Atmospheric dynamics and meteorology

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II Potential vorticity, tropopause and baroclinic instability

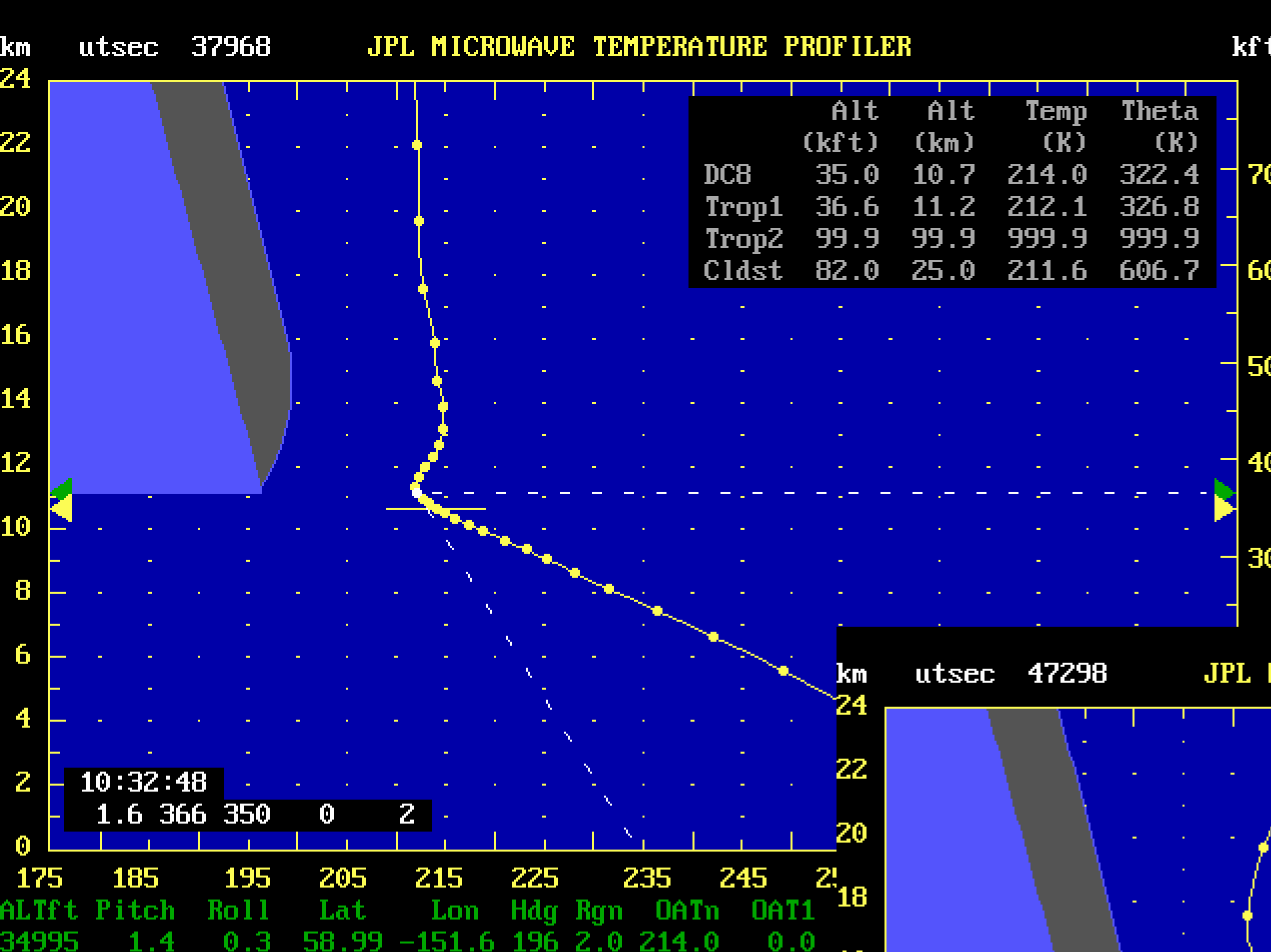
(supposed to be known: notions on the conservation of potential vorticity, simple model of the baroclinic instability (Eady / Phillips))

Recommended documents:
- Hoskins, McIntyre & Robertson, 1985, On the use and significance of isentropic potential vorticity maps, *Quart. J. Met. Soc.*, **111**, 877-946

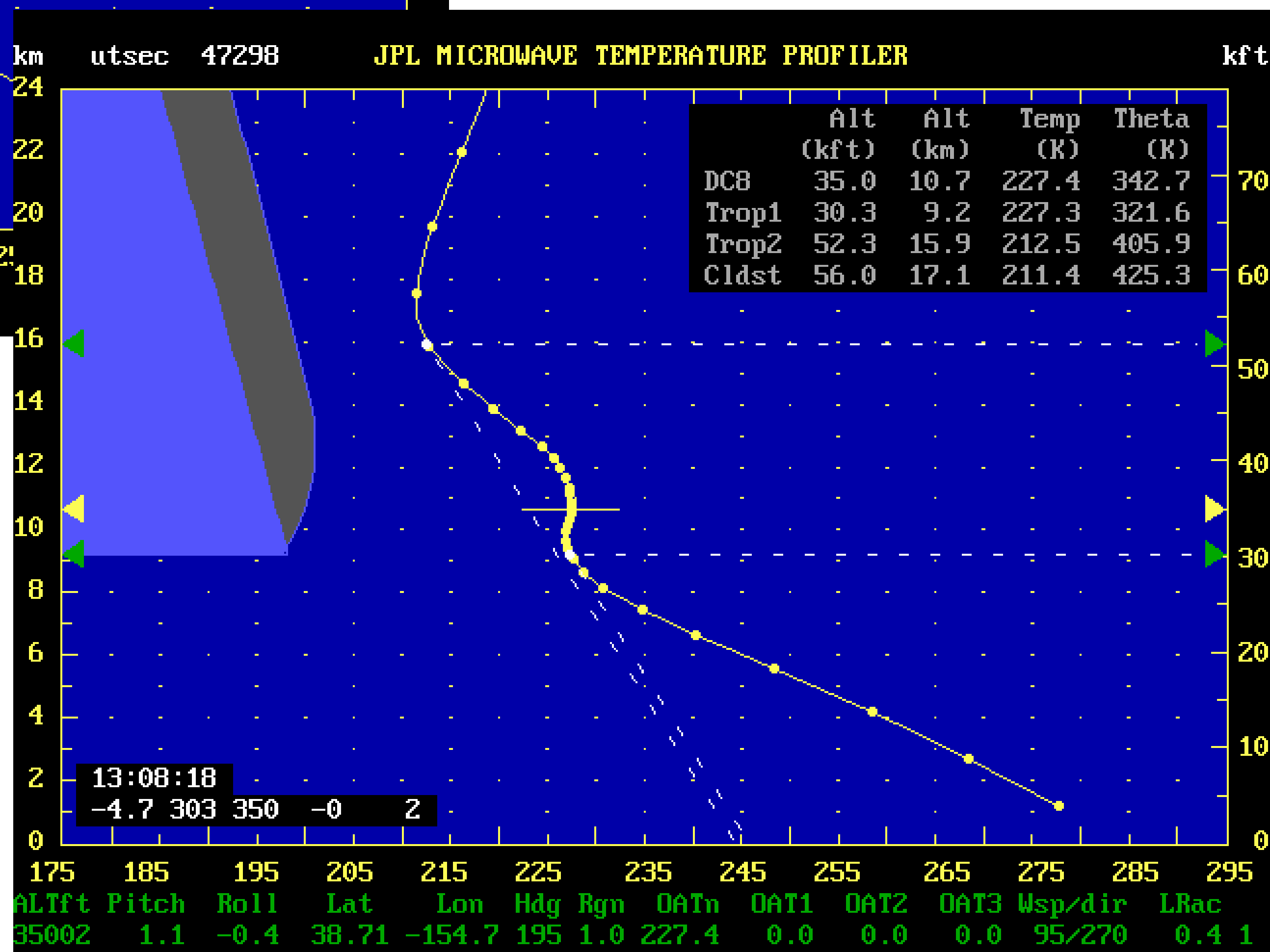
Updated
09/11/2023

Outline

1 The thermal tropopause
2 The potential vorticity
3 The dynamic tropopause
4 The anomalies of potential vorticity
5 Mutual amplification
6 The baroclinic instabillity revisited



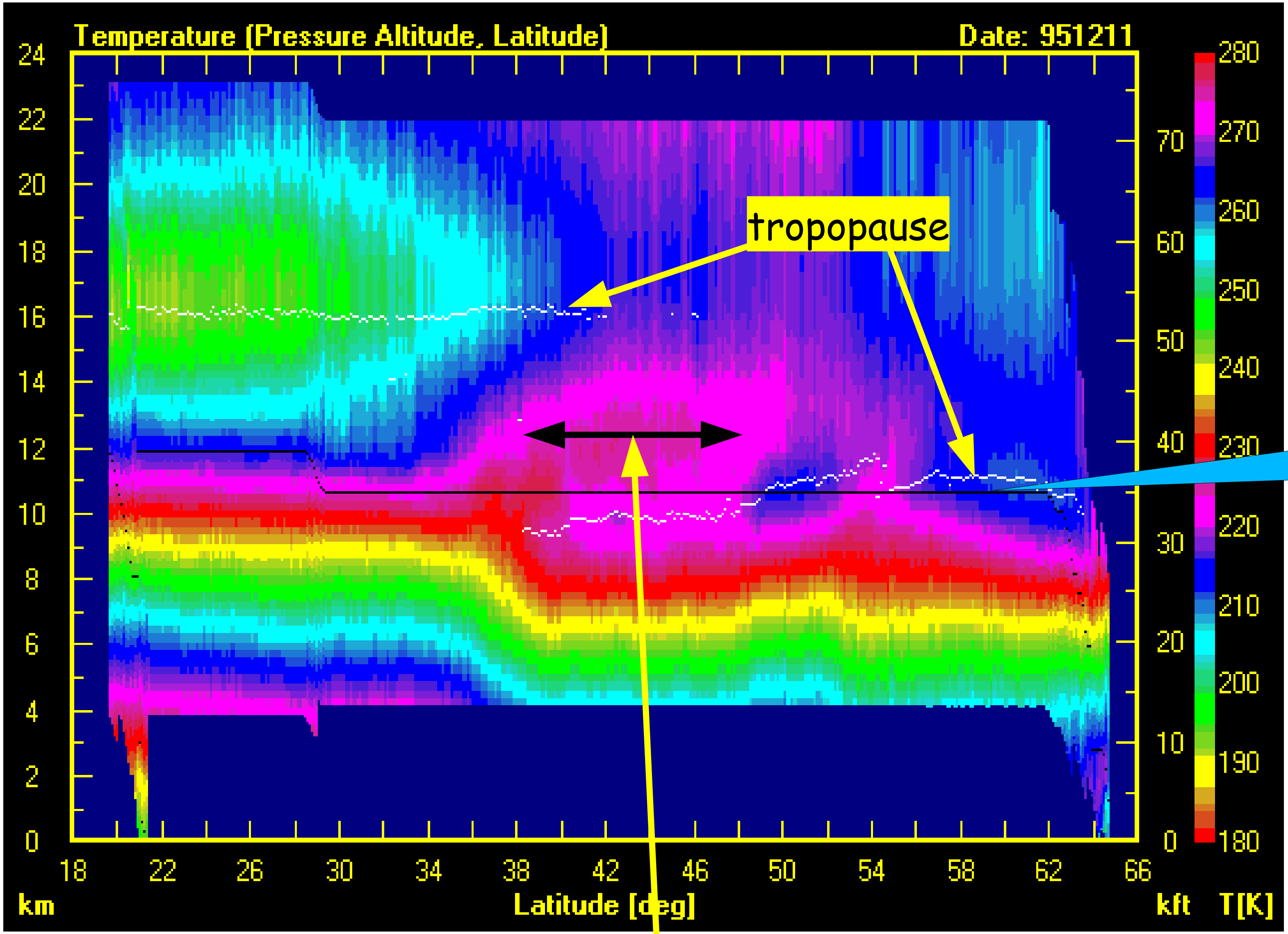
1The thermal tropopause



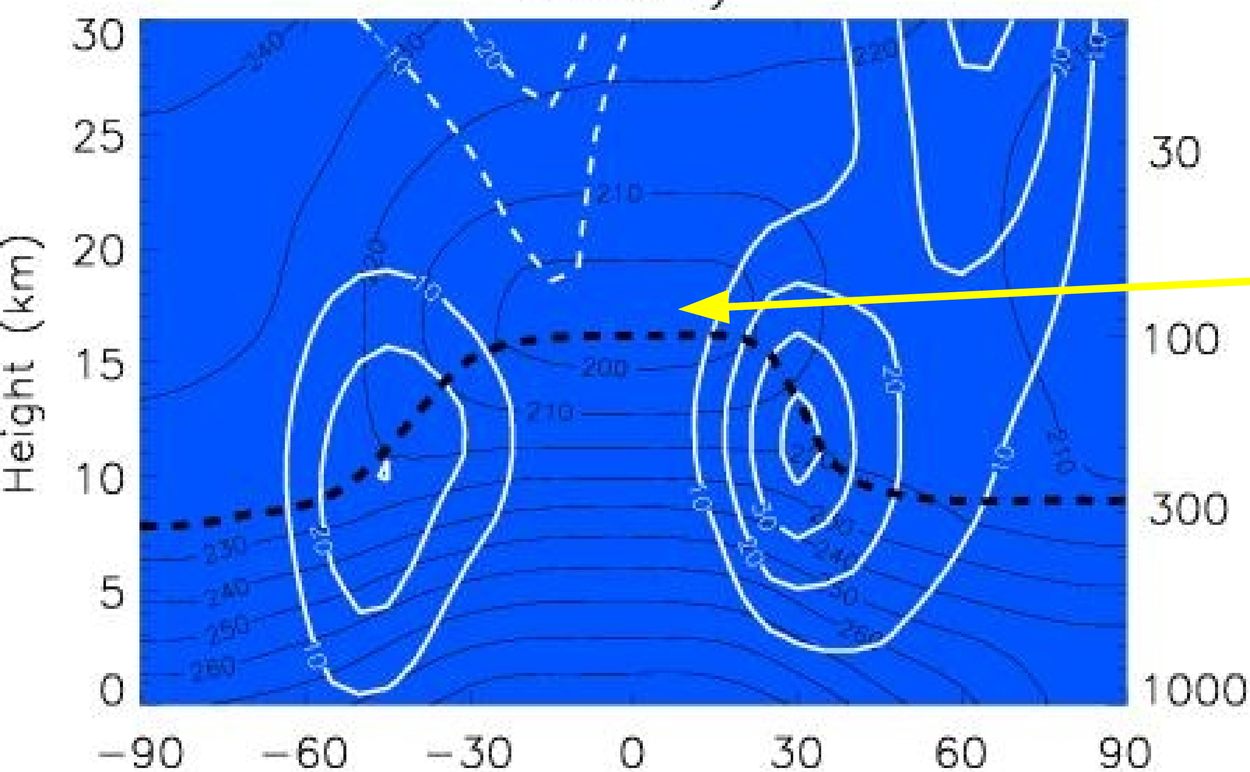
Simple or double tropopause

Meridional variation of the tropopause

Airborne measurement by radiometer.



Region of the double tropopause in the subtropics



Wind and temperature at the tropopause

Temperature minima at the tropical tropopause

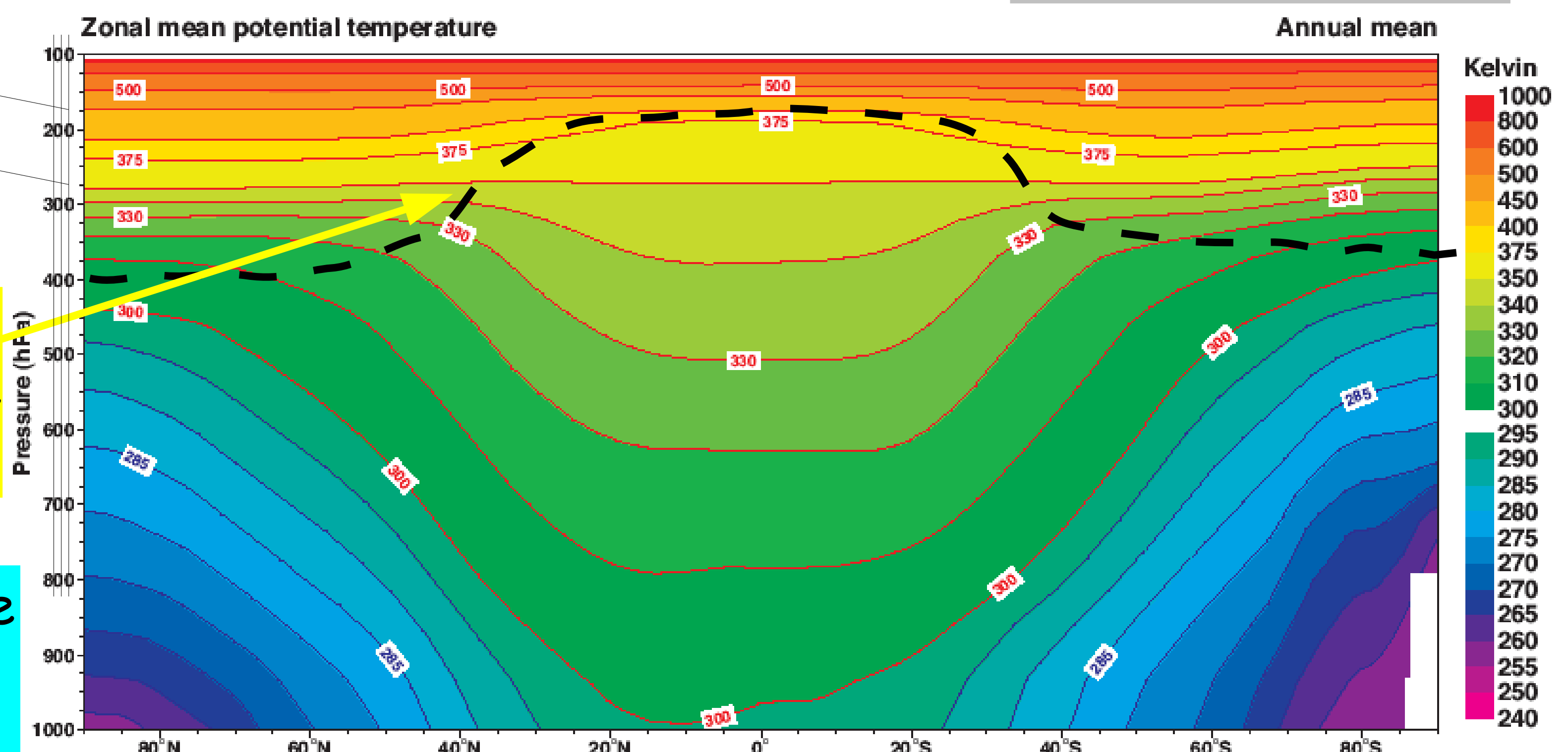
Subtropical jet winds associated with tropopause drop

ERA-40 Atlas, 2005

B. Randell

100 Hpa

200 Hpa



Isentropic surfaces crossing the tropopause in the subtropics

Potential temperature and the tropopause

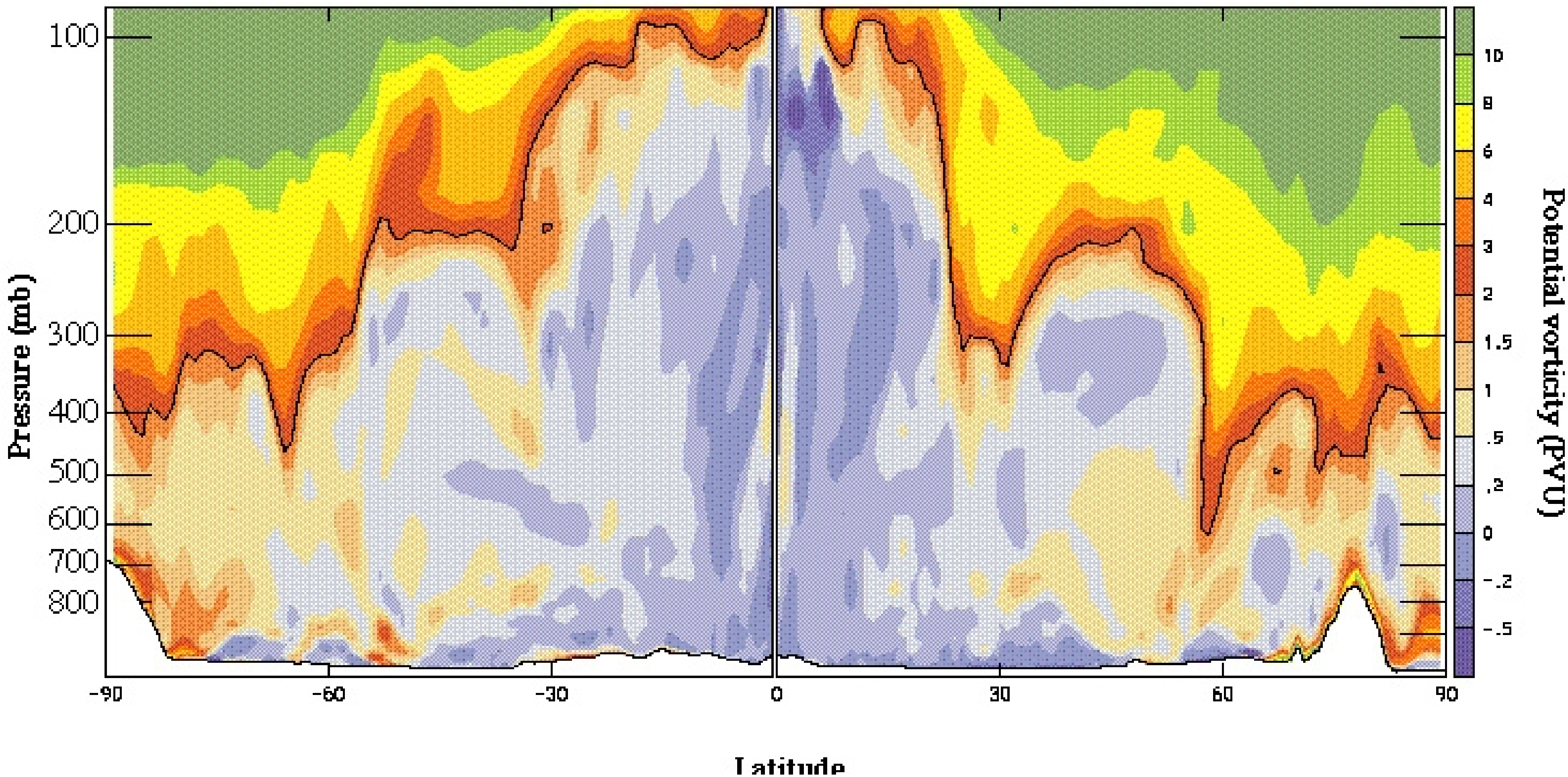
2 The potential vorticity

Ertel potential vorticity

$$\frac{D}{Dt}\left(\frac{\vec{\zeta}_a \cdot \vec{\nabla}\theta}{\rho}\right) = 0$$

for a flow without friction or heating.

Simplified forms:.



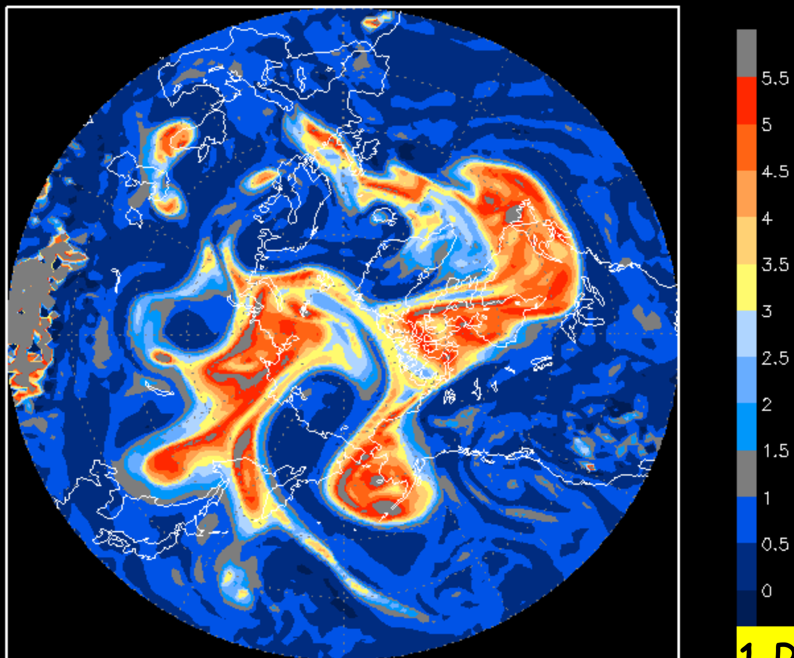
Instantaneous height-latitude cross section of potential vorticity along a single longitude (55W), with the tropopause marked (in black) as the 2PVU contour.

Courtesy of H. Wernli, ETH Zurich

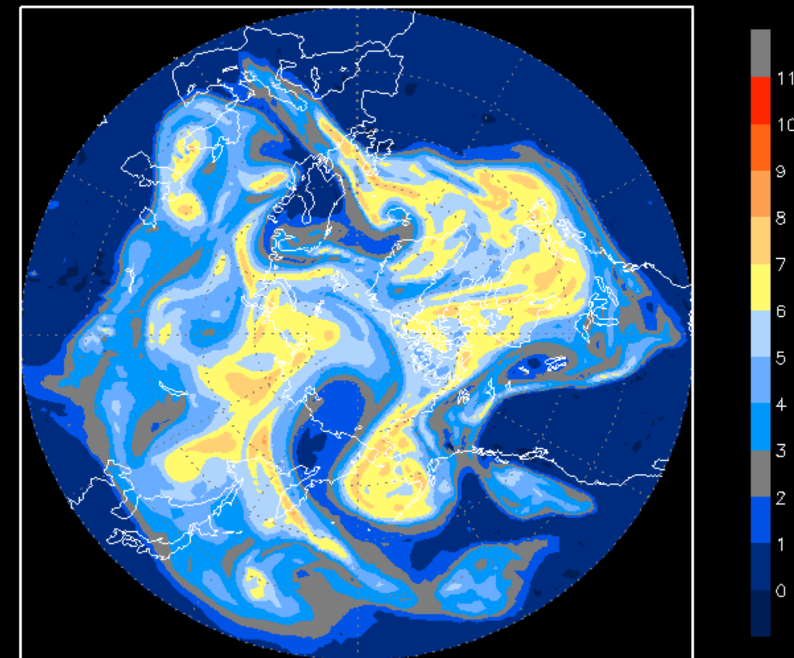
PV increases in the stratosphere due to increase of static stability. It increases in latitude due to f.

PV definition of the tropopause

Potential Vorticity at 300K (PVU) 18Z09JAN2004

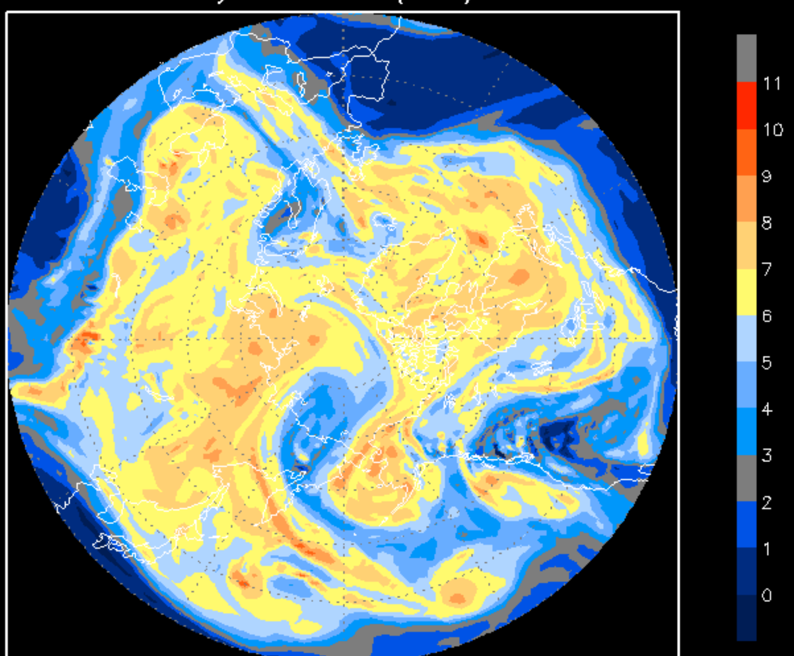


Potential Vorticity at 315K (PVU) 18Z09JAN2004

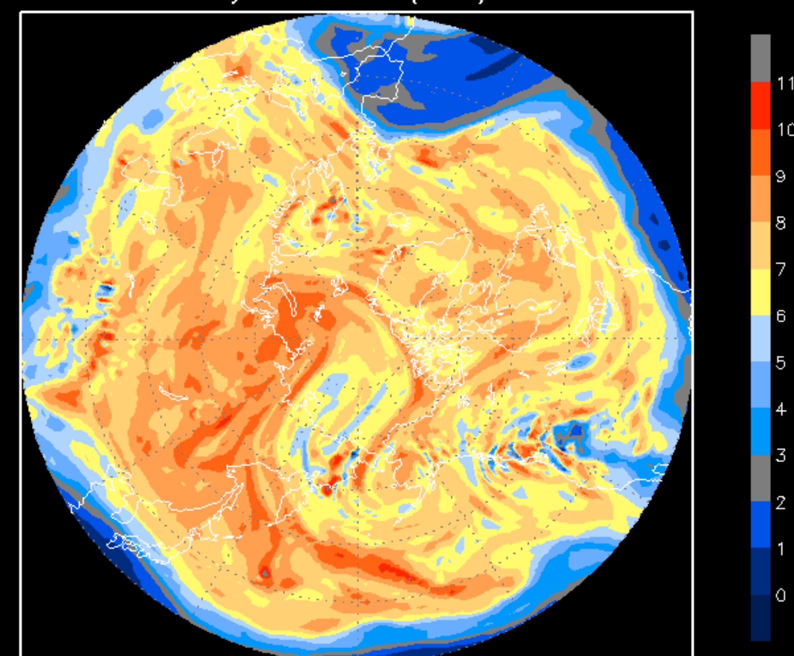


1 PVU = 10^{-6} K kg^{-1} m^2 s^{-1}

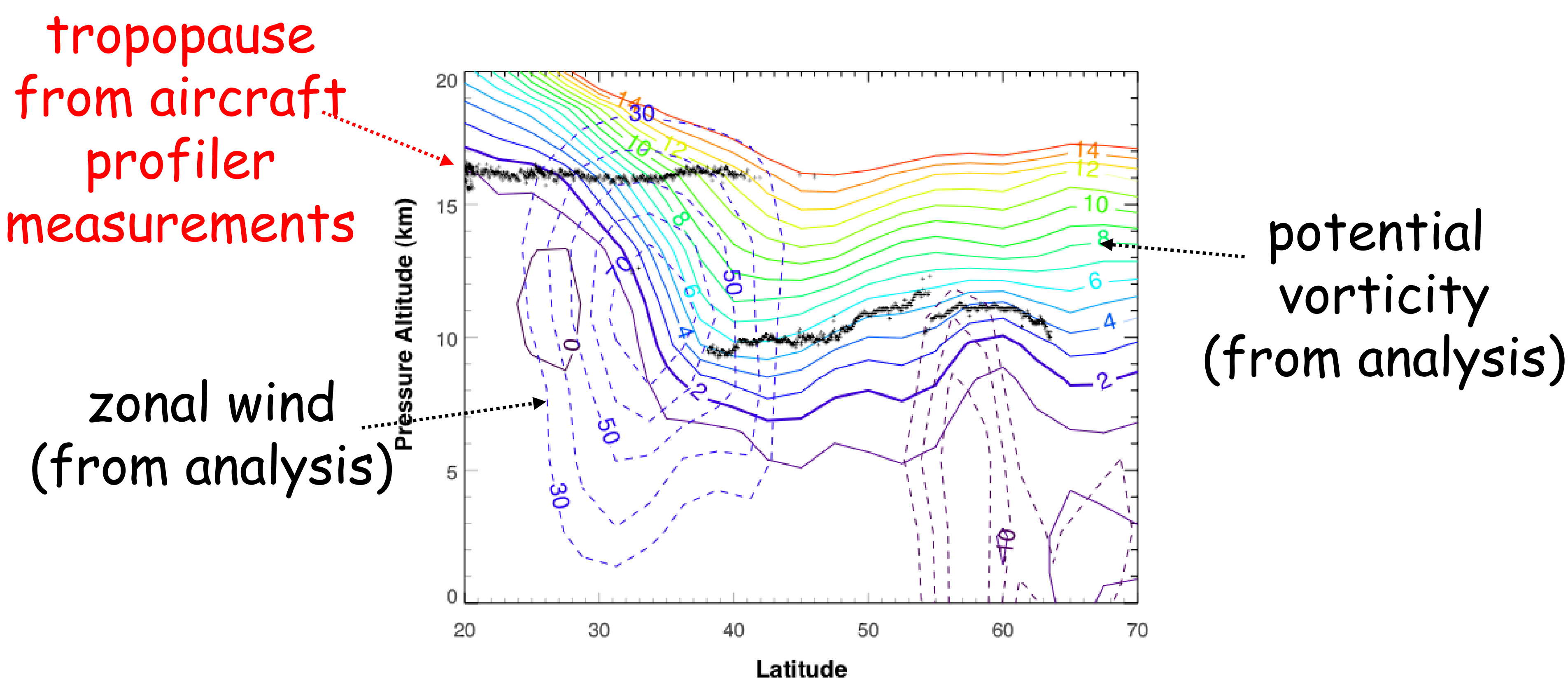
Potential Vorticity at 330K (PVU) 18Z09JAN2004



Potential Vorticity at 350K (PVU) 18Z09JAN2004



Relation between the double tropopause and the PV distribution

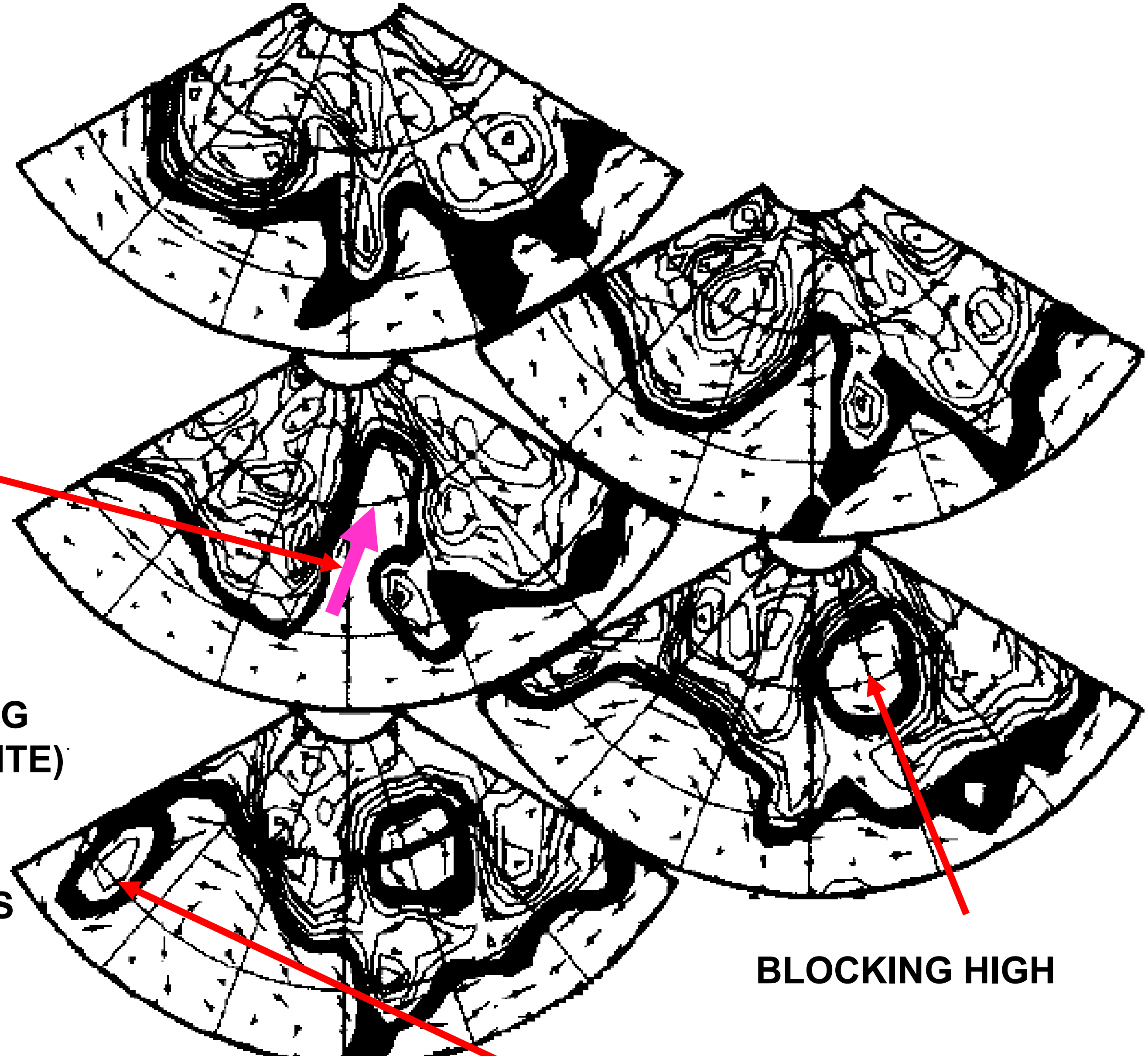


from Pan et al., JGR, 2004

3 Potential vorticity anomalies

Cut-off Lows and Blocking Highs

IPV on 330K for consecutive days. 1-2PVU filled in black



ADVECTION OF LOW IPV

IPV A BETTER WAY OF DEFINING "CUT-OFF" THAN 500hPa HEIGHT

CUT-OFF LOWS AND BLOCKING HIGHS HAVE SIMILAR (OPPOSITE) STRUCTURES

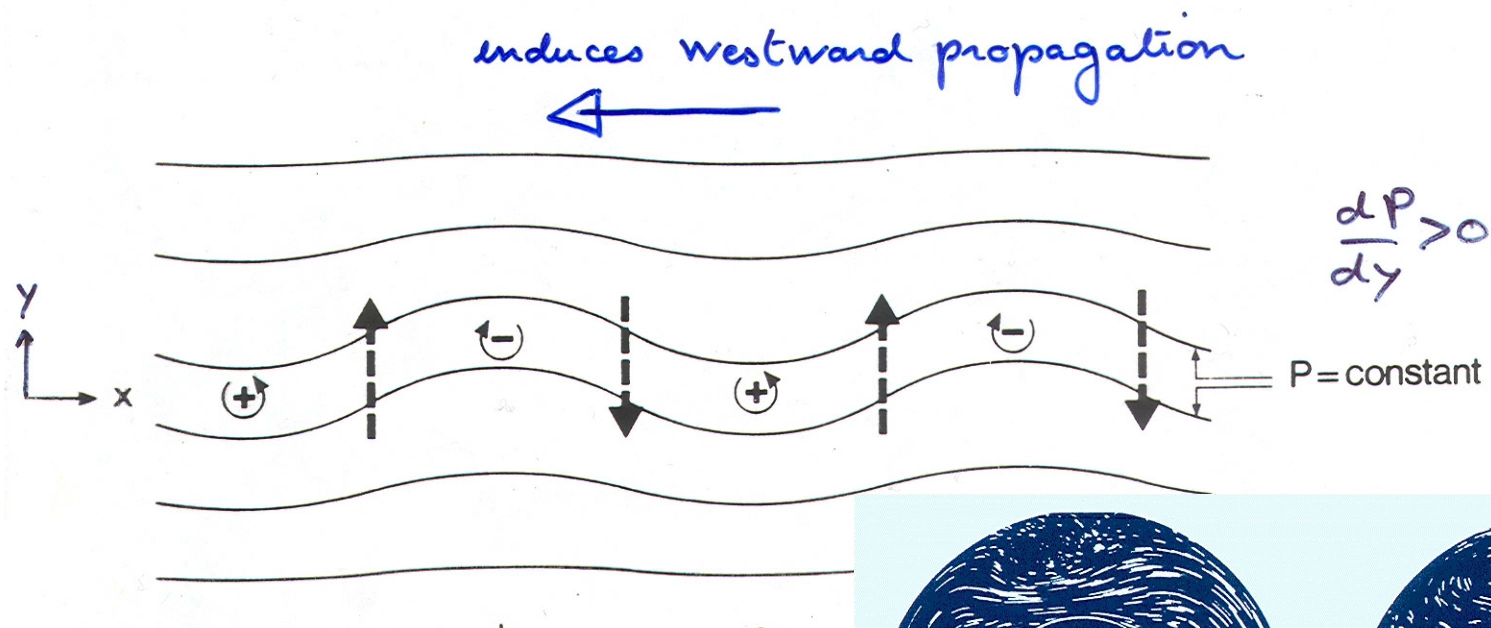
CONVECTIVE HEATING DAMPS CYCLONE FASTER THAN RADIATIVE COOLING DAMPS ANTICYCLONE

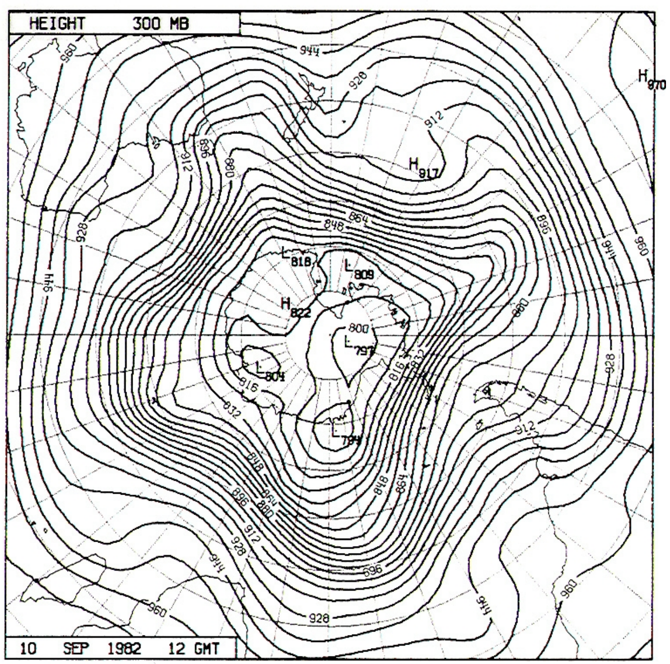
BLOCKING HIGH

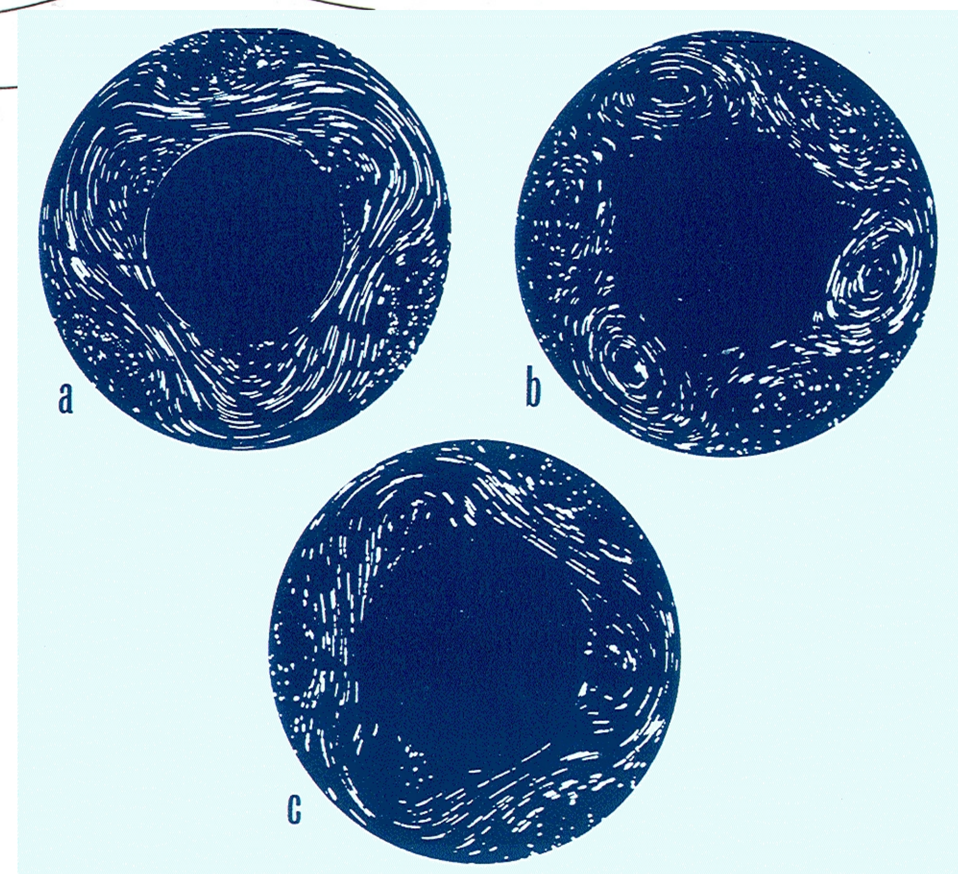
CUT-OFF LOW (HIGH PV)

Hoskins, McIntyre and Robertson (1985) Fig. 11

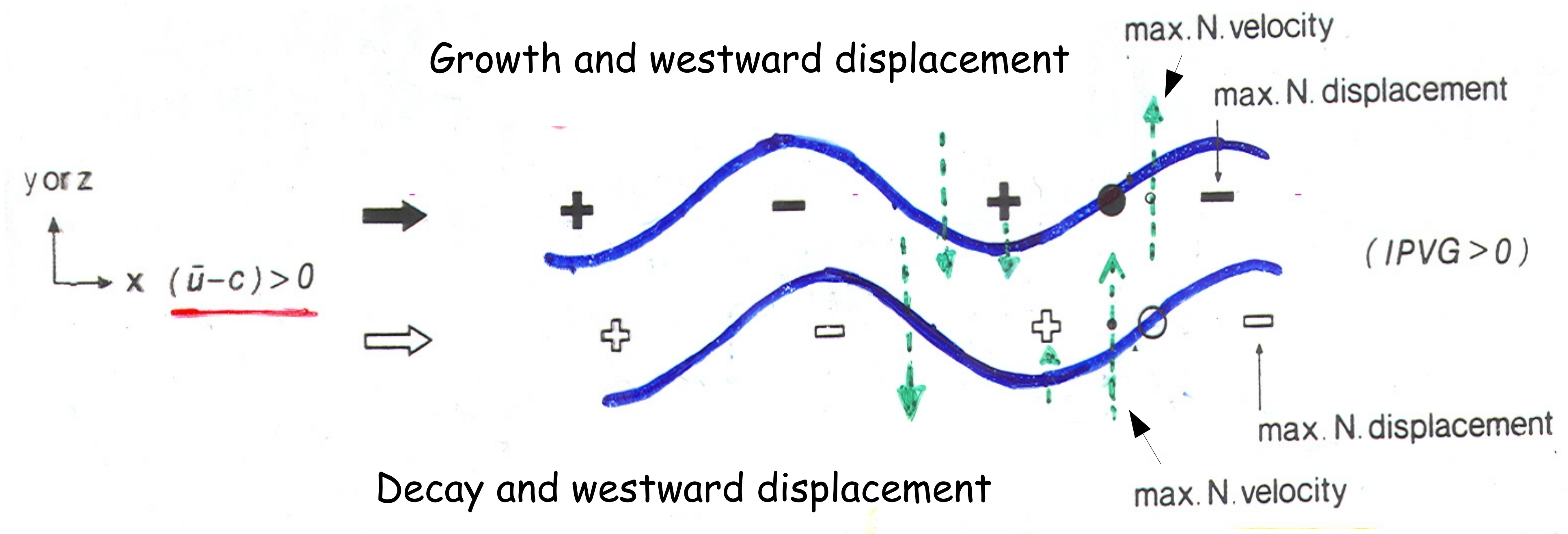
Rossby waves in a PV gradient







Vertical or lateral propagation of a wave



Effect of an axisymmetric PV anomaly localized at the tropopause.

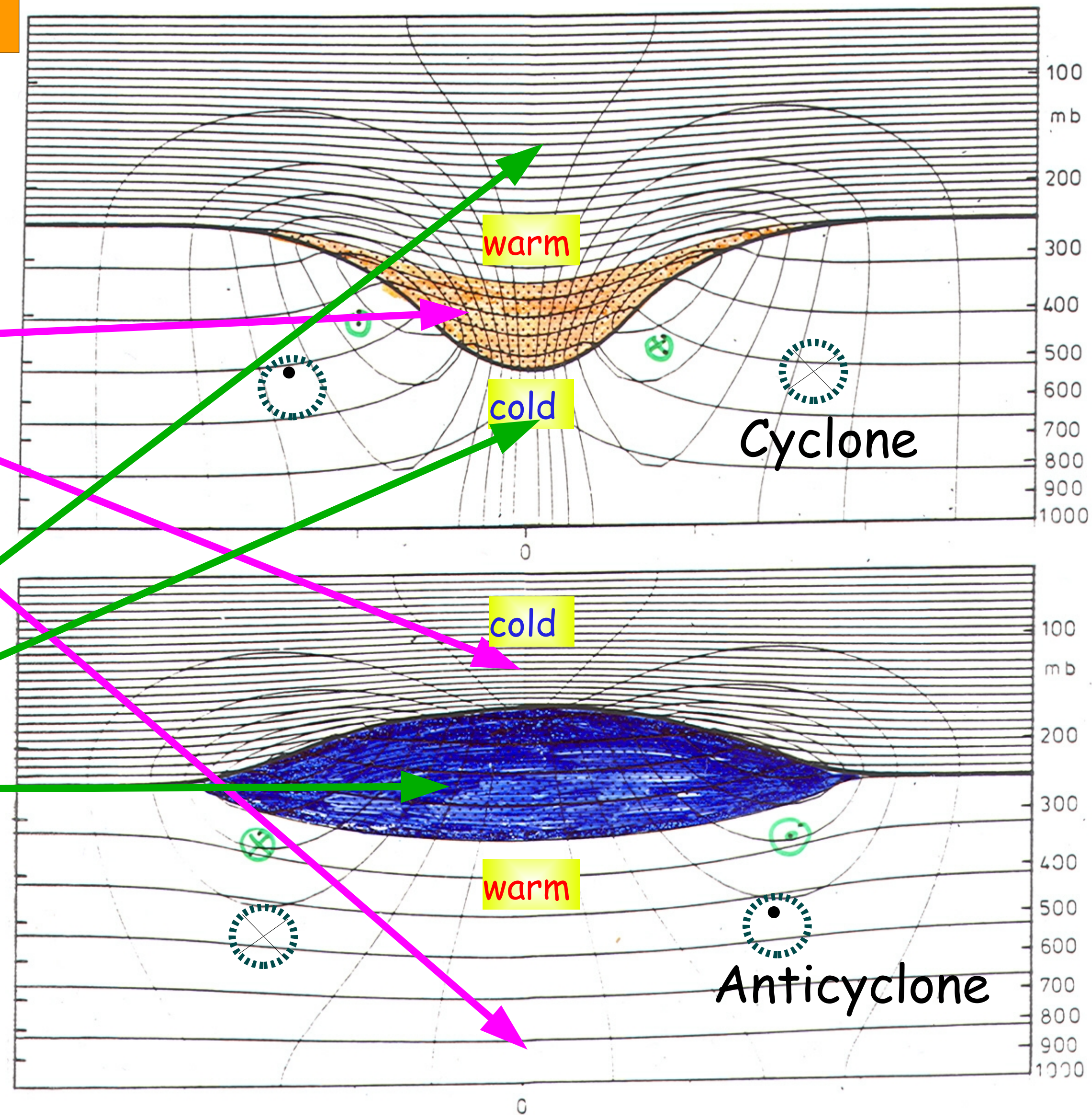
Upper panel : positive anomaly
Lower panel : negative anomaly

Augmented stability

Reduced stability

Inside the colored area: PV anomaly. Outside : no anomaly.de PV.
Background PV : uniform stratification in the horizontal with a discontinuity at the tropopause. Surfaces are drawn with an interval Δθ=5K

Section of the PV distribution
+ wind (contours)



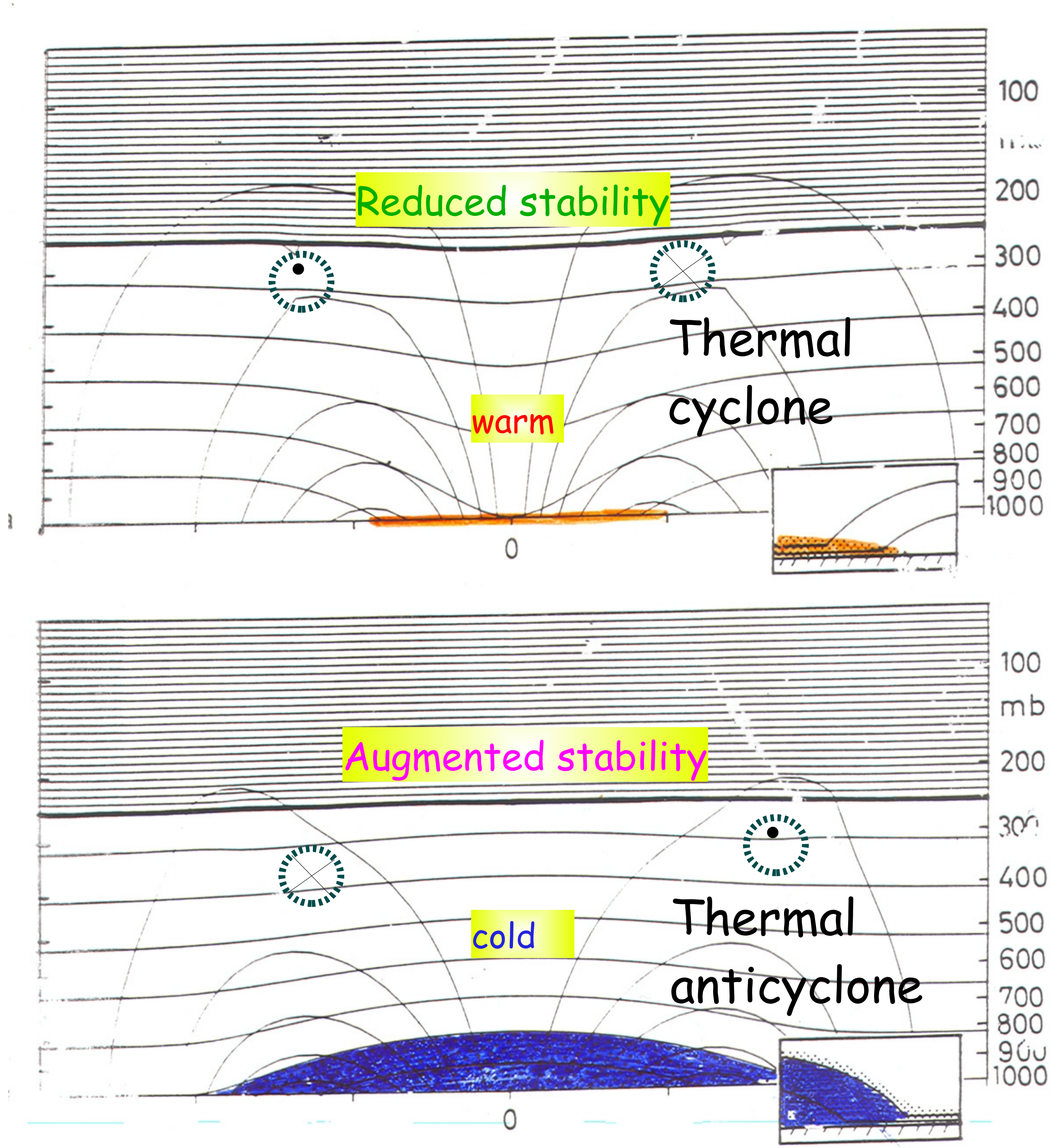
PV +

PV -

Hoskins, McIntyre and Robertson, 1985

Effect of a ground temperature anomaly
Upper panel: warm anomaly, <> positive PV anomaly
Lower panel : cold anomaly, <> negative PV anomaly

Inside colored areas: PV anomaly.
Outside : no PV anomaly.
Potential temperature surfaces every 5K.
Contours : azimuthal horizontal wind.



Amplification of a PV anomaly by mutual advection

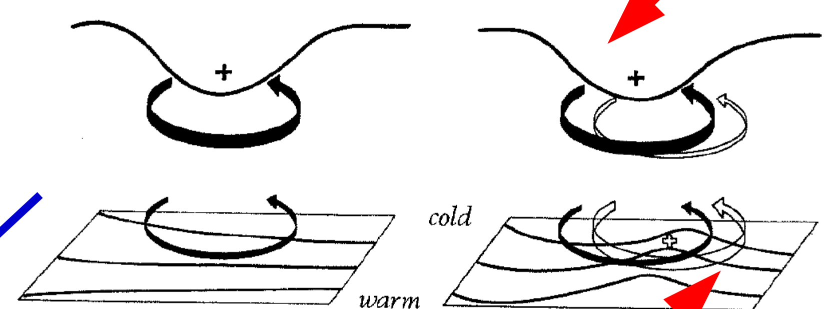
- The warm surface anomalies acts as positive PV anomalies near the surface
- PV anomalies induce a flow in the same way an electric charge induces an electric field.
- The vertical penetration and the magnitude of the flow induced by a PV anomaly grow with its horizontal scale.
- The flows induced by 2 PV anomalies can be approximately superimposed
- The induced flows can generate mutual amplification if the positions of the two anomalies are shifted with the upper one on the west of the lower one and if they get locked.

top $\frac{\partial}{\partial t} PV + v_h \frac{\partial}{\partial y} PV = 0$

bottom $\frac{\partial}{\partial t} T + v_b \frac{\partial}{\partial y} T = 0$

meridional PV gradient at the tropopause

+ PV advection



T surface gradient

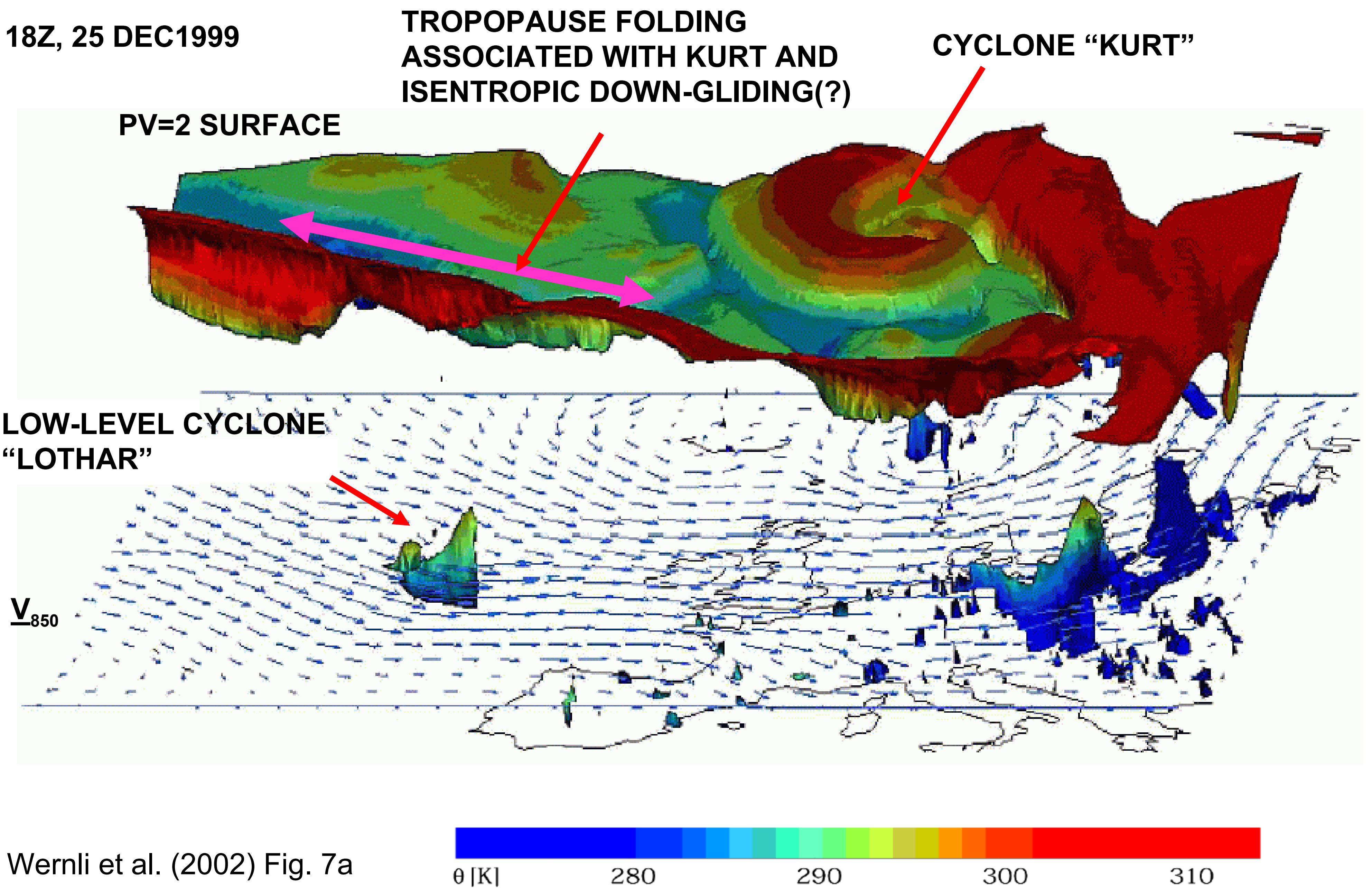
T advection

Hoskins, McIntyre and Robertson (1985) Fig. 21

$(+)\mathrm{IPV}\ (\equiv -g\,\zeta_\theta\,\partial\theta/\partial p)$

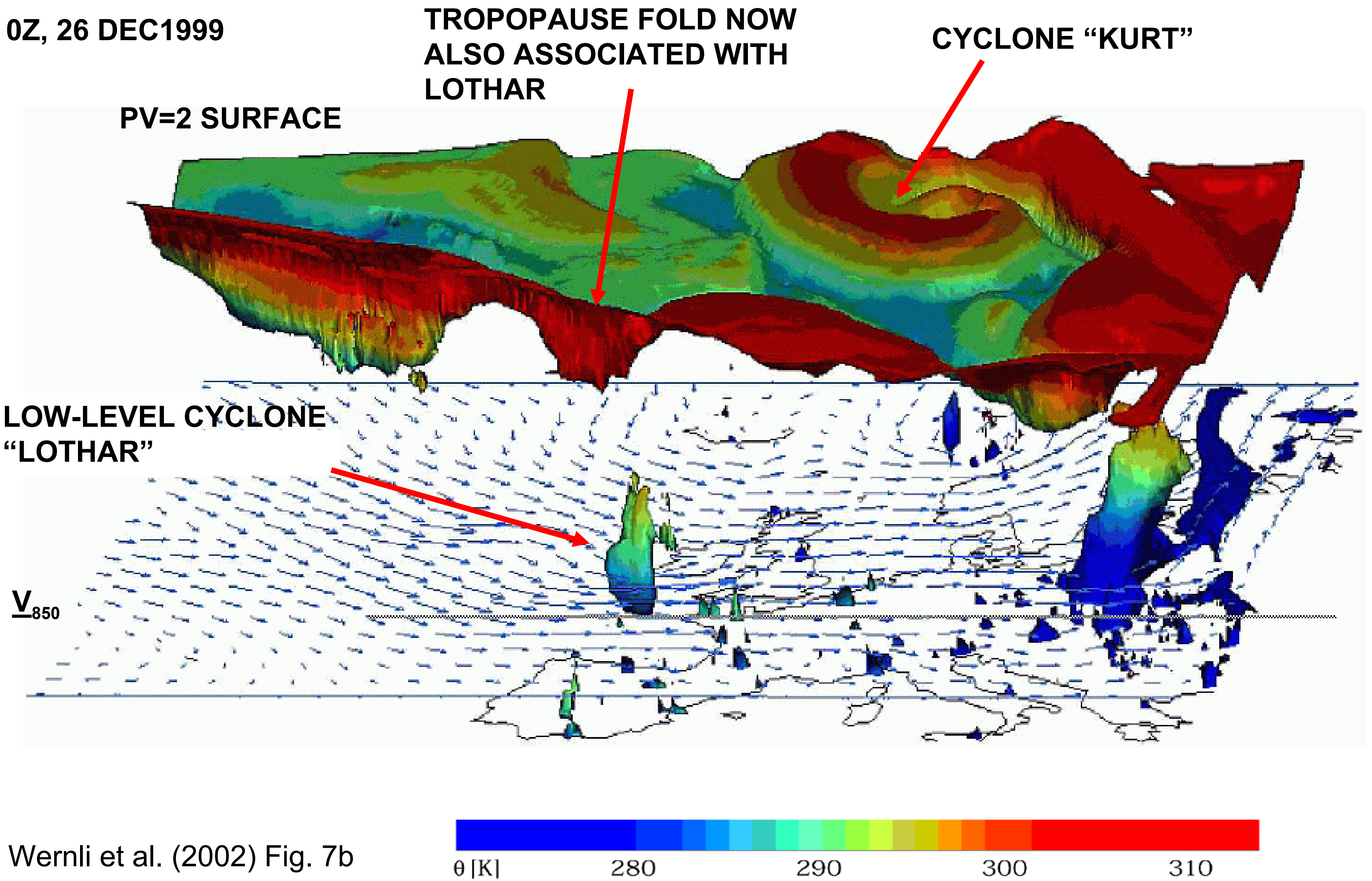
4. Mutual amplification

Analysis of winter storm “Lothar”



Wernli et al. (2002) Fig. 7a

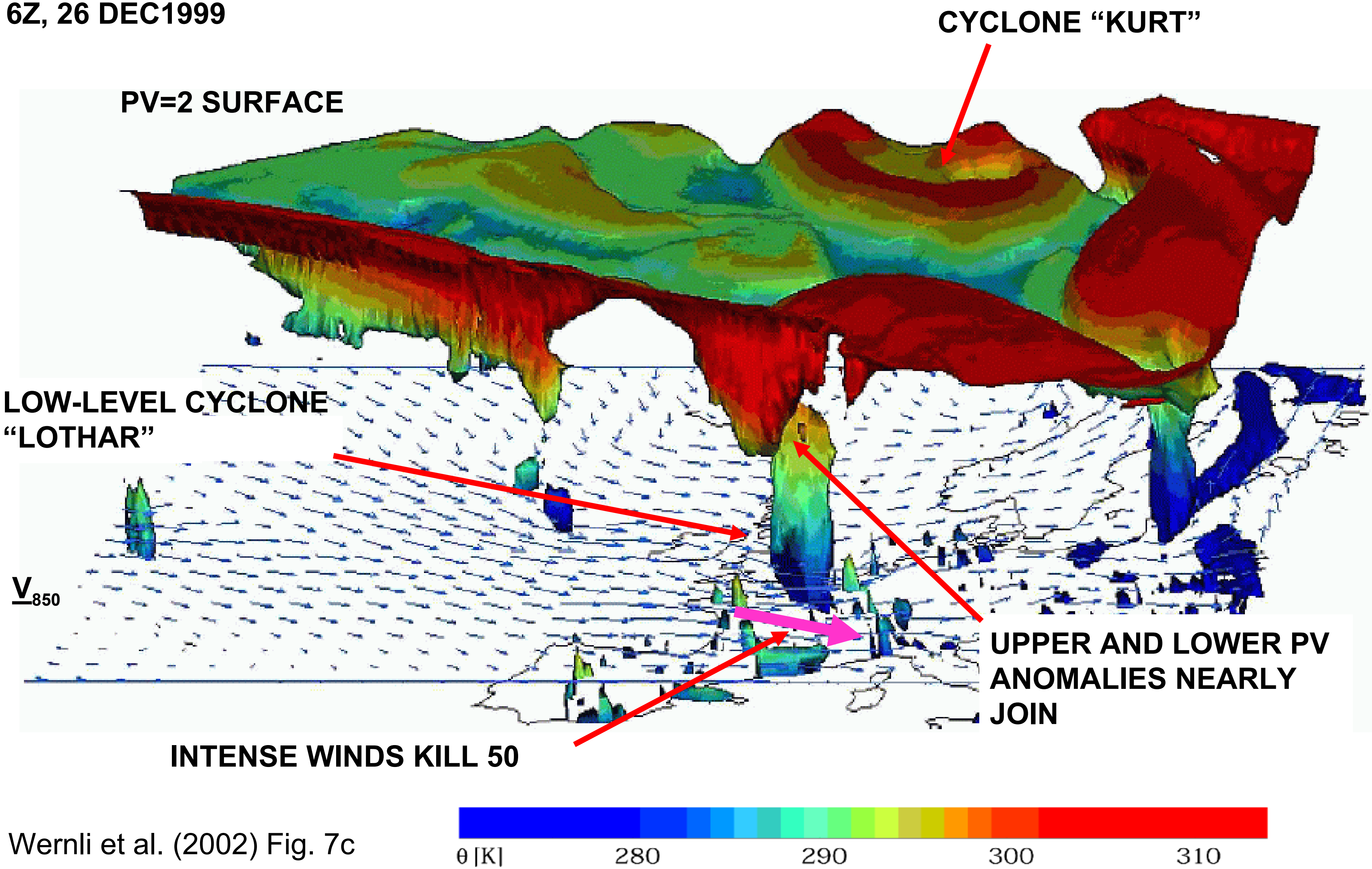
Analysis of winter storm “Lothar”



Wernli et al. (2002) Fig. 7b

Analysis of winter storm "Lothar"

6Z, 26 DEC1999



Wernli et al. (2002) Fig. 7c

Growth of a barotropic or baroclinic mode coupling two waves within opposite gradients of PV.

Mutual amplificaton
Locking by mutual increase of the phase speed.

5. Baroclinic instability

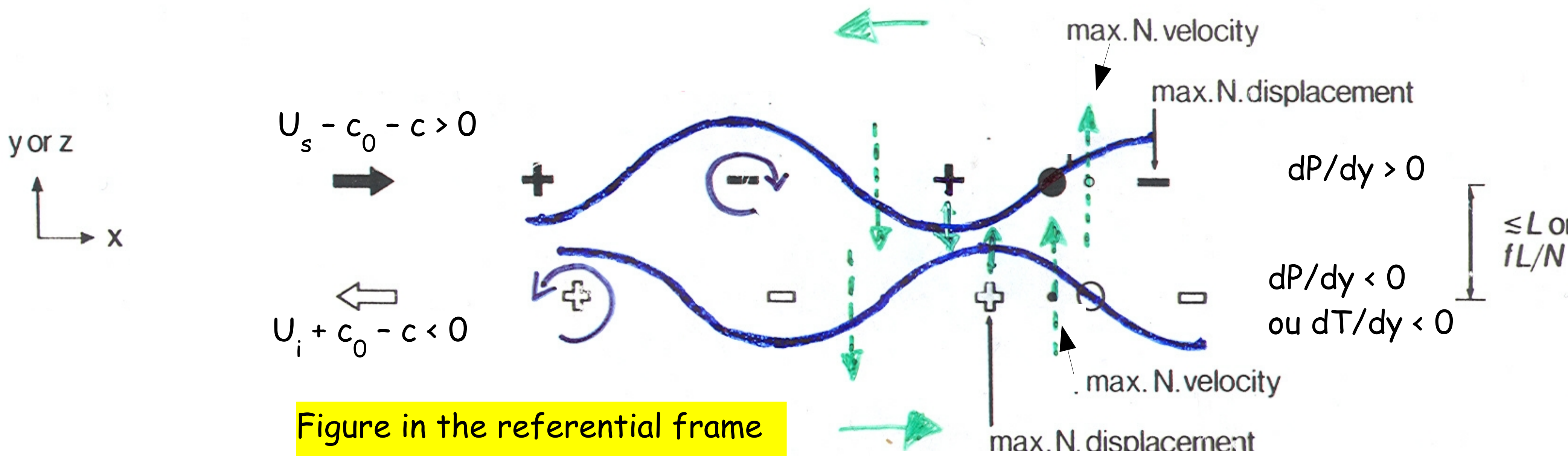


Figure in the referential frame moving at speed c in which the two waves are stationary.
c_0 is the phase speed of each wave with respect to rest (supposed to be opposite at top and bottom)

The instability requires
- PV gradients of opposite sign.
- Westward tilt.
- $U_s > c > U_i$

Simplified Phillips layer model (1)

z_4

z_3

z_2

z_1

z_0

Layer thickness $z_{i+1} - z_i = h$

In this model, the temperature anomaly is vanishing on the boundaries and the PV disstribution is discretized inside. This is complentary to Eady model where all the dynamics occurs on the boundaries.

$$\frac{D}{Dt}\left(\nabla^2 \psi + \mu^2 \frac{\partial^2 \psi}{\partial z^2} + \beta y \right) = 0$$

$$\frac{\partial \psi}{\partial z} = 0 \quad \text{en} \quad z = z_{0,} \;\; z_4$$

Equations on levels 1 et 3

$$\left(\frac{\partial}{\partial t} + \nabla \psi_1 \cdot \nabla \right)\left(\nabla^2 \psi_1 + \beta y + S(\psi_3 - \psi_1) \right) = 0$$

$$\left(\frac{\partial}{\partial t} + \nabla \psi_3 \cdot \nabla \right)\left(\nabla^2 \psi_3 + \beta y + S(\psi_1 - \psi_3) \right) = 0$$

with $S = \frac{\mu^2}{4 h^2}$

Basic state $U_{1,} \quad U_{3,} \quad \Psi_1 = -U_1 y , \quad \Psi_3 = -U_3 y$

Perturbed state $\psi_1 = \Psi_1 + \psi_1 ' , \quad \psi_3 = \Psi_3 + \psi_3 '$

Linearized equations

$$O = \left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} \right)\left(\nabla^2 \psi_1 ' + S(\psi_3 ' - \psi_1 ') \right) + \frac{\partial \psi_1 '}{\partial x} S(U_1 - U_3) + \beta \frac{\partial \psi_1 '}{\partial x}$$

$$O = \left(\frac{\partial}{\partial t} + U_3 \frac{\partial}{\partial x} \right)\left(\nabla^2 \psi_3 ' + S(\psi_1 ' - \psi_3 ') \right) + \frac{\partial \psi_3 '}{\partial x} S(U_3 - U_1) + \beta \frac{\partial \psi_3 '}{\partial x}$$

Simplified Phillips layer model (2)

Linearized equations

$$0=\left(\frac{\partial}{\partial t}+U_1\frac{\partial}{\partial x}\right)\left(\nabla^2\psi_1'+S(\psi_3'-\psi_1')\right)+\frac{\partial\psi_1'}{\partial x}S(U_1-U_3)+\beta\frac{\partial\psi_1'}{\partial x}$$

$$0=\left(\frac{\partial}{\partial t}+U_3\frac{\partial}{\partial x}\right)\left(\nabla^2\psi_3'+S(\psi_1'-\psi_3')\right)+\frac{\partial\psi_3'}{\partial x}S(U_3-U_1)+\beta\frac{\partial\psi_3'}{\partial x}$$

We define $\psi_m=\frac{1}{2}(\psi_1'+\psi_3'),\quad \psi_T=\frac{1}{2}(\psi_3'-\psi_1'),\quad U_m=\frac{1}{2}(U_1+U_3),\quad U_T=\frac{1}{2}(U_3-U_1)$

such that $$0=\left(\frac{\partial}{\partial t}+U_m\frac{\partial}{\partial x}\right)\nabla^2\psi_m+\beta\frac{\partial\psi_m}{\partial x}+U_T\frac{\partial\nabla^2\psi_T}{\partial x}$$

$$0=\left(\frac{\partial}{\partial t}+U_m\frac{\partial}{\partial x}\right)(\nabla^2\psi_T-2S\psi_T)+\beta\frac{\partial\psi_T}{\partial x}+U_T\frac{\partial}{\partial x}(\nabla^2\psi_m+2S\psi_m)$$

We choose $\psi_m=A\,e^{ik(x-ct)}\cos l\,y$ et $\psi_T=B\,e^{ik(x-ct)}\cos l\,y$ avec $K^2=k^2+l^2$

Thus $$ik\left((c-U_m)K^2+\beta\right)A-ikK^2U_TB=0$$

$$-ikU_T(K^2-2S)A+ik\left((c-U_m)(K^2+2S)+\beta\right)B=0$$

After calculation, the dispersion relation is

$$(c-U_m)^2(K^4+2SK^2)+2\beta(K^2+S)(c-U_m)+\beta^2+U_T^2(2S-K^2)=0$$

In other words $\boldsymbol{c=U_m+\frac{\beta(K^2+S)}{K^2(K^2+2S)}\pm\delta}$

with $\boldsymbol{\delta^2=\frac{\beta^2S^2}{K^4(K^2+2S)^2}-U_T^2\frac{2S-K^2}{K^2+2S}}$

Simplified Phillips layer model (3)

Dispersion relation $c = U_m + \dfrac{\beta(K^2+S)}{K^2(K^2+2S)} \pm \delta$

with $\delta^2 = \dfrac{\beta^2 S^2}{K^4(K^2+2S)^2} - U_T^2 \dfrac{2S-K^2}{K^2+2S}$

a) $U_T = 0$, in this case c is real (pure propagation) with possible values

$c_1 = U_m - \dfrac{\beta}{K^2}$ (external mode) and $c_2 = U_m - \dfrac{\beta}{K^2+2S}$ (internal mode)

b) $\beta = 0$, in this case $c = U_m \pm U_T \left(\dfrac{K^2-2S}{K^2+2S}\right)^{1/2}$

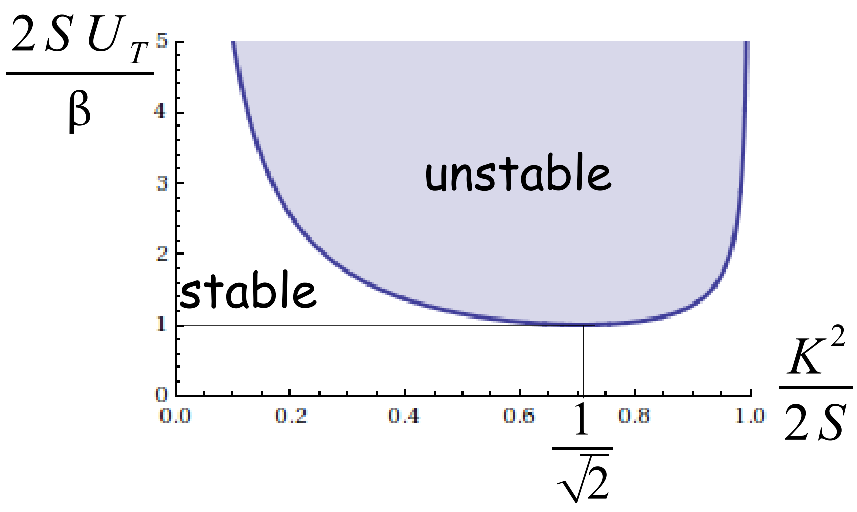
Instability occurs for $K < \sqrt{2S}$

c) general case, the instability threshold $\delta = 0$ is then given by

$$\frac{K^4}{2S^2} = 1 \pm \left(1 - \frac{\beta^2}{4S^2U_T^2}\right)^{1/2}$$

Notice that β produces a stabilizing effect

With $U_T = 4\,\mathrm{m\,s^{-1}}$
between 750 and 250 hPa.
Most unstable mode:
$K = 1/4000$ km



Simplified Phillips layer model (4)

The basic flow vorticity is

$$Q_3 = \beta\, y + S\, y (U_3 - U_1)$$
$$Q_1 = \beta\, y + S\, y (U_1 - U_3)$$

Knowing that $U_1 < U_3$, the necessary instability condition on the potential vorticity gradients is

$$\frac{\partial Q_1}{\partial y} < 0 < \frac{\partial Q_3}{\partial y}$$

that is $S(U_1 - U_3) \; < \; -\beta \; < \; S(U_3 - U_1)$

or in other words $\frac{2 S U_T}{\beta} > 1$

This is identical to the instability criterion established in the previous slides

Since the wind is uniform over each level,
the gradient is only due to the planetary vorticity
and the thermal structure.