frac{T_p - \bar{T}}{\bar{T}} = \text{buoyancy}$ acceleration, the second equality being obtained by using the perfect gas law. 19

If T_s is the temperature of the parcel and the ambient air at ist initial location, the temperatures of the parcel and ambient air at the displaced location δz_p are perturbations of T_s , that is $\bar{T} = T_s + \delta \bar{T}$ and $T_p = T_s + \delta T_p$, hence $T_p - \bar{T} = \delta T_p - \delta \bar{T}$.

The potential temperature, being defined as $\theta = T \left(\frac{p_0}{p}\right)^{\kappa}$, leads to $\frac{\delta \bar{\theta}}{\bar{\theta}} = \frac{\delta \bar{T}}{\bar{T}} - \kappa \frac{\delta \bar{p}}{\bar{p}}$

for the ambient air et $\mathbf{0} = \frac{\delta T_p}{\overline{T}} - \kappa \frac{\delta \overline{p}}{\overline{p}}$ for the adiabatically displaced parcel

hence
$$\frac{d^2 \delta z_p}{dt^2} = -g \frac{\delta \overline{\theta}}{\overline{\theta}} = -N^2 \delta z_p$$
 with $N^2 = \frac{g}{\overline{\theta}} \left(\frac{\partial \overline{\theta}}{\partial z} \right)$.

N is the Brünt -Vaissala frequency

For $\frac{\partial \theta}{\partial z} > 0$, the Brünt-Vaissala is real and the parcel oscillates in the vertical with this pulsation.

For $\frac{\partial \overline{\theta}}{\partial z} < 0$, the atmosphere is unstable and convective motion is generated that stirs the air until mixing is performed and neutral equilibrium is restored.



In the troposphere, N is of the order of 10^{-2} s⁻¹ (10 minutes period) and twice larger in the stratosphere



Courtesy of A. Kottayil, ACARR





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- The vertical atmospheric displacements are adiabatic over a time scal comparable with convection, which leads to use potential temperature as a conserved quantity.
- The vertical stabillity of the atmosphere depends on the value of the actual lapse rate with respect to the adiabatic lapse rate. Instability occurs when the temperature decays with height faster than for the adiabatic lapse rate or when the potential temperature decays with altitude.

I Introduction et principes de base <u>II Convection humide et nuages</u>

II.1 Thermodynamic diagram of pure water



The conditions of Earth atmosphere are such that water can be found under its three phases.

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III.2 Moisture condensation

Water vapour in the air can be characterized by its partial presure soit par sa pression partielle e, or by its mass mixing ratio

$r_v = \rho_v / \rho_d = (e/p_d)(R_d/R_v)$

where the index d refers to dry air. The <u>saturation</u> partial pressure depends on the temperature through the Clausius-Clapeyron law d ln(e_s)/dT = L/R_vT² Approximate formula (in hPa) $e_{s}^{liquide} = 6,112 \exp(17,67 (T-273.15) / (T-29.65))$ $e_{s}^{glace} = \exp(23,33086-6111,72784/T + 0,15215 \ln(T))$

Exemples of saturating ratios at 1000hPa and T=20°C: $r_s = 14,5 g/kg$, at 800 hPa (2000m) and T = 7°C: $r_s = 7,8 g/kg$, at 500 hPa and T=-30°C $r_s = 0,47 g/kg$, at 100 hPa and T =-80°C $r_s = 0,003 g/kg$, (the atmospheric water content is divided by approximately 4 orders of magnitude between the ground and 100 hPa in the tropics)



LCL (lifting condensation level): level at which parcels rising from the ground condensate

III.3 Formation of convective clouds



cumulus

cumulonimbus







III.6 Large-scale cloud organisation

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IR false colour composite image, obtained from 5 Geostationary satellites on 22/09/2005 18:00TU (GOES-10 (1350), GOES-12 (750), METEOSAT-7 (OE), METEOSAT-5 (63E), MTSAT (140E))



Cloud cover from ISSCP

January – July comparison



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Observed distribution of clouds





CALIPSO lidar (launched 2006)



(b) CLOUD FRACTION JFM GOCCP



Chepfer et al., JGR 2010

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In the mid and high latitudes, poleward isentropic motion is accompanied by upward motion. Hence the cloud bands.

In the tropic, vertical upward motion needs heating. Convection organizes as compact clusters.

Moist air thermodynamics

Two gas phases, dry air(d), water vapour (v), one liquid phase (l) and one ice phase (i) pressure $p = p_d + e$ (dry air + water vapour pressure denoted as e) mass mixing ratio $r = \frac{\rho_v}{\rho_l}, r_l = \frac{\rho_l}{\rho_l}, r_i = \frac{\rho_i}{\rho_l}, r_T = r + r_l + r_i$ $M_{d} = 29 g$ $M_{y} = 18 g$ $R_{d} = 287 J kg^{-1} K^{-1} \qquad R_{v} = 461.5 J kg^{-1} K^{-1}$ $C_{pd} = 1005.7 J kg^{-1} K^{-1} \qquad C_{pv} = 1870 J kg^{-1} K^{-1}$ $C_{i} = 4190 J kg^{-1} K^{-1}$ at $T > 0 \circ C$ $C_{i} = 2106 J kg^{-1} K^{-1}$ at $T \approx 0 \circ C$ $\frac{R_d}{R_v} = \epsilon = 0.622 \qquad \qquad \frac{C_{pv}}{C_{pd}} = \beta = 1.86 \qquad \qquad \kappa = \frac{R_d}{C_{pd}} = 0.285$ $r = \frac{e/(R_v T)}{p_d/(R_d T)} = \epsilon \frac{e}{p-e} \qquad \qquad p_d = p \frac{\epsilon}{\epsilon + r} \qquad \qquad e = p \frac{r}{\epsilon + r} = p_d \frac{r}{\epsilon}$ saturation pressure e^s , saturating mixing ratio r^s (function of p and T) relative humidity $H \equiv \frac{e}{e^{S}} = \frac{r}{r^{S}} \left(\frac{1 + r^{S}/\epsilon}{1 + r/\epsilon} \right)$ specific volume $\alpha \equiv \frac{1}{\rho} = \frac{V_a + V_l + V_i}{m_d + m_u + m_l + m_l} = \alpha_d \left(\frac{1 + r_l(\alpha_l / \alpha_d) + r_i(\alpha_l / \alpha_d)}{1 + r_u} \right) \simeq \frac{\alpha_d}{1 + r_u}$

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 $\alpha \simeq \frac{R_d T}{p_d} \frac{1}{1+r_T} = \frac{R_d T}{p} \frac{1+r/\epsilon}{1+r_T}$

Non saturated air: notion of virtual temperature $T_{\nu} \equiv T \frac{1+r/\epsilon}{1+r} \simeq T(1+0,608 r)$ Saturated air: notion of density temperature $T_{\rho} \equiv T \frac{1+r^{S}/\epsilon}{1+r_{T}} = T_{\nu} \frac{1+r^{S}}{1+r_{T}}$ T_{ν} is the temperature of dry air with the same density as moist air

in the <u>unsaturated case</u>: $p = \rho R_d T_v$

 T_{ρ} is the temperature of the dry air with the same density as moist air in the <u>saturated case</u>: $p = \rho R_d T_{\rho}$

In the tropical regions where r may reach 0,04, the difference between T and T_{v} may reach 2,5%.

 T_{ν} is always larger than T. This is not always true for T_{ρ} which may be smaller than T when the load in liquid water is high.

In the unsaturated case, $\frac{1}{p}\frac{dp}{dz} = -\frac{g}{R_d T_v}$

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Moist saturated air thermodynamics - Basic laws

Equilibrium of temperature T, and free energy g at constant pressure between the two phases, with $g=u+p\alpha-Ts=h-Ts$, $h=u+p\alpha$, u et h being only function of T for a perfect gas and with $p=e^{s}$ for the water vapour phase. The latent heat is $L=h_{v}^{s}-h_{l}=T(s_{v}^{s}-s_{l})$

Kirchhoff law

$$d L = dT \left[\left(\frac{\partial h_{v}}{\partial T} \right)_{p} - \left(\frac{\partial h_{l}}{\partial T} \right)_{p} \right] + dp \left[\left(\frac{\partial h_{v}}{\partial p} \right)_{T} - \left(\frac{\partial h_{l}}{\partial p} \right)_{T} \right]$$
Perfect gas law
$$= dT \left[C_{pv} - C_{l} \right] + dp \left[-\alpha_{l} - p \left(\frac{\partial \alpha_{l}}{\partial p} \right)_{T} \right]$$
Negligible specific volume of the condensed phase
$$\frac{d L}{dT} = C_{pv} - C_{l}$$
vaporization $L_{0} = 2,5 \times 10^{6} \text{ J kg}^{-1} \text{ à } 0^{\circ}C.$

<u>Clausius-Clapeyron law</u>

For a variation of the equilibrium of the two phases: $dg_v = dg_l$ Using the definition of g and the first law of thermodynamics $T ds = du + p d\alpha$ $-s_v dT + \alpha_v de^s = -s_l dT + \alpha_l de^s$ $\frac{de^s}{dT} = \frac{s_v - s_l}{\alpha_v - \alpha_l} = \frac{L}{T(\alpha_v - \alpha_l)} \approx \frac{Le^s}{R_v T^2}$

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Application :

Lifting condensation level for a parcel lifted adiabatically from the surface knowing the temperature and the moisture at the surface.

Let define the relative humidity as $H = e/e_s$. Saturation is reached when H = 1. The mass mixing ratio at the ground (preserved during unsaturated ascent) is rWe have $d \ln(H) = d \ln(e) - d \ln(e_s)$

Using the conservation of potential temperature $d \ln(e) = d \ln(p) = \frac{1}{\kappa} \frac{1+r\beta}{1+r/c} d \ln(T)$ Using Clapeyron and Kirkhhoff laws: $d \ln(e_s) = \frac{L}{R T^2} dT = \frac{L_0 + (C_{pv} - C_l)(T - T_0)}{R T^2} dT$ Hence $0 -\ln(\boldsymbol{H}_g) = \left(\frac{1}{\kappa}\frac{1+r\beta}{1+r/\epsilon} + \frac{\boldsymbol{C}_l - \boldsymbol{C}_{pv}}{\boldsymbol{R}_v}\right)\ln\frac{\boldsymbol{T}^*}{\boldsymbol{T}_s} + \left(\frac{\boldsymbol{L}_0 + (\boldsymbol{C}_l - \boldsymbol{C}_{pv})\boldsymbol{T}_0}{\boldsymbol{R}_v}\right)\left(\frac{1}{\boldsymbol{T}^*} - \frac{1}{\boldsymbol{T}_s}\right)$ where T^* is the temperature of the parcel at the LCL and the indice g refers to values at the ground. Approximate solution $T^* = \frac{2840}{3.5 \ln(T_a) - \ln(e_a) - 4.805} + 55$ with e_g in hPa. The pressure at the LCL p^* is then given by $\ln \frac{p^*}{p} = \frac{1}{\kappa} \frac{1+r\beta}{1+r/\epsilon} \ln \frac{T^*}{T}$ and the altitude at the LCL can be determined by integrating $\frac{dT}{dz} = \frac{-g}{C} \frac{1+r}{1+r\beta}$ hence $z^* - z_g = \frac{C_{pd}}{\sigma} \frac{1 + r\beta}{1 + r} (T_g - T^*)$

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Equivalent potential temperature

For a parcel of humid air, the entropy per unit mass of dry air is $s = s_d + r s_v + r_l s_l$ with r_l and r_v the mixing ratios for liquid and vapour water # $s_d = C_{pd} \ln(T/T_0) - R_d \ln(p_d/p_0)$ for the dry air, # $s_v = C_{pv} \ln(T/T_0) - R_v \ln(e/p_0)$ for water vapour, # $s_1 = C_1 \ln(T/T_0)$ for liquid water. Using $L = T(s_v^S - s_l)$ and $H = e/e^S$, and after a few manipulations (using $r_T = r_l + r_V$): $s = s_d + r s_v + r_l s_l = s_d + r (s_v - s_v^S) + r (s_v^S - s_l) + r_T s_l = s_d + r_T s_l + \frac{Lr}{T} + r (s_v - s_v^S)$ = $(C_{pd} + r_T C_l) \ln(T/T_0) - R_d \ln(p_d/p_0) + \frac{Lr}{T} - r R_v \ln(H)$ The equivalent potential temperature θ_e can be defined such that $s = (C_{pd} + r_T C_l) \ln(\theta_e / T_0)$ hence $\theta_e = T\left(\frac{p_0}{p_d}\right)^{R_d/(C_{pd}+r_TC_l)} (H)^{-rR_v/(C_{pd}+r_TC_l)} \exp\left(\frac{Lr}{(C_{pd}+r_TC_l)T}\right)$ This quantity is conserved under both saturated and non saturated moist adiabatic transforms where condensates are carried aloft. For a saturated parcel, $\theta_e = T\left(\frac{p_0}{p_d}\right)^{R_d/(C_{pd}+r_TC_l)} \exp\left(\frac{Lr^S}{(C_{pd}+r_TC_l)T}\right)$ function of (T, p_d, r_T) . case, Since $r = r_T$ in the unsaturated, θ_e is always a function of (T, p_d, r_T)

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Question : How is defined an equivalent potential temperature for a non saturated air outside of the cloud layer.

Response : It is defined as the equivalet potential temperature of the saturated air obtained by a diabatic detent of the air parcel until its level of condensation.

In this way, the equivalent potential temperature is conserved along the whole trajectory of the parcel from its initial unsaturated state below the cloud to its saturated state inside, under the hypothesis

- adiabatic : no heat exchange

- quasi-static reversible: going through a series of thermodynamic equilibrium states

- transport of the condensates within the parcel: no rain Any violation of these hypothesis generate an augmentation of the entropy of the parcel + environment (the entropy of the parcel itself can increase or decrease)

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The negative slope of the equivalent potential temperature in the first five km of the tropical troposphere induces ubiquitous deep convective activity.





Observed cloud fraction (CALIOP lidar) in the tropical region (Fu et al., GRL 2007)

Probability density and cumulated probability of θ_e in tropical region

0,5% of the clouds reach the tropical troppause (17,5 km, 100 hPa, T=200K, θ =380K)

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