MEC 654
Polytechnique-UPMC-Caltech
Year 2014-2015

Turbulence

chapter 12

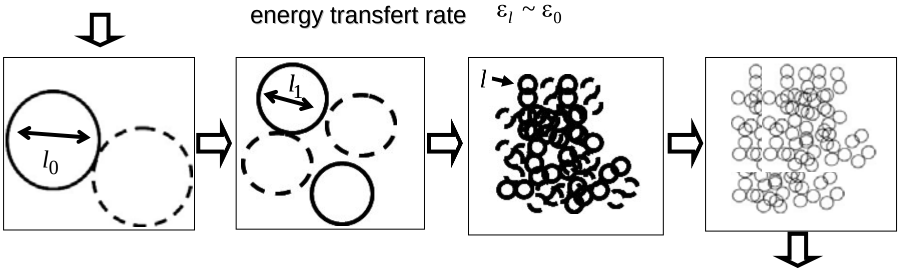
3D vortices : an introduction

12.1 2D / 3D : scales and energy (remainder)

• **3D decay**

ε_0

energy transfert rate $\varepsilon_l \sim \varepsilon_0$



dissipation rate ε_0

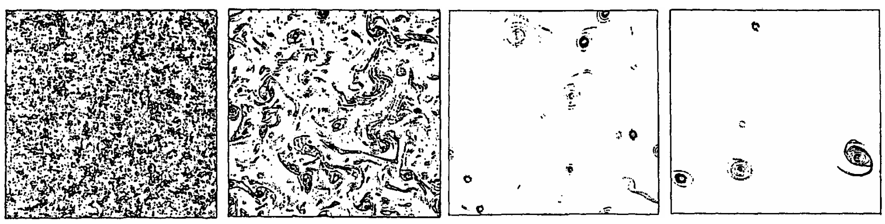
✓ scales : breakdown

✓ energy : without injection $\lim_{\nu \to 0} \iiint_V \frac{1}{2} \underline{u}^2 dV = 0$

⇨ energy is transfered towards smaller scales where it is entirely transformed into heat (direct cascade)

12.1 2D / 3D : scales and energy remainder (…)

• **2D decay**



Mc Williams 1982

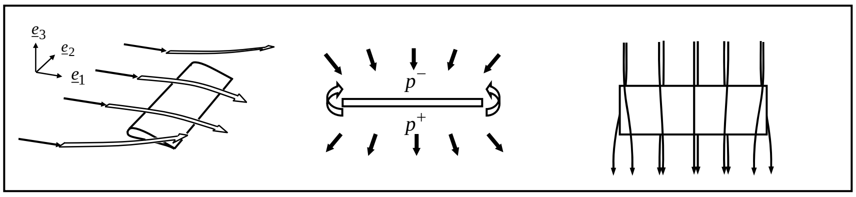
✓ scales : they grow

✓ energy $\iint_S \frac{1}{2} \underline{u}^2 \, dS$ conserved (in the limit $\nu \to 0$)

⇨ energy is transfered towards larger scales (inverse cascade)

12.2 transition 2D → 3D : the example of lift vortices

- **lift vortices**



12.2 transition 2D → 3D : the example of lift vortices (…)

b = span

x = donwnstream distance

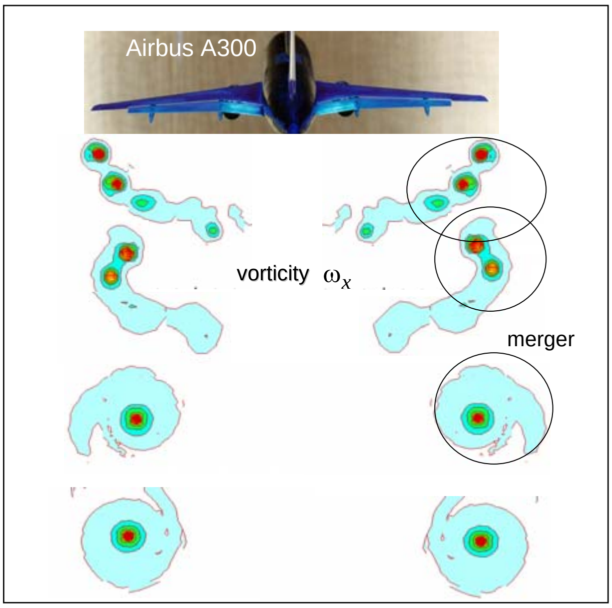
$x/b = 0.5$

$x/b = 1$

$x/b = 3$

⋮

$x/b = 10$



2D dynamics

Jacquin et al. 2000

12.2 transition 2D → 3D : the example of lift vortices

• lift vortices

12. 2 transition 2D → 3D : the example of lift vortices

film : vortex encounter

12. 2 transition 2D $\rightarrow$ 3D : the example of lift vortices

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12. 2 transition 2D $\rightarrow$ 3D : the example of lift vortices

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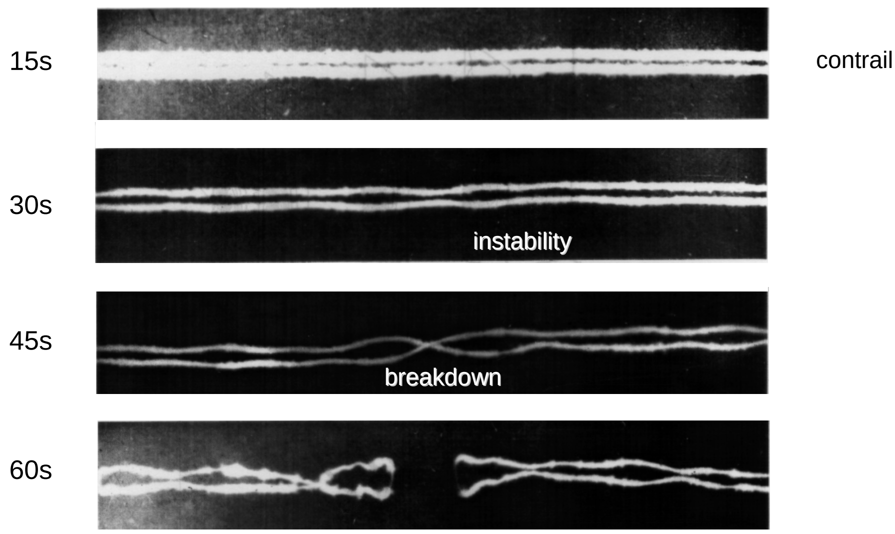
12. 2 transition 2D → 3D : the example of lift vortices

12. 2 transition 2D → 3D : the example of lift vortices



Crow 1970

⇨ the start of 3D turbulence

12. 2 transition 2D → 3D : the example of lift vortices

VIDEO0057.mp4

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12.3 vortex distorsion

- **remainder**

✓ Helmoltz's equation (chapter 9)

$$\frac{d\,\underline{\omega}}{dt} + \underline{\omega}\, div\, \underline{u} - \nabla \underline{u}\,.\,\underline{\omega} = \underline{rot}\ \underline{f} + \frac{1}{\rho^2}\,\underline{grad}\ \rho \wedge \underline{grad}\ p + \underline{rot}\left[\ \frac{1}{\rho} div\, \underline{\underline{\tau}}\ \right]$$

✓ simplifications (chapter 9)

$$\frac{d\,\underline{\omega}}{dt} = \underbrace{\nabla \underline{u}\,.\,\underline{\omega}} + \nu\ \ \Delta\underline{\omega}$$

vortex distorsion

incompressibilty $div\left(\underline{u}\right) = 0$
homogeneity $\rho = const.$
constant viscosity $\eta = const.$
conservative forces $\underline{f} = -\underline{grad}\ \phi$

✓ 2D flows (chapter 11)

$$\frac{d\,\underline{\omega}}{dt} = \nu\ \ \Delta\underline{\omega}$$

- **observations**

as soon as the 2D constraint is released, vortices tend to become 3D under the action of the vortex distorsion term $\nabla \underline{u}\,.\,\underline{\omega}$

12.3 vortex distorsion (…)

- **vortex distorsion** $\frac{d\underline{\omega}}{dt} = \boxed{\nabla\underline{u}.\underline{\omega}} + \nu \ \Delta\underline{\omega}$

✓ velocity gradient tensor décomposition

$$\nabla\underline{u} = \underline{\underline{d}} + \underline{\underline{\Omega}} \quad \begin{cases} \underline{\underline{d}} = \frac{1}{2}\left(\nabla\underline{u} + {}^{t}\nabla\underline{u}\right) & \text{- deformation rate tensor} \\ \underline{\underline{\Omega}} = \frac{1}{2}\left(\nabla\underline{u} - {}^{t}\nabla\underline{u}\right) & \text{- rotation rate tensor} \end{cases}$$

✓ rotation has no effect

$$\nabla\underline{u}.\underline{\omega} = \underline{\underline{d}}.\underline{\omega} + \underline{\underline{\Omega}}.\underline{\omega} = \underline{\underline{d}}.\underline{\omega} \quad \text{because} \quad \underline{\underline{\Omega}}.\underline{\omega} = \underline{\Omega} \wedge \underline{\omega} = \frac{1}{2}\underline{\omega} \wedge \underline{\omega} = 0$$

- **conclusion :** $\frac{d\underline{\omega}}{dt} = \boxed{\underline{\underline{d}}.\underline{\omega}} + \nu \ \Delta\underline{\omega}$

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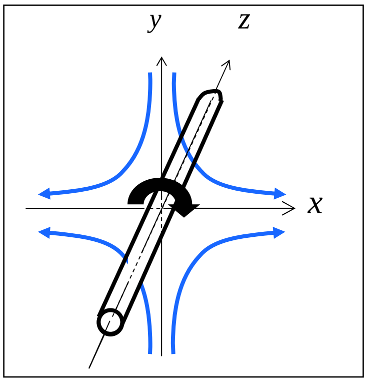
12.3 vortex distorsion (…)

- **vorticity equation** $\frac{d\underline{\omega}}{dt} = \underline{\underline{d}}.\underline{\omega} + \nu\Delta\underline{\omega}$ (1)
- **distorsion effect : example**

✓ an **<u>infinitesimal vortex tube</u>** immersed in a base flow corresponding to an **<u>irrotationnal deformation</u>** of rate α :

$$\boxed{\begin{cases} \underline{\omega} = (0 \ ,0 \ , \ \omega_z) \\ \underline{u} = (\alpha x, -\alpha y, \ 0), \alpha > 0 \end{cases}}$$

$$\Rightarrow \underline{\underline{d}}.\underline{\omega} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & -\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



✓ linearization : the vorticity is supposed to sufficiently weak so as to neglect its impact on the base flow ($\underline{\underline{d}} \approx const.$)

✓ we also neglect viscosity

(1) $\Rightarrow \frac{d\underline{\omega}}{dt} = \underline{\underline{d}}.\underline{\omega} = 0 \Rightarrow \boxed{\omega_z(t) = \omega_z(0)}$

12.3 vortex distorsion (…)

• **vorticity equation** $\frac{d\underline{\omega}}{dt} = \underline{\underline{d}}.\underline{\omega}$ (1)

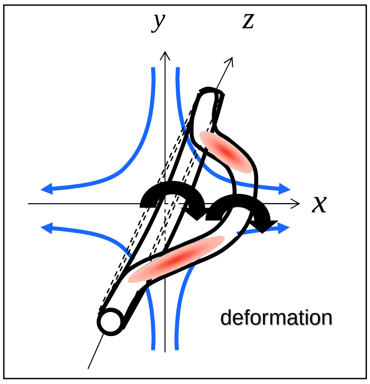
✓ suppose now that the vortex tube is slightly deformed so that a small component ω_x emerges

$$\begin{cases} \underline{\omega} = (\omega_x, 0, \omega_z) \\ \underline{u} = (\alpha x, -\alpha y, 0), \alpha > 0 \end{cases}$$

$$\Rightarrow \underline{\underline{d}}.\underline{\omega} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & -\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ 0 \\ \omega_z \end{bmatrix} = \begin{bmatrix} \alpha\,\omega_x \\ 0 \\ 0 \end{bmatrix}$$

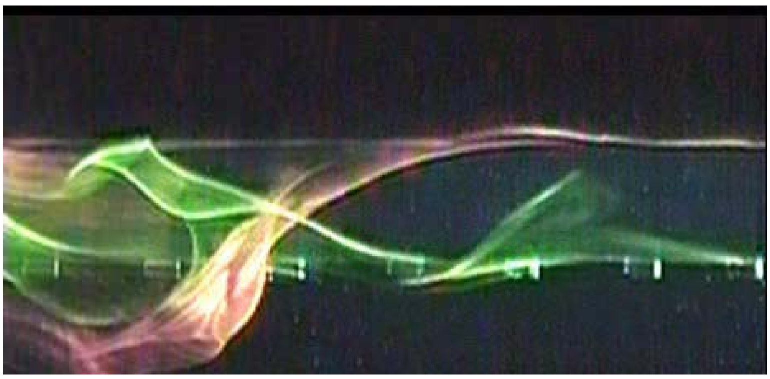
$$(1) \Rightarrow \begin{cases} \omega_x(t) \sim \omega_x(0)e^{\alpha t} \\ \omega_z(t) = \omega_z(0) \end{cases}$$

amplification ⇩ growth of a 3D component



12.3 vortex distorsion (…)

• **a pair of co-rotating vortices (…)**



Meunier & Leweke (IRPHE)

⇨ merger due to a 3D instability

⇨ emergence of 3D turbulence

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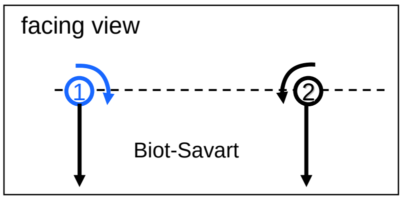
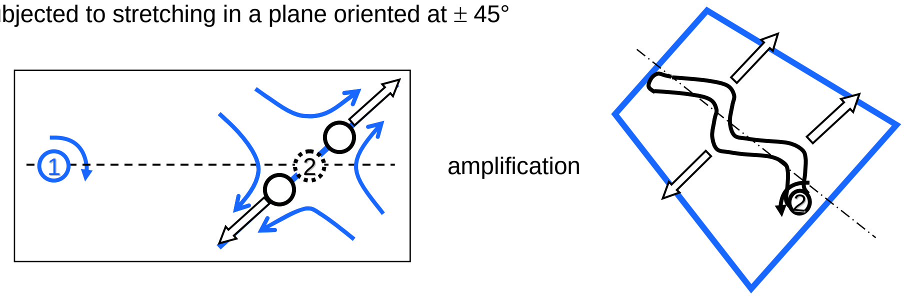
12.4 back to lift vortices

• **a pair of counter - rotating vortices**

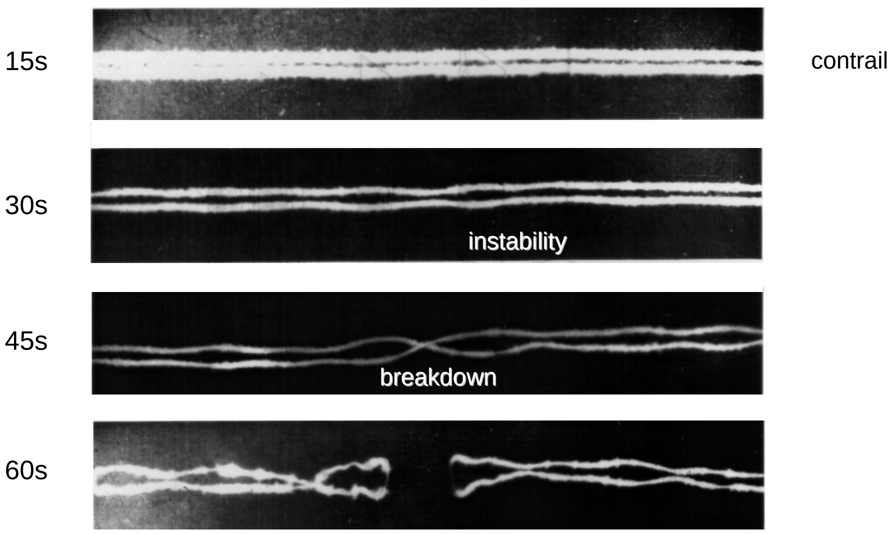


✓ you will show (training lesson today) that in the frame moving with the system, each vortex is subjected to stretching in a plane oriented at ± 45°



12.4 back to lift vortices (…)

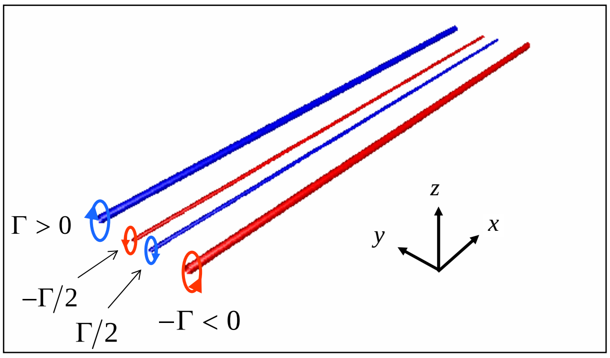
• **a pair of counter - rotating vortices**



Crow 1970

12.4 back to lift vortices (…)

• **a double pair of counter - rotating vortices**

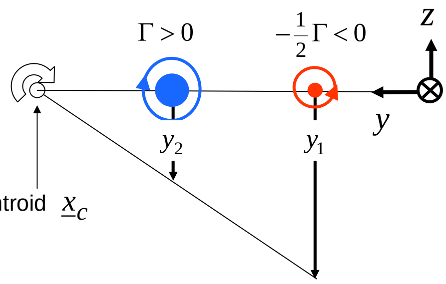


✓ note : this 4 vortex system is not chaotic due to mirror symmetry

✓ explain on the blackboard : vortices introduced by outer flaps

12.4 back to lift vortices (…)

• **a double pair of counter - rotating vortices (…)**



• **centroid : back to chapter 11 (§11.7)**

✓ linear moment $\quad I_x = \sum_i y_i \Gamma_i = \Gamma\, y_2 - \frac{1}{2}\Gamma\, y_1 = const.$

✓ centroid $\quad y_c = \dfrac{\sum_i y_i \Gamma_i}{\sum_i \Gamma_i} = \dfrac{\Gamma\, y_2 - \frac{1}{2}\Gamma\, y_1}{\Gamma - \frac{1}{2}\Gamma} = \dfrac{y_2 - \frac{1}{2} y_1}{1 - \frac{1}{2}} = 2y_2 - y_1$

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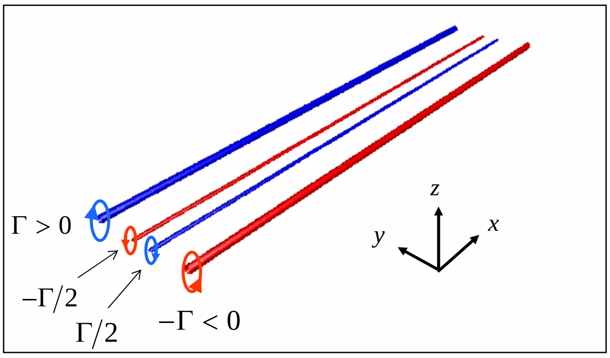
12.4 back to lift vortices (…)

• a double pair of counter - rotating vortices

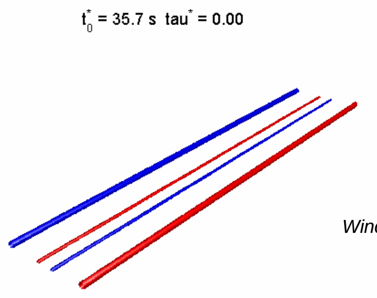


⇨ let's perturb the system

12.4 back to lift vortices (…)

• a double pair of counter-rotating vortices (…)

✓ inviscid computation

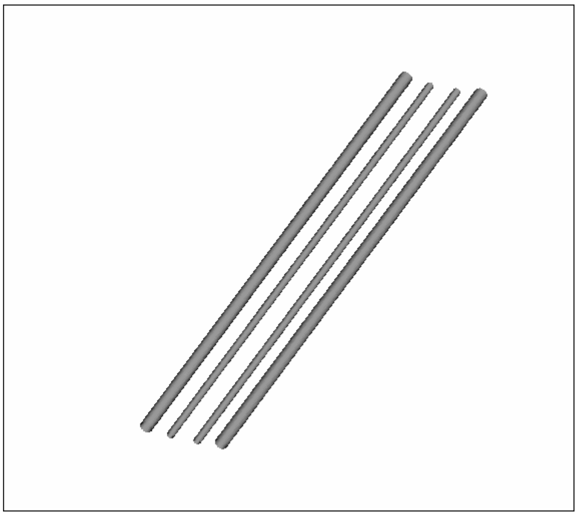


Winckelmans, 2005

12.4 back to lift vortices (…)

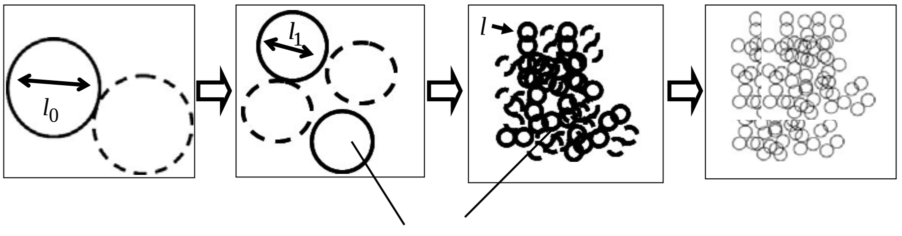
• **a double pair of counter-rotating vortices (…)**

✓ Navier-Stokes computation



Stumpf, 2005

12.5 back to the Richardson – Kolmogorov's cascade



inertial regime :
3D vortices ?

⇨ how are they made ?

12.5 back to the Richardson - Kolmogorov cascade (…)

• scale breakdown and dissipation : the role of stretching

✓ vorticity in an irrotational deformation flow

$$\begin{cases} \dfrac{d\underline{\omega}}{dt} = \underline{\underline{d}}\,.\underline{\omega} + \nu\Delta\underline{\omega} \\ \underline{u} = (\alpha_1 x,\ \alpha_2 y,\ \alpha_3 z) \end{cases}$$

✓ incompressibility $\quad \mathrm{div}\,\underline{u} = \alpha_1 + \alpha_2 + \alpha_3 = 0 \quad \Rightarrow$ streching in 1 or 2 directions where α_i is positif

✓ inviscid $(\nu = 0)$ $\quad \boxed{\dfrac{d\omega_i}{dt} = \alpha_i \omega_i \ , i = 1,2,3} \quad \Rightarrow \quad \boxed{\omega_i(t) = \omega_i(0)\ e^{\alpha_i t}}$ (1)

(with no indice contraction)
production / destruction

✓ pseudo-dissipation rate per unit mass : $\epsilon_2(t) = \nu\,\underline{\omega}^2(t) = \nu\left[\,\omega_1^2 + \omega_2^2 + \omega_3^2\,\right](t)$

(1) $\Rightarrow$ $\boxed{\lim_{\alpha_i t >> 1} \epsilon_2(t) \sim \nu\,\omega_i^2(0)\,e^{2\alpha_i t}}$ $\quad \alpha_i = \max_j \{\alpha_j\},\ j = 1,2,3$

(with no indice contraction)

12.5 back to the Richardson - Kolmogorov cascade (…)

• scale breakdown and dissipation : the role of stretching (…)

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✓ scales ?

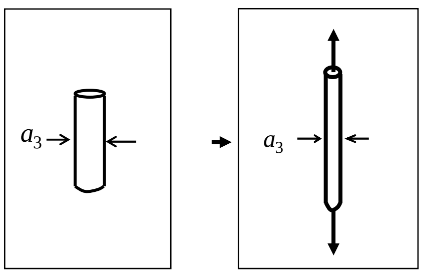
✓ let be a vortex aligned with axis Ox_3

✓ introduce the circulation $\Gamma = \iint_S \omega_3\, dS \sim \omega_3\, a_3^2$ (with no indice contraction) where a_i characterizes the vortex thickness

$\boxed{\Gamma = const. \quad \Rightarrow \quad \omega_3\, a_3^2 = const.}$ $\quad \boxed{\omega_3(t) = \omega_3(0)\ e^{\alpha_3 t} \ \Rightarrow \ a_3(t) = a_3(0)\, e^{-\alpha_3 t/2}}$

✓ exemple $\quad \underbrace{\alpha_1 = -\frac{1}{2}\alpha, \alpha_2 = -\frac{1}{2}\alpha}_{\text{contraction}},\ \underbrace{\alpha_3 = \alpha \quad > 0}_{\text{streching}}$

$$\Rightarrow \begin{cases} \text{vorticité} & \omega_3(t) = \omega_3(0)e^{\alpha t} \\ \text{scales} & a_3 \propto a_3(0)e^{-\alpha t/2} \end{cases}$$



<u>nota</u> $\quad div\,\underline{u} = \sum_i \alpha_i = 0$

$\Rightarrow$ stretching increases vorticity, so dissipation, as the flow sturructures become thinner

12.5 back to the Richardson - Kolmogorov cascade (…)

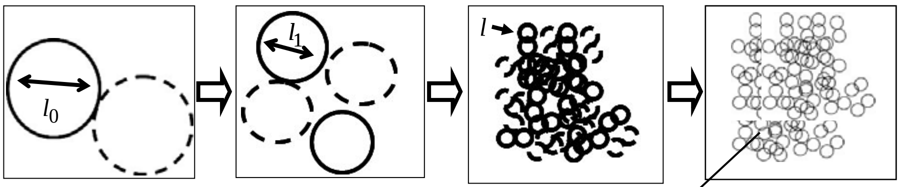
• scale breakdown and dissipation : the role of stretching (…)

✓ stretching is the non-viscous process which reduces the scales and increases dissipation

✓ this is the « engine » of the Richardson-Kolmogorov cascade

✓ the inertial regime of the Richardson-Kolmogorov cascade can be seen as a process of stretching of small eddies by larger ones

✓ how does this process stop ?



✓ the smallest vortices scale as : $\eta = \left(\nu^3/\varepsilon_0\right)^{1/4}$ (Kolmogorov scale)

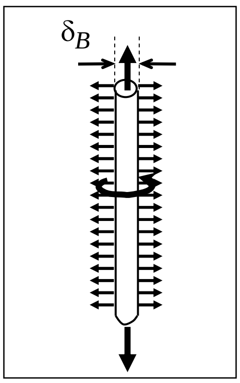
⇨ how are they made ?

12.5 back to the Richardson - Kolmogorov cascade (…)

• the role of viscosity : the Burger's scale

$$\begin{cases} \dfrac{d\underline{\omega}}{dt} = \underline{\underline{d}}.\underline{\omega} + \nu\Delta\underline{\omega} \\ \underline{u} = \left(\alpha_1 x,\ \alpha_2 y,\ \alpha_3 z\right) \end{cases}$$

✓ search for a steady solution : $0 = \underbrace{\alpha_i\omega_i}_{\text{stretching}} + \underbrace{\nu\,\Delta\omega_i}_{\text{spreading}}$



⇨ two conflicting mechanisms

✓ time scales $\begin{cases} \text{stretching} & \tau_\alpha \sim \alpha_i^{-1} \\ \text{spreading} & \tau_\nu \sim a_i^2/\nu \end{cases}$

✓ equilibrium $\tau_\alpha \sim \tau_\nu$ ⇨ $a_i = \sqrt{\nu/\alpha_i} = \delta_B$

Burger's scale

⇨ the Burger's scale characterizes the size of eddies where viscous diffusion and stretching are in equilibrium

⇨ what is the Burger's scale in the 3D cascade ?

12.5 back to the Richardson - Kolmogorov cascade (…)

• the role of viscosity : the Burger's scale (…)

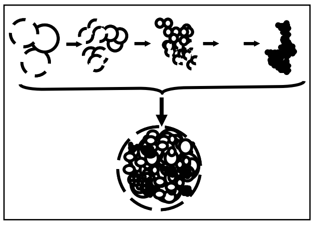
✓ Burger's scale $a_i = \sqrt{\nu / \alpha_i} = \delta_B$

α_i stretching rate

✓dissipation $\epsilon = 2\nu \underline{\underline{d}} : \underline{\underline{d}} \sim \nu\, \alpha^2$

⇨ $\alpha \sim \sqrt{\epsilon / \nu}$

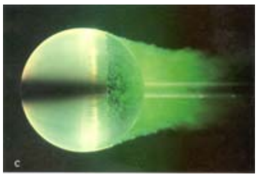
⇨ $\boxed{\delta_B = \sqrt{\nu / \alpha} = \left(\nu^3 / \epsilon\right)^{1/4} = \eta}$



cascade = stretching of small scales by larger scales

⇨ the burger's scale of turbulence is the kolmogorov scale

REM -



$\mathrm{Re} = \frac{U_0 D}{\nu} \approx 300000$

$\nu_{air} \sim 10^{-5}\, m^2/s$

$l_0 \sim D = 30cm$

(football)

$U_0 \sim 10ms^{-1}$

$\frac{\eta}{D} \sim \left(\frac{\nu}{U_0 D}\right)^{3/4} = \mathrm{Re}^{3/4}$

⇨ $\boxed{\eta \approx 8\, \mu m}$

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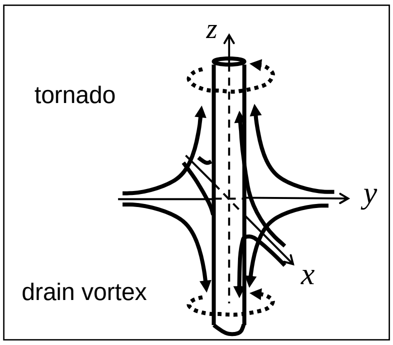
12.5 back to the Richardson - Kolmogorov cascade (…)

• a model for a dissipative structure : the Burger vortex

✓ vortex in an axysimmetric deformation

$$\begin{cases} \underline{\omega} = \left(0, 0, \omega(r,t)\right) \\ \underline{u} = \left(-\frac{1}{2}\alpha r,\ u_\theta,\ \alpha z\right) \end{cases}$$



✓ solution = gaussian vortex of thickness $\delta_B = \sqrt{\nu / \alpha}$

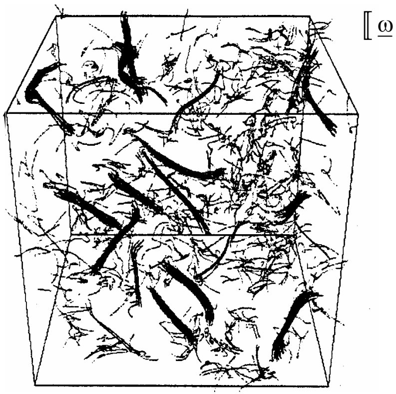
⇨ training course

12.5 back to the Richardson - Kolmogorov cascade (…)

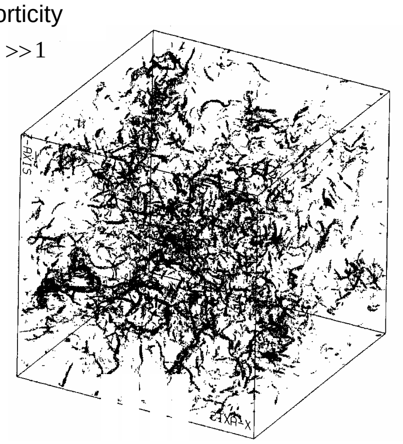
✓ the end of the cascade : tornadoes ?

iso-vorticity

$[\![\underline{\omega}]\!] >> 1$



« small » Reynolds

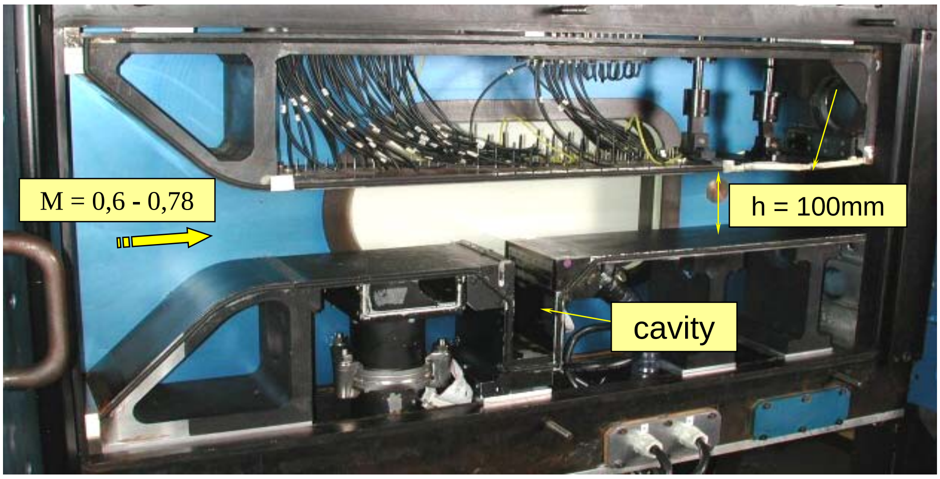


« large » Reynolds

12.6 vortex dynamics and turbulence : summary

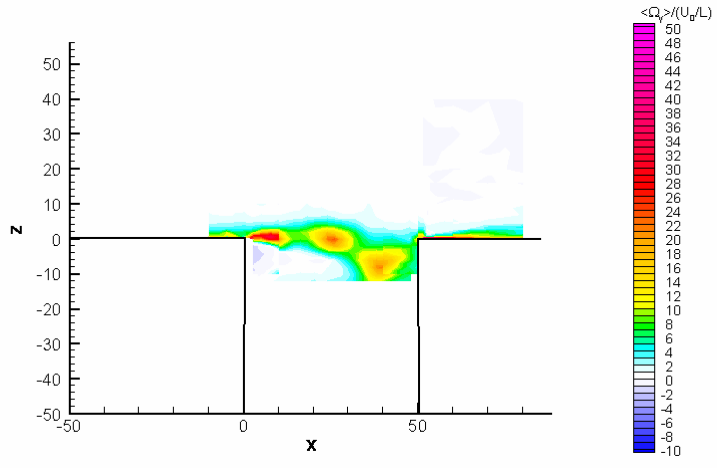
- ✓ 2D flow dynamics at high Reynolds numbers are chaotic dynamics and result in a growth of the energetic scales, namely an inverse energy cascade compared to that of 3D flows
- ✓ in the infinite Reynolds number limit, 2D flow energy is a constant
- ✓ mechanisms that control this process are : chaotic convection, filamentation and vortex merging
- ✓ 3D flows are dominated by stretching which reduces and fragments eddies and which imposes a direct energy cascade (towards smaller scales)
- ✓ through this direct cascade, 3D flow energy is always dissipated in the infinite Reynolds number limit.
- ✓ Most of the dissipation occurs at the Burgers scale which balances stretching and viscous diffusion

12.7 vortex dynamics and turbulence : an example



ONERA-DAFE transonic wind tunnel S8Ch

12.7 vortex dynamics and turbulence : an example



Forestier et al., 2002

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12.7 vortex dynamics and turbulence : an example

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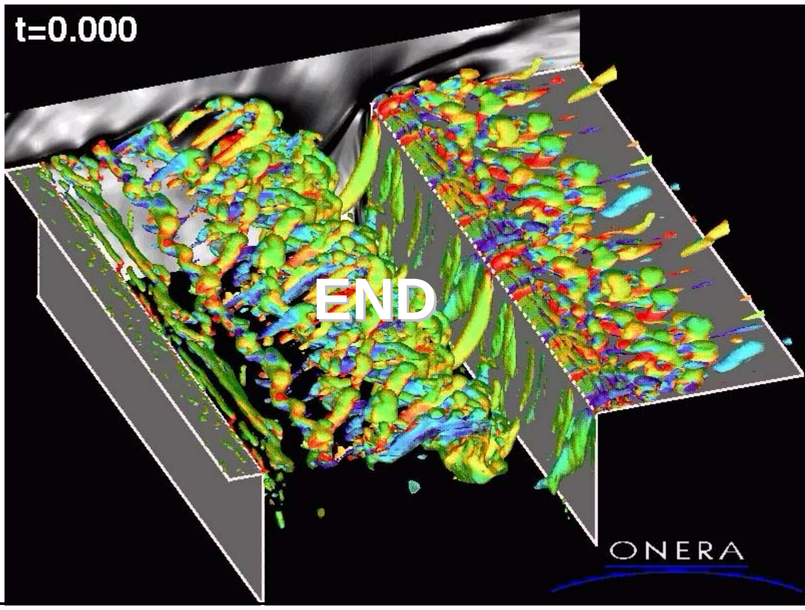
Larchevèque et al., 2002

12.7 vortex dynamics and turbulence : an example



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