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MEC 654
Polytechnique-UPMC-Caltech
Year 2014-2015

Turbulence

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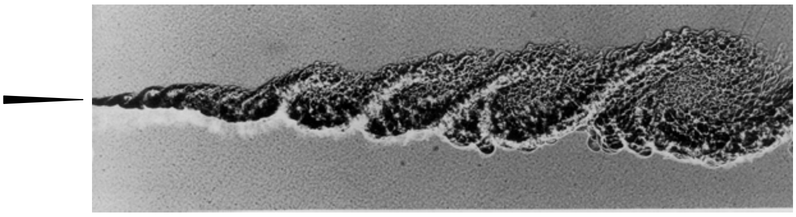
chapter 16

turbulent shear flows

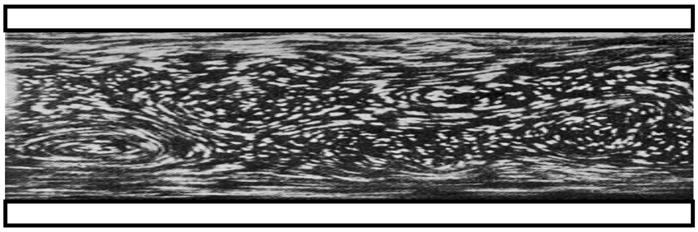
16.1 shear flows

• **two families**



free shear flows

a mixing layer



wall bounded flows

a channel flow

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16.1 shear flows (...)

• **back to chapter 13**

• **Reynolds decomposition** $\underline{u} = \langle \underline{u} \rangle + \underline{u}', \; p = \langle p \rangle + p'$

• **Reynolds equation**

$$\frac{D\langle \underline{u} \rangle}{Dt} = -\frac{1}{\rho}\underline{grad}\langle p \rangle + \underline{div}\left(2\nu \langle \underline{\underline{d}} \rangle - \langle \underline{u}' \otimes \underline{u}' \rangle \right)$$

Reynolds stress tensor

• **kinetic energy**

$$K = \frac{1}{2}\langle \underline{u} \rangle^2$$

$$k = \frac{1}{2}\langle \underline{u}'^2 \rangle$$

$$\frac{DK}{Dt} = K_{,t} + \langle \underline{u} \rangle . \underline{grad}\, K = div\, \underline{\phi}_K - \varepsilon_K - P$$

convection, diffusion, dissipation, production

$$\frac{Dk}{Dt} = k_{,t} + \langle \underline{u} \rangle . \underline{grad}\, k = div\, \underline{\phi}_k - \varepsilon_k + P$$

Reynolds stress power $P = -\langle \underline{u}' \otimes \underline{u}' \rangle : \nabla \langle \underline{u} \rangle$

>0 (in general)

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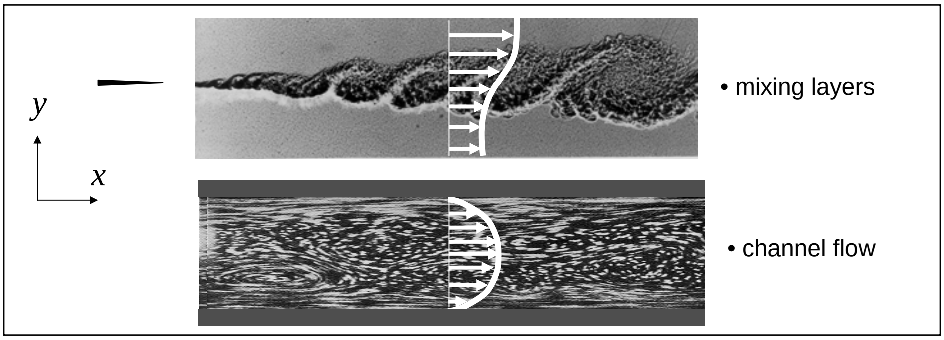
16.1 shear flows (...)



✓ equilibrium shear flows
- statistical stationnarity
- x - wise statistical homogeneity

✓ turbulent kinetic energy (TKE) budget

$$\frac{Dk}{Dt} = \cancel{\frac{\partial k}{\partial t}} + \cancel{\langle \underline{u} \rangle . \underline{grad}\, k} = \boxed{div\, \underline{\phi}_k \quad - \ \varepsilon_k \qquad + P \qquad = \ 0}$$

convection — diffusion (in the non-homogeneity directions) — dissipation — production

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16.2 scales

$$\boxed{div\, \underline{\phi}_k - \varepsilon_k + P = 0}$$

$$P = -\langle \underline{u}' \otimes \underline{u}' \rangle : \nabla \langle \underline{u} \rangle$$

- **turbulent stress** $\langle \underline{u}' \otimes \underline{u}' \rangle \sim u_0^2$
- **dissipation** $\varepsilon_k = \nu \langle |\nabla \underline{u}'|^2 \rangle \sim u_0^3 / l_0$
- **diffusion** $d = div(\ \underline{\phi}_k\ ) \sim u_0^3 / l_0$
- **production of** k $P = -\langle \underline{u}' \otimes \underline{u}' \rangle : \nabla \langle \underline{u} \rangle \sim u_0^2 |\nabla \langle \underline{u} \rangle|$

$\boxed{u_0\ ?}$ $\boxed{l_0\ ?}$ $\boxed{\nabla \langle \underline{u} \rangle ?}$

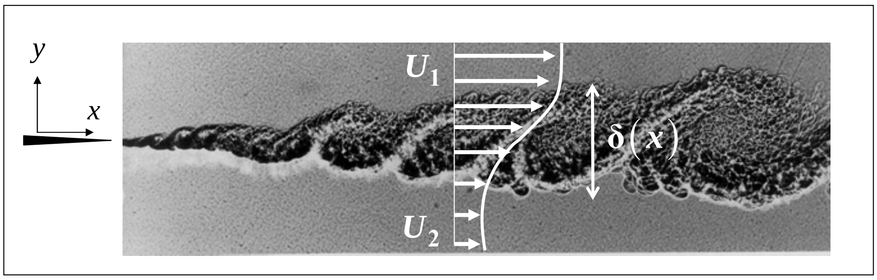
- **cascade in equilibrium**

cascade in equilibrium
production ~ dissipation

⇨ $\boxed{|\nabla \langle \underline{u} \rangle| \sim \frac{u_0}{l_0}}$ the turbulent scales set the mean field gradients

16.3 scales : free shear flows (…)

• **mixing layer**



• **scales**

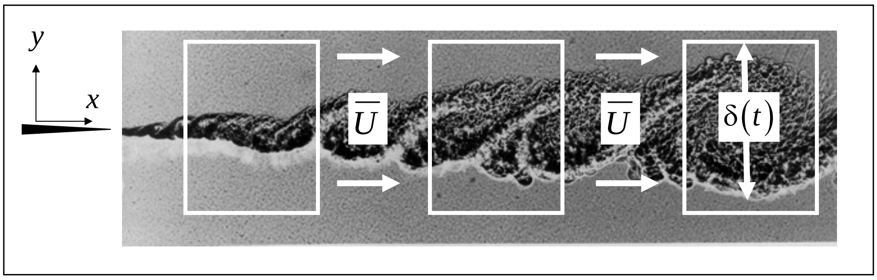
✓ velocities $\quad \overline{U} = (U_1 + U_2)/2 \quad$ convection

$\Delta U = U_1 - U_2 \quad$ production $\quad \Rightarrow \quad \boxed{u_0 \sim \Delta U}$ constant

✓ length $\quad l_0 \sim \delta(x) \quad \Rightarrow \quad \boxed{l_0 \sim \delta(x)}$ variable

16.3 scales : free shear flows (…)

• **approximation : Taylor hypothesis**



✓ expansion rate :

$$\left.\begin{aligned} \frac{\partial \delta}{\partial x} &\approx \frac{1}{\overline{U}} \frac{\partial \delta}{\partial t} \\ \frac{\partial \delta}{\partial t} &\sim u_0 \sim \Delta U \end{aligned}\right\} \Rightarrow \boxed{l_0 \sim \delta(x) \sim \frac{\Delta U}{\overline{U}} x = 2 \frac{U_1 - U_2}{U_1 + U_2} x}$$

affine

✓ note : $\quad 0 \le \frac{\Delta U}{\overline{U}} = 2 \frac{U_1 - U_2}{U_1 + U_2} \le 2$

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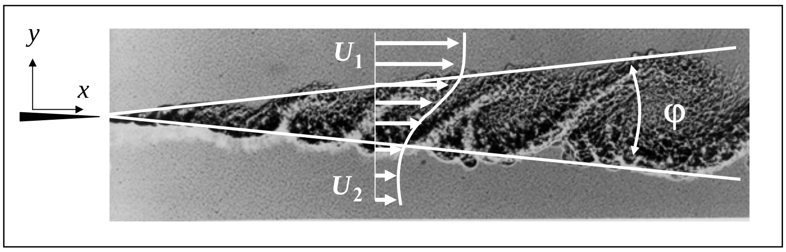
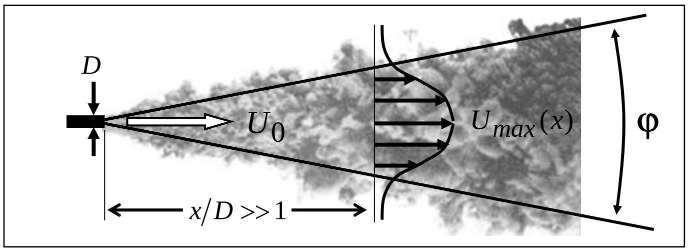
16.3 scales : free shear flows (...)

- **conclusion** $l_0 \sim \delta(x) \sim \frac{\Delta U}{\overline{U}} x$ affine



$\varphi = \varphi\left(\frac{\Delta U}{\overline{U}} \right)$

- **the case of a free jet : far field**



$\frac{\Delta U}{\overline{U}} = \frac{U_{max} - 0}{\frac{1}{2}(U_{max} + 0)} = 2$

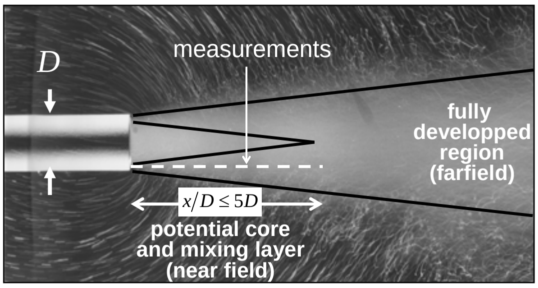
$\Rightarrow$ $\boxed{\varphi = const.}$

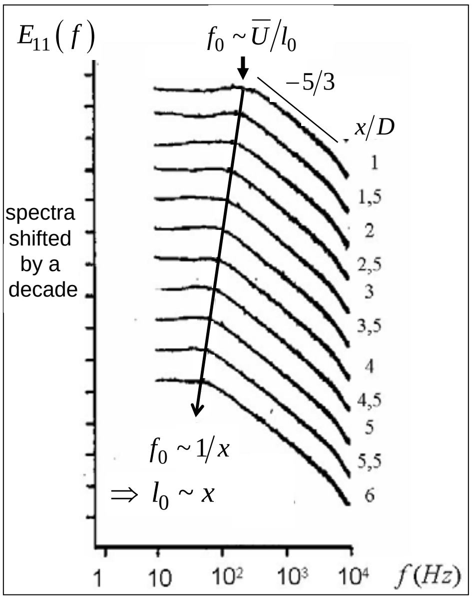
universal ?

probably

16.3 scales : free shear flows (...)

- **the case of a free jet : near field**

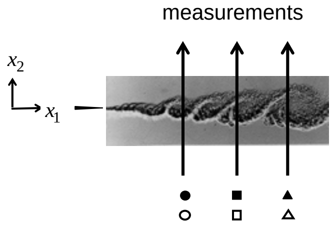




Leuchter (1988)

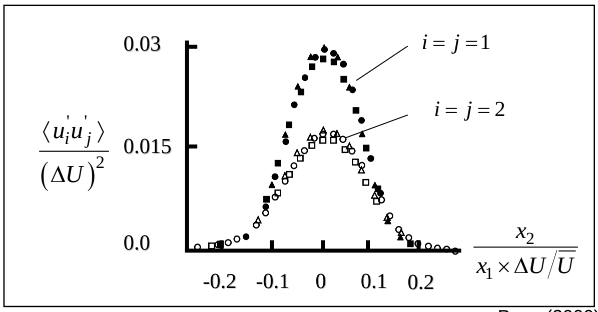
16.3 scales : free shear flows (…)

- **self-similarity in mixing layers**



✓ turbulent field

$$\frac{\langle u_i' u_j' \rangle}{u_0^2} = f\left(\frac{x_2}{l_0}\right), \quad u_0 \sim \Delta U, \quad l_0 \sim \frac{\Delta U}{\overline{U}} x_1 \quad \Rightarrow \quad \boxed{\frac{\langle u_i' u_j' \rangle}{(\Delta U)^2} = f\left(\frac{x_2}{x_1 \times \Delta U / \overline{U}}\right)}$$



see Pope (2000)

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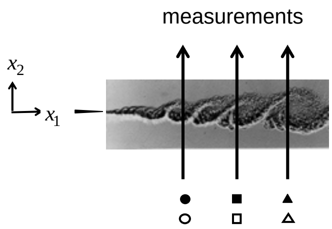
16.3 scales : free shear flows (…)

- **orders of magnitude**



✓ turbulence rate $\boxed{\frac{u_0}{\Delta U} \approx const.}$ *const.*= ?

✓ thickness $\boxed{\frac{l_0(x_1)}{x_1 \times \Delta U / \overline{U}} \approx const.}$ *const.*= ?

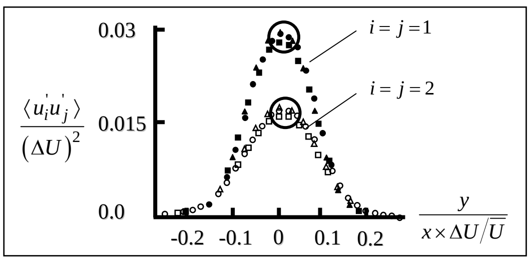
- **turbulence rate**

✓ one choose $\boxed{\frac{u_0}{\Delta U} = \frac{\sqrt{k_{max}}}{\Delta U}}$

turbulent kinetic energy (TKE)

$$\langle u_3'^2 \rangle \approx \langle u_2'^2 \rangle$$

$$k = \frac{1}{2}\left(\langle u_1'^2 \rangle + \langle u_2'^2 \rangle + \langle u_3'^2 \rangle \right) \approx \frac{1}{2}\left(\langle u_1'^2 \rangle + 2\langle u_2'^2 \rangle \right)$$



$\Rightarrow$ $\frac{\sqrt{k_{max}}}{\Delta U} \approx \sqrt{0.03} = 0.17$

$\Rightarrow$ turbulent rate ≈ 15-20%

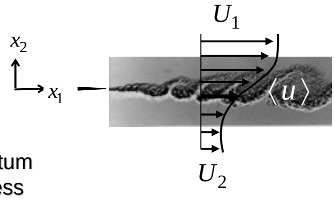
16.3 scales : free shear flows (...)

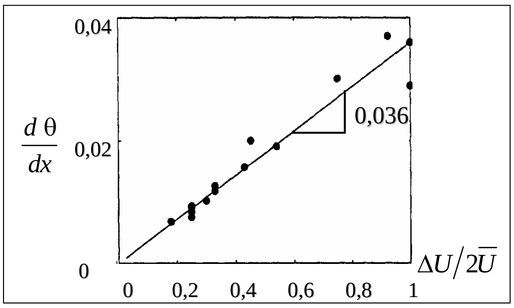
- **thickness** $\dfrac{l_0(x)}{x \times \Delta U / \overline{U}} \approx const. = ?$

✓ one choose :

$$l_0(x) = \theta(x) = \int_{-\infty}^{\infty} \frac{\langle u \rangle - U_2}{\Delta U} \left(1 - \frac{\langle u \rangle - U_2}{\Delta U} \right) dy$$

momentum thickness





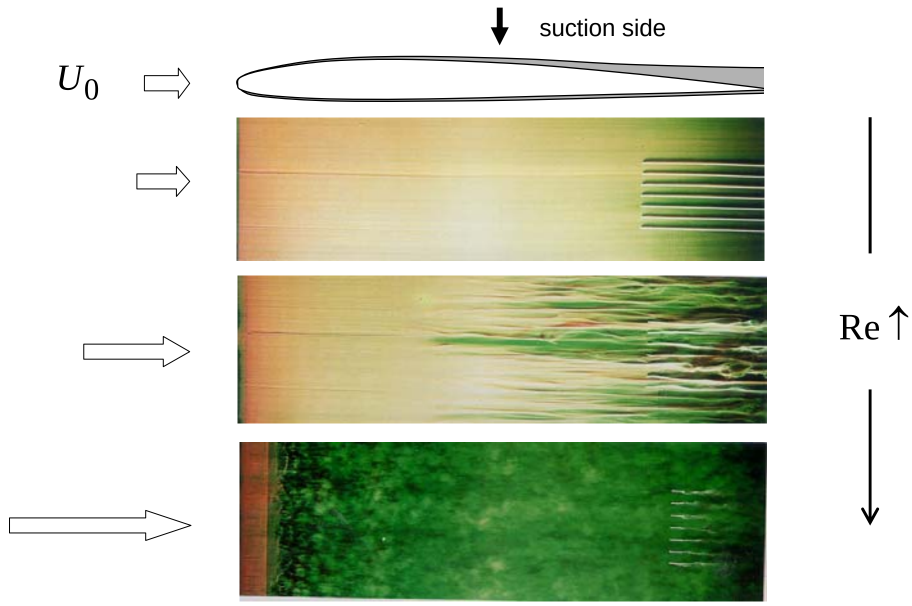
Jimenez (2000)

⇨ $\dfrac{l_0(x)}{x} \approx 0,02 \times \dfrac{\Delta U}{\overline{U}}$ ≈ few % (*)

(*) Rem : $0 \le \dfrac{\Delta U}{\overline{U}} = 2\dfrac{U_1 - U_2}{U_1 + U_2} \le 2$

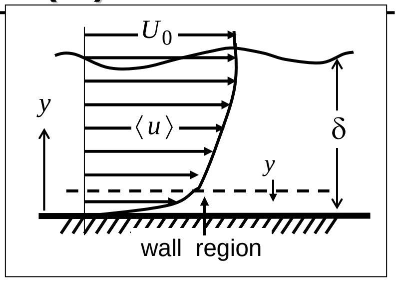
⇨ a turbulent shear layer is a thin layer

16.4 scales : wall bounded shear flows



Werlé, 1987

16.4 scales : wall bounded shear flows (…)



- **outer scales** (same as in a free shear flow**)**

velocity U_0 constant

length δ variable

⇨ not suited because this does not account for the "blocage effect" by the wall. Turbulence originates at the wall where it is produced by the wall shear stress

- **inner scales : wall region**

⇨ region where turbulence is constrained by the wall

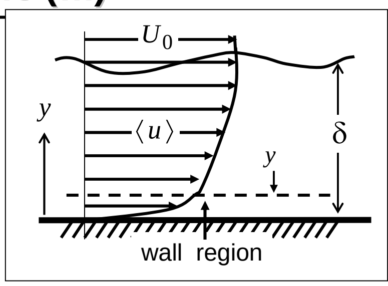
velocity $\boxed{u_0 \sim u_\tau = \sqrt{\tau_p/\rho}}$ **friction velocity** where $\tau_p(x) = \eta \dfrac{\partial \langle u \rangle}{\partial y}(x, y=0)$ = wall shear stress

length $\boxed{l_0 = l_0(y) \sim y}$ at a given height y turbulence scales cannot exceed y due to a blocage effect

- **boundary layer variables**

✓ introducing the viscous lenthscale $\delta_\nu = \nu / u_\tau$ ⇨ $\begin{cases} y^+ = \dfrac{y}{\delta_\nu} = \dfrac{u_\tau\, y}{\nu} = \dfrac{u_0\, l_0}{\nu} = \mathrm{Re}_0 \\ u^+ = \langle u \rangle / u_\tau \end{cases}$ **boundary layer variables**

16.4 scales : wall bounded shear flows (…)



- **scales : wall region**

velocity $u_0 \sim u_\tau = \sqrt{\dfrac{\tau_p}{\rho}} = \sqrt{\nu \dfrac{\partial \langle u \rangle}{\partial y}}(y=0)$

length $l_0 \sim y$

- **boundary layer variables**

$$\begin{cases} y^+ = \dfrac{y}{\delta_\nu} = \dfrac{u_\tau\, y}{\nu} = \dfrac{u_0\, l_0}{\nu} = \mathrm{Re}_0 \\ u^+ = \langle u \rangle / u_\tau \end{cases}$$

- **laminar region (viscous sublayer) :** $y^+ \sim 1 \quad (\mathrm{Re}_0 \sim 1)$

$\nu \dfrac{\partial \langle u \rangle}{\partial y} \approx \nu \dfrac{\partial \langle u \rangle}{\partial y}(y=0) = \dfrac{\tau_p}{\rho} = u_\tau^2$ ⇨ $\boxed{u^+ = \dfrac{\langle u \rangle}{u_\tau} = \dfrac{u_\tau y}{\nu} = y^+}$ **linear law**

- **turbulent region (log region) :** $y^+ >> 1 \quad (\mathrm{Re}_0 >> 1)$

production $P = -\nabla \langle \underline{u} \rangle : \langle \underline{u}' \otimes \underline{u}' \rangle \sim \dfrac{\partial \langle u \rangle}{\partial y} u_\tau^2$

dissipation $\varepsilon \sim \dfrac{u_0^3}{l_0} \sim \dfrac{u_\tau^3}{y}$

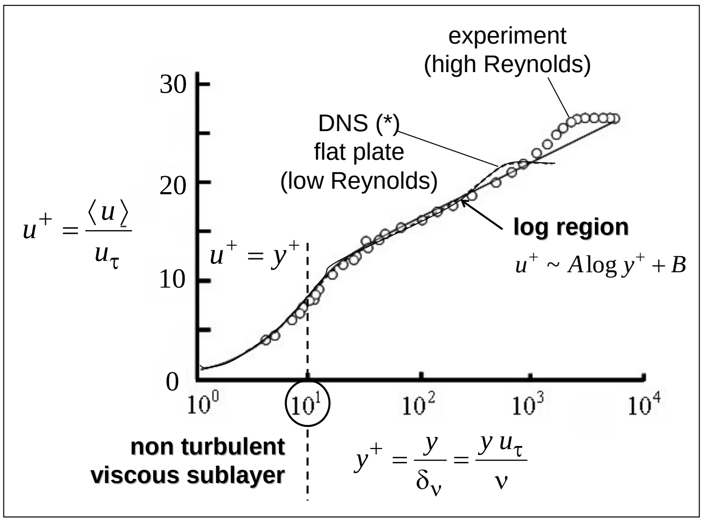
⇨ $\boxed{\text{cascade : } P \sim \varepsilon}$ ⇨ $\dfrac{\partial \langle u \rangle}{\partial y} u_\tau^2 \sim \dfrac{u_\tau^3}{y}$ ⇧ $\dfrac{\partial u^+}{\partial y^+} \sim \dfrac{1}{y^+}$ ⇧

log law

$\boxed{u^+ \sim A \log y^+ + B}$

16.4 scales : wall bounded shear flows (…)

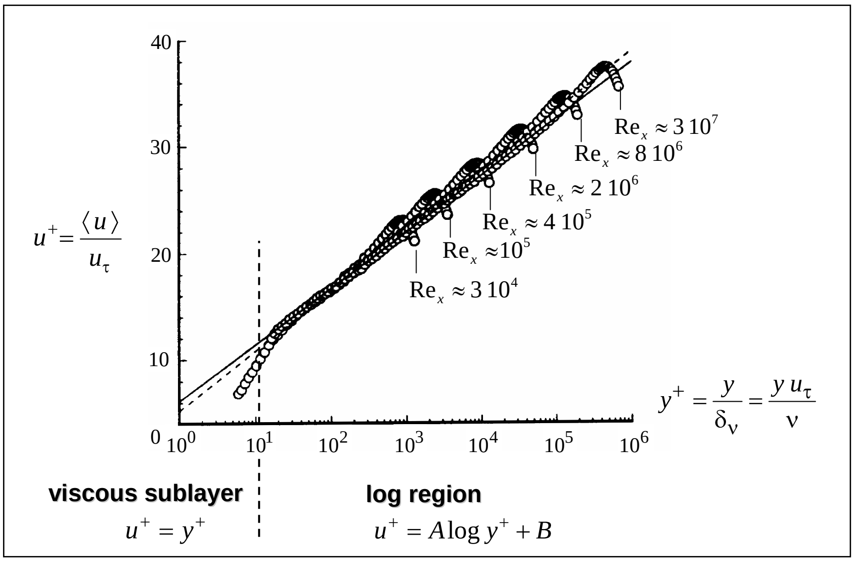
• **the log-law**



(*) DNS = Direct Numerical Simulation (see later)

16.4 scales : wall bounded shear flows (…)

• **the log-law : variation with the Reynolds number** $\mathrm{Re}_x = \frac{U_0 x}{\nu}$



Zagora & Smits 1997, see also Pope 2000

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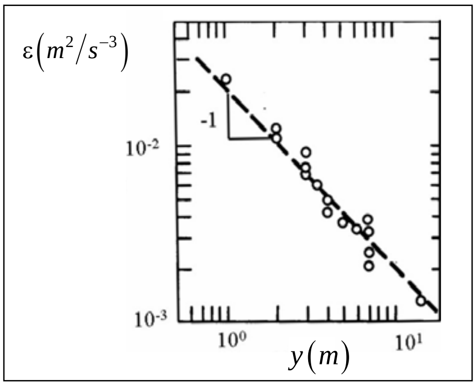
16.4 scales : wall bounded shear flows (…)

- **dissipation** $\varepsilon \sim \nu \langle |\nabla \underline{u}|^2 \rangle \sim \frac{u_0^3}{l_0} \sim \frac{u_\tau^3}{y}$ true ?

atmospheric boundary layer



Gibson & Williams 1969

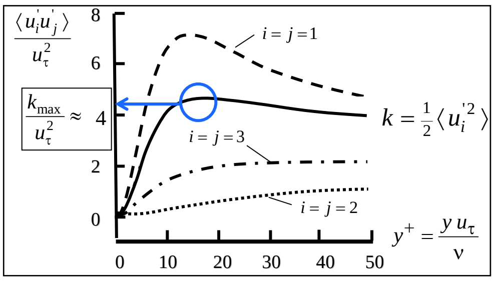
16.4 scales : wall bounded shear flows (…)

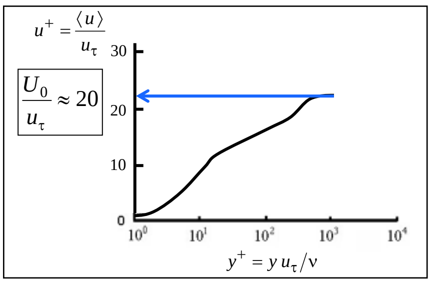
- **orders of magnitude**
  - ✓ turbulence rate $\boxed{\frac{u_0}{U_0} = const. \times \frac{u_\tau}{U_0}}$ $const. =$?
  - ✓ thickness see later

- **turbulence rate**
  - ✓ one choose $\boxed{\frac{u_0}{U_0} = \frac{\sqrt{k_{max}}}{U_0}}$

turbulent kinetic energy (TKE)

$$k = \frac{1}{2}\langle u_i'^2 \rangle = \frac{1}{2}\left(\langle u_1'^2 \rangle + \langle u_2'^2 \rangle + \langle u_3'^2 \rangle \right)$$





Spalart – DNS (1988)

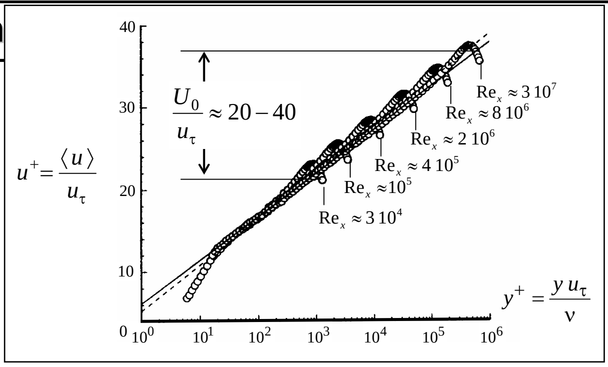
$\Rightarrow$ $u_0 \sim \sqrt{k_{max}} \approx 2u_\tau$ $\Rightarrow$ $u_0 \sim U_0/10$ $\Rightarrow$ turbulent rate ≈ 10%

16.5 boundary layer : rem

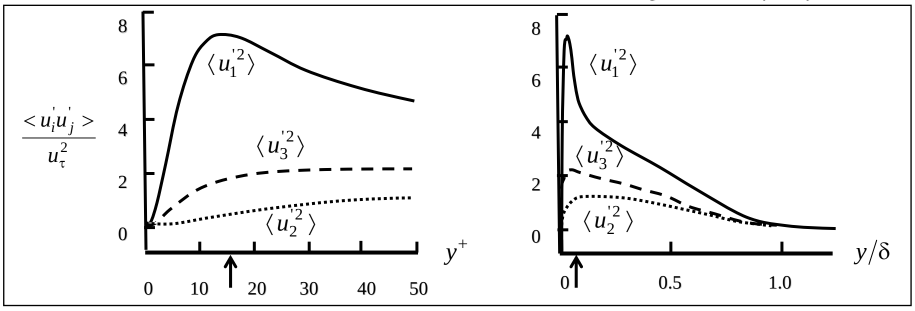
✓ the friction velocity varies weakly with Reynolds number Re_x

✓ boundary layer turbulence is strongly anisotopic , such as $\langle u_1'^2 \rangle >> \langle u_3'^2 \rangle > \langle u_2'^2 \rangle$

✓ turbulence reach its maximum at the bottom of the log region, at $y^+ \approx 15$



Zagora & Smits (1997), see also Pope (2000)



Spalart – DNS (1988), see also Pope (2000)

16.5 boundary layer : remarks (…)

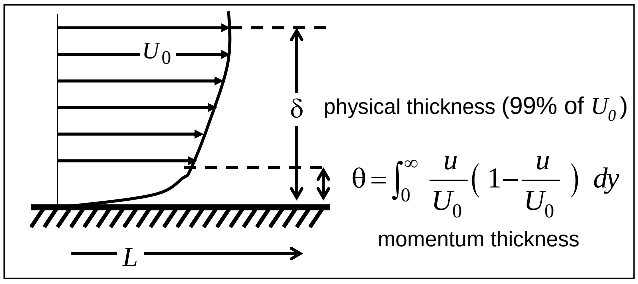
• the different Reynolds numbers in a boundary layer

$$\mathrm{Re}_x = \frac{U_0\, x}{\nu}$$

$$\mathrm{Re} = \frac{U_0\, L}{\nu}$$

$$\mathrm{Re}_\theta = \frac{U_0\, \theta}{\nu}$$

$$\mathrm{Re}^+ = \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$$



relations

$$\mathrm{Re}^+ = \mathrm{Re}\frac{u_\tau}{U_0}\frac{\delta}{L}$$

$$\mathrm{Re}_\theta = \mathrm{Re}\frac{\theta}{\delta}\frac{\delta}{L}$$

typical

$$\frac{U_0}{u_\tau} \approx 20-40$$

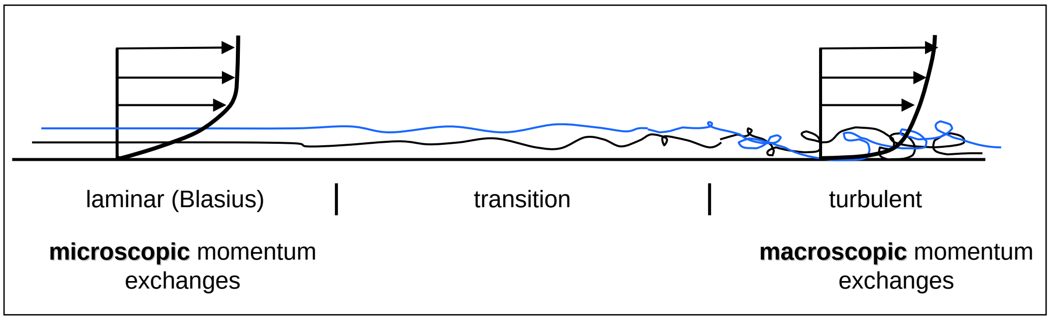
$$\frac{\theta}{\delta} \approx \frac{1}{10}$$

exemple : $U_0 = 10\ m/s$, $L = 1\ m$, $\delta \approx 10^{-2}\ m$

$$\begin{cases} \mathrm{Re} \approx 10^6 \\ \mathrm{Re}^+ \approx 3000 \\ \mathrm{Re}_\theta \approx 1000 \end{cases}$$

16.5 boundary layer : remarks (…)

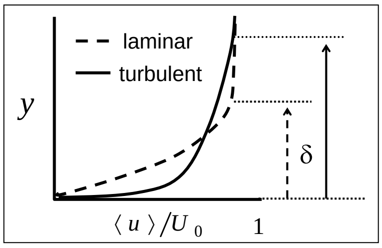
• **from laminarity (Blasius) to turbulence**



• **due to turbulence :**

✓ increase of the wall shear stress

✓ increase of the thicknesses



laminar (Prandtl's theory) $\frac{\delta}{x} \sim Re_x^{-1/2}$

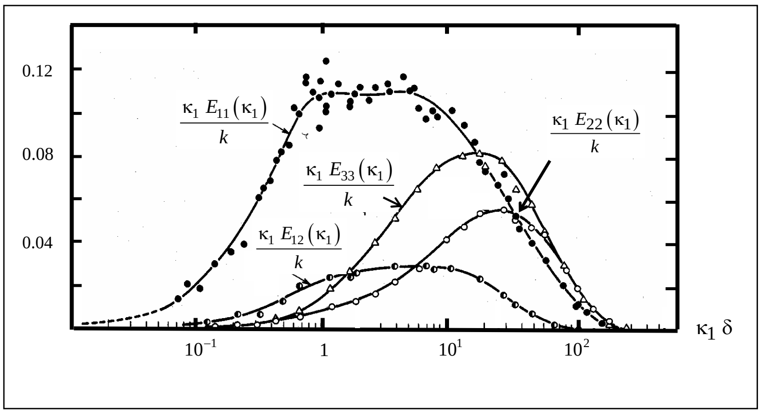
turbulent (empirical) $\frac{\delta}{x} \sim Re_x^{-1/6}$

16.5 boundary layer : remarks (…) not projected

• **scale anisotropy**

1D - spectra (hot-wires) in a boundary layer at $y_0/\delta = 0.11, y_0^+ = 217$

<u>Note</u> : this is a premultiplied semi-logarithmic plot of the 1D spectra $\kappa_i\ E_{ij}(\kappa_1) = f(\log \kappa_1)$ (see annex)



Fulachier 1972

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annex : premultiplied semi-logarithmic plots of spectra

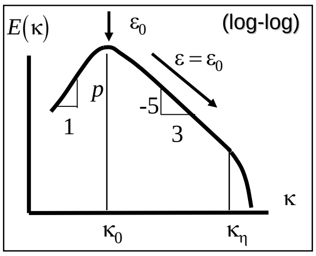
• **TKE and energy spectrum** $k = \frac{1}{2}\langle \underline{u}'^2 \rangle = \int_0^\infty E(\kappa)\, d\kappa$ — $E(\kappa)$ = distribution of energy among the wave numbers

• **self similarity :** the reason for the appearance of power laws such as $E_\kappa \sim \kappa^{-5/3}$ is that the inviscid equations are invariant to geometric scaling, so that the important relations are those between a **given length scale and its multiples**, rather than between scales which differ by a fixed amount. It is for this reason that spectra are usually plotted in **logarithmic**

not projected

• **but :** in doing so, that representation loses one of the useful graphic properties of the spectrum, which is to represent energies by integrals or by areas :

$$k = \int_0^\infty E(\kappa)\, d\kappa \neq \int_{-\infty}^{\infty} log\, E(\kappa)\, d\, log\, \kappa$$



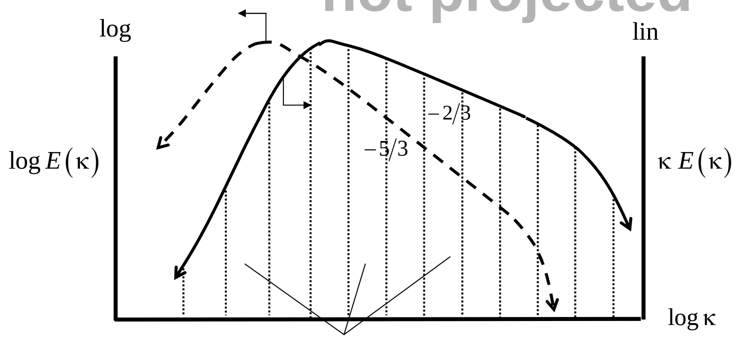
annex : premultiplied semi-logarithmic plots of spectra (…)

• **to remidy that :** it is useful to use semi-logarithmic plots of the pre-multiplied spectrum, writing

$$k = \frac{1}{2}\langle \underline{u}'^2 \rangle = \int_0^\infty E(\kappa)\, d\kappa = \int_0^\infty \kappa\, E(\kappa)\, d \log \kappa$$

where the premultiplied factor κ in front of the spectrum compensates for the differential of the logarithm, and the integral property is restored ⇨ integral under the curve gives the energy k

not projected



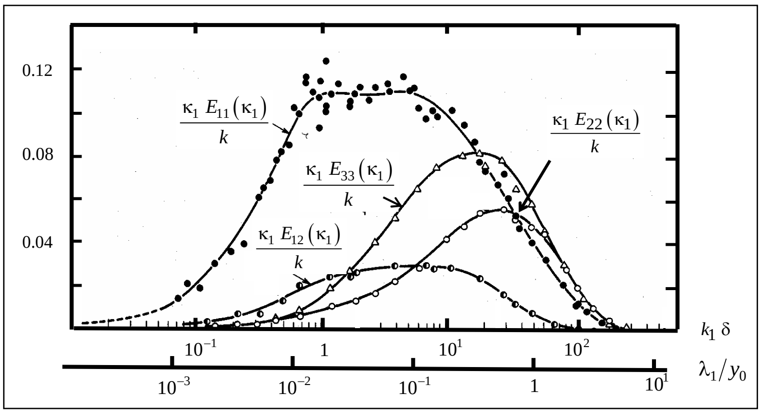
constant relative bandwidth $\delta \log \kappa = \frac{\delta\kappa}{\kappa} = const.$

annex : premultiplied semi-logarithmic plots of spectra (…)

- **scale anisotropy (…)**

not projected

1D - spectra (hot-wires) in a boundary layer at $y_0/\delta = 0.11, y_0^+ = 217$



Fulachier 1972

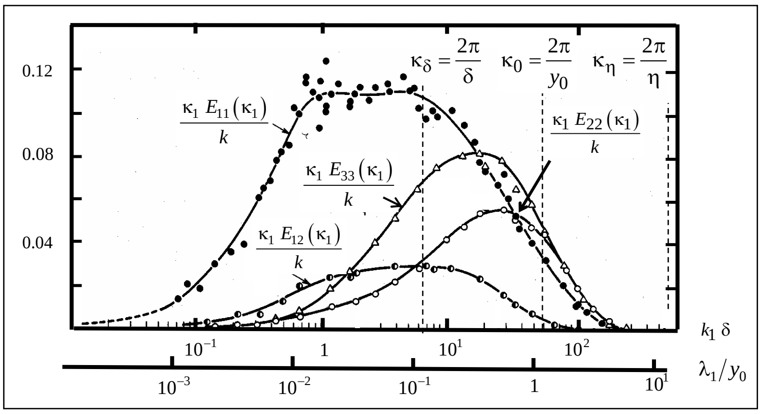
$$\Rightarrow \quad \frac{\langle u_i u_j \rangle}{k} = \int_0^\infty \frac{\kappa_1 E_{ij}(\kappa_1)}{k} d\log\kappa_1 , \quad k = \tfrac{1}{2}\langle u_i^2 \rangle$$

annex : premultiplied semi-logarithmic plots of spectra (…)

- **scale anisotropy (…)**

not projected

1D - spectra (hot-wires) in a boundary layer at $y_0/\delta = 0.11, y_0^+ = 217$



Fulachier 1972

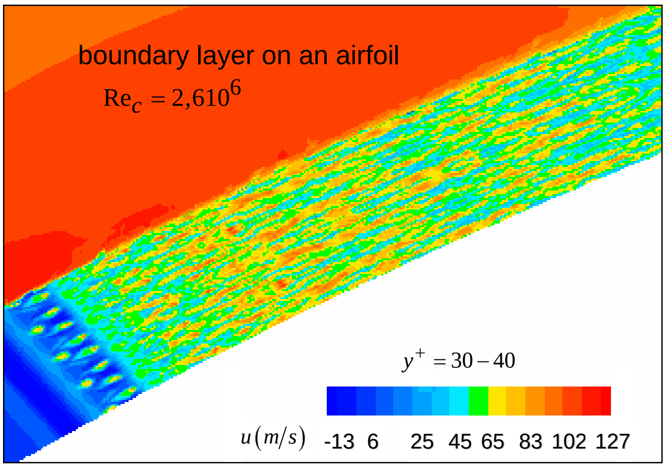
⇨ confirms the strongly anistropic nature of the boundary layer turbulence

⇨ existence of strong longitudinal velocity fluctutations of scales larger than δ

16.5 boundary layer : remarks (…)

• **scale anisotropy (…)** not projected

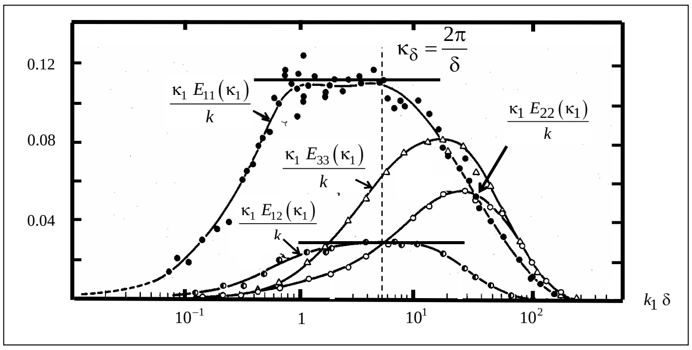
DNS of a boundary layer showing the presence of elongated structures, named streaks



Y. Mary (2000)

16.5 boundary layer : remarks (…) not projected

• **production scales**



Fulachier 1972
$\begin{cases} y_0/\delta = 0.11 \\ y_0^+ = 217 \end{cases}$

✓ note the following trend : maximum of $\kappa_1\, E_{11}(\kappa_1)$ and $\kappa_1\, E_{12}(\kappa_1)$ such that $\kappa\, E(\kappa) \sim const.$

✓ this conflicts the Richardson-Kolmogorov $E(\kappa) \sim k^{-5/3}$ law

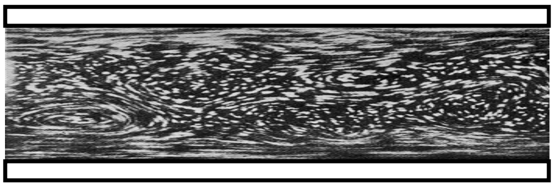
✓ such energy producing large scale structures lie outside the scope of an self-similar isotropic cascade model

✓ on dimensional grounds, to get this we must write $E = f(\kappa, u_\tau)$ instead of $E = f(\kappa, \varepsilon)$

✓ so $E \sim u_\tau^2\, \kappa^{-1} \Rightarrow \kappa\, E \sim const.$ ⇨ these structures contribute a lot to the wall shear stress u_τ

16.6 channel flow

• **turbulent channel flow : remainder**



✓ far away from the entrance, if the Reynolds number is large enough, an equilibrium turbulent shear flow is obtained

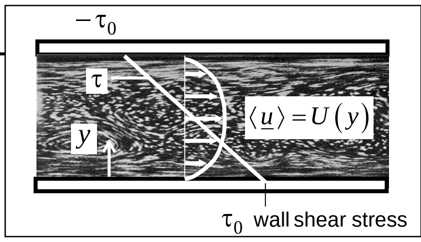
✓ a seen in chapter 6, the dissipation rate per unit mass averaged on time and in a volume V, $\varepsilon_V = \overline{\langle \varepsilon \rangle_V}^T$, fulfils :

$$\frac{\varepsilon_V}{\overline{U_b}^T / L} \approx const.$$

mean dissipation rate per unit mass

where $\overline{U_b}^T$ denotes the time averaged bulk velocity

16.6 channel flow (…)



• **the Reynolds equation**

$$\frac{D\langle \underline{u} \rangle}{Dt} = \frac{\partial \langle \underline{u} \rangle}{\partial t} + \nabla \langle \underline{u} \rangle . \langle \underline{u} \rangle$$

$$= -\frac{1}{\rho} \underline{grad} \langle p \rangle + div\left(2\nu \langle \underline{\underline{d}} \rangle - \langle \underline{u}' \otimes \underline{u}' \rangle \right)$$

• **x- component**

H1 - unidirectional steady mean flow $\langle \underline{u} \rangle = U(y) \underline{e}_x$

H3 - x - wise and z - wise statistical homogeneity

$$\Rightarrow \quad \cancel{\frac{\partial U}{\partial t}} + U \cancel{\frac{\partial U}{\partial x}} = \underbrace{-\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x}}_{G>0} + \nu \frac{\partial^2 U}{\partial y^2} - \cancel{\frac{\partial}{\partial x} \langle u'^2 \rangle} - \frac{\partial}{\partial y} \langle u' v' \rangle - \cancel{\frac{\partial}{\partial z} \langle u' w' \rangle}$$

$$\Rightarrow \quad 0 = G + \frac{\partial}{\partial y} \underbrace{\left(\nu \frac{\partial U}{\partial y} - \langle u' v' \rangle \right)}_{\tau \;=\text{shear stress}} \quad \Rightarrow \quad \boxed{\frac{\partial \tau}{\partial y} = -G}$$

τ affine

$$\begin{cases} \tau(y) = \tau^{visc}(y) + \tau^{turb}(y) & \text{- total} \\ \tau^{visc}(y) = \nu \, \partial U / \partial y & \text{- viscous} \\ \tau^{turb}(y) = -\langle u' v' \rangle & \text{- turbulent} \end{cases}$$

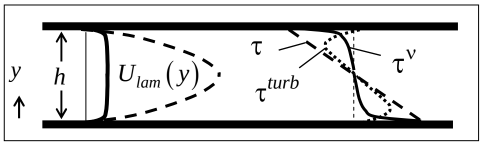
16.6 channel flow (…)

• **the x- component of the mean momentum equation (…)**

$$\boxed{\frac{\partial \tau}{\partial y} = -G}$$

τ affine

$$\begin{cases} \tau(y) = \tau^{visc}(y) + \tau^{turb}(y) \\ \tau^{visc}(y) = \nu\, \partial U/\partial y \\ \tau^{turb}(y) = -\langle u' v' \rangle \end{cases}$$



• **laminar regime :** $\partial\tau/\partial y = \partial\tau^{visc}/\partial y = \nu\, \partial^2 U/\partial y^2 = -G$ ⇨ $\boxed{U_{lam}(y) = -\frac{G}{2\nu}\left(y^2 - \frac{h^2}{4}\right)}$

poiseuille

• **turbulent regime :** $\partial\left(\tau^{visc} + \tau^{turb}\right)/\partial y = -G$

• **shear stress boundary conditions and symmetry**

$$\begin{cases} \tau^{turb}(0) = -\rho\langle u'v'\rangle(0) = 0 \\ \tau^{turb}(h) = -\rho\langle u'v'\rangle(h) = 0 \\ \tau^{turb}\left(\frac{1}{2}h\right) = 0 \end{cases}$$

(first two: no sleep; third: flow symmetry)

⇨ the turbulent stress $\tau^{turb}(y) = -\langle u' v' \rangle$ can freely develop away from the wall

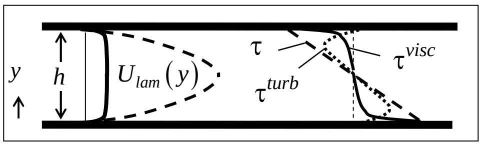
⇨ the viscous stress $\tau^{visc}(y) = \nu\, \partial U/\partial y$ concentrates close to the wall

⇨ integrating $\tau^{visc}(y) = \nu\, \partial U/\partial y$ leads to a U profile flatter than Poiseuille's

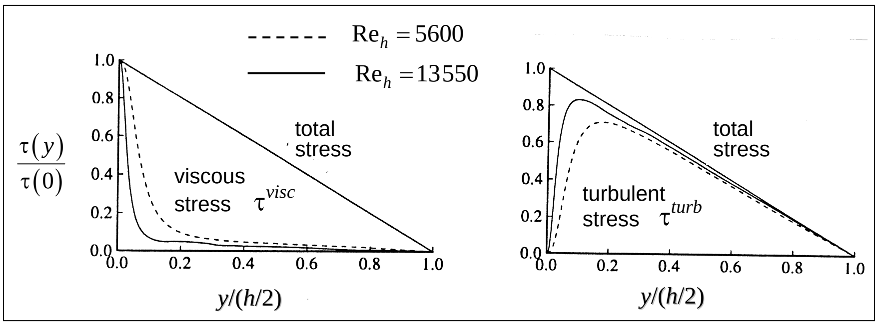
⇨ turbulence slows down the flow

16.6 channel flow (…)

• **observations : DNS**



$$\mathrm{Re}_h = \frac{U_b\, h}{\nu}$$

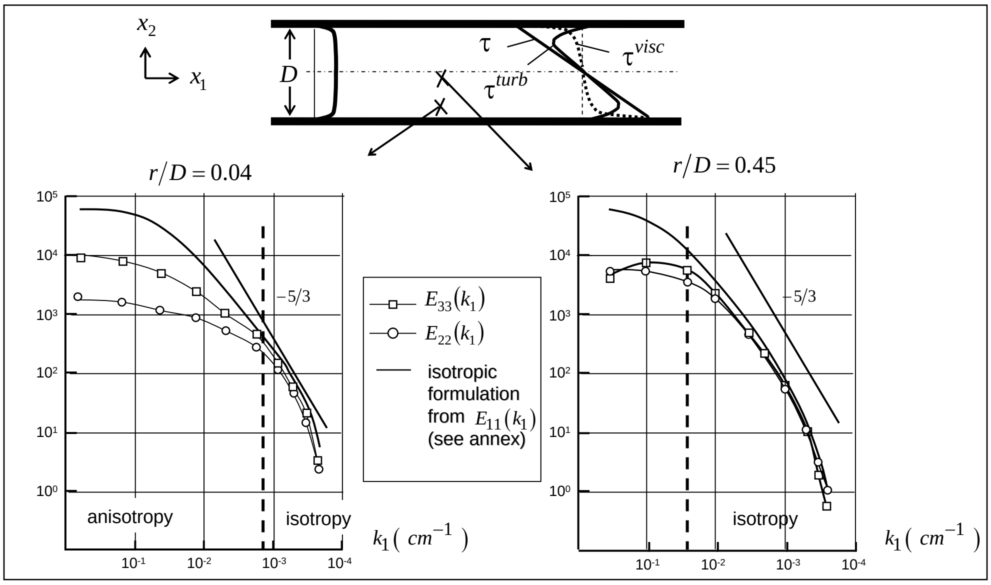


Pope – DNS (2000)

16.6 channel flow (…)

not projected

- **observations : spectra (experiment)**



Comte Bellot et al. (1971)

annex – isotropic formulation of the 1D spectra (remainder)

- **back to chapter 15**

not projected

✓ knowing the true longitudinal 1D spectrum $E_{11}(\kappa_1)$, one calculates the isotropic energy $E(\kappa)$ by means of the formulae :

$$E(\kappa = \kappa_1) = \frac{1}{2}\kappa_1^3 \frac{d}{d\kappa_1}\left(\frac{1}{\kappa_1}\frac{dE_{11}(\kappa_1)}{d\kappa_1} \right)$$

✓ starting from the three 1D spectra , one can also use :

$$E(\kappa = \kappa_1) = -\frac{1}{2}\kappa_1 \frac{dE_{ii}(\kappa_1)}{d\kappa_1}, E_{ii}(\kappa) = E_{11}(\kappa) + E_{22}(\kappa) + E_{33}(\kappa)$$

✓ then we can go back to an isotropic formulation of the spectra $E_{22}(\kappa_1), E_{33}(\kappa_1)$ as done in the figure of the previous slide (Comte-Bellot et al. 1971), using :

$$E_{22}^{iso}(\kappa_1) = E_{33}^{iso}(\kappa_1) = \int_{\kappa_1}^{\infty} \frac{E(\kappa)}{\kappa}\left(1 + \frac{\kappa_1^2}{\kappa^2} \right) d\kappa$$

16.7 turbulent shear flows : summary not projected

- **the two main classes of turbulent flows have been inspected : free or wall bounded shear flows**

- **the cascade model still works in such flows as far as the two following main hypotheses are respected :**

✓ scale decoupling (large Reynolds number $\mathrm{Re}_0 = u_0 l_0 / \nu$)
✓ statistical stationnarity of energy injection

- **problematic cases are those where turbulence is put out of equilibrium**

✓ turbulence extinction close to walls
✓ interfaces separating turbulent – laminar regions (intermittency)
✓ rapid variations in the mean field ($\Rightarrow$ variations of the energy input)

- **away from such situations, « cascade based » models of turbulence may be imagined**

16.8 turbulent shear flows : research briefs

- **free shear flows : sensitivity to initial conditions**

✓ initial conditions of free shear flows … are fixed by wall bounded shear flows

✓ this can be used to control the jet

film

16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation or acoustic forcing**

not projected

R4_0001-2010.avi

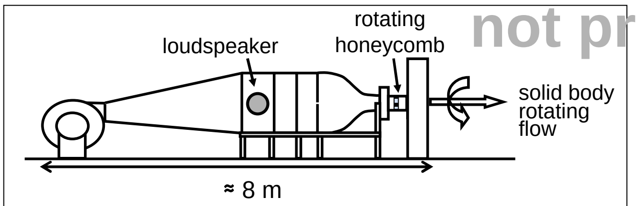
film

16.8 turbulent shear flows : research briefs (…)

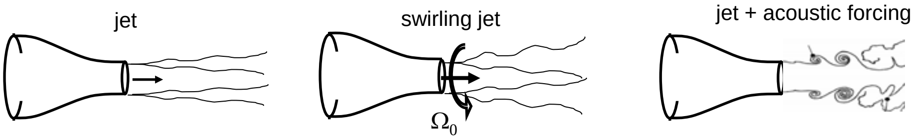
- **jet : turbulent shear flows under rotation or acoustic forcing (…)**

✓ apparatus

not projected



R4Ch wind-tunnel, ONERA / DAFE



exit velocity :	$U_0 = 21.6$ m/s
exit diameter :	$D = 0.15$ m
Reynolds number :	$Re = 2.14\ 10^5$
swirl number :	$0 \le S = \dfrac{\Omega_0 D}{2U_0} \le 0.8$

@ Master MEC654 - Turbulence - Laurent Jacquin - 2014 - Polytechnique

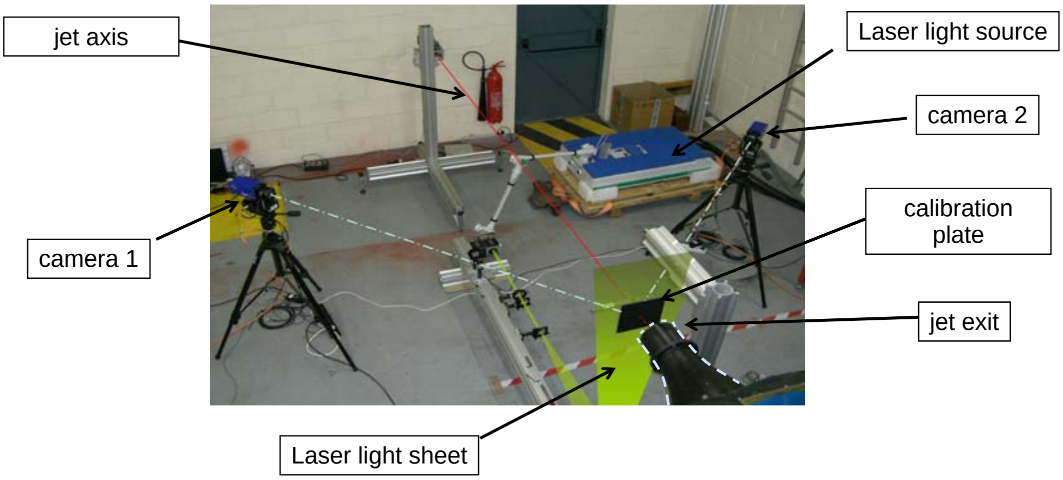
16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation (…)**

not projected

✓ measurements

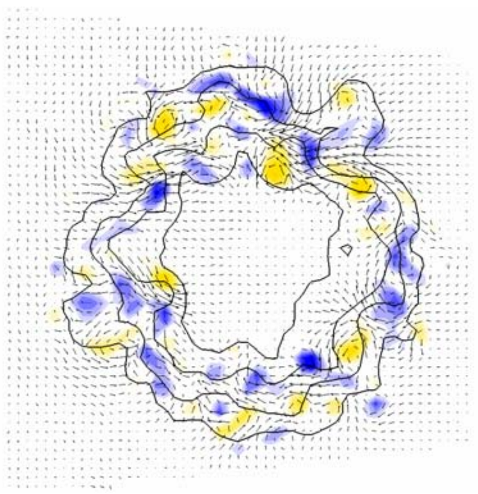
PIV : standard and time resolved



16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation (…)**

not projected

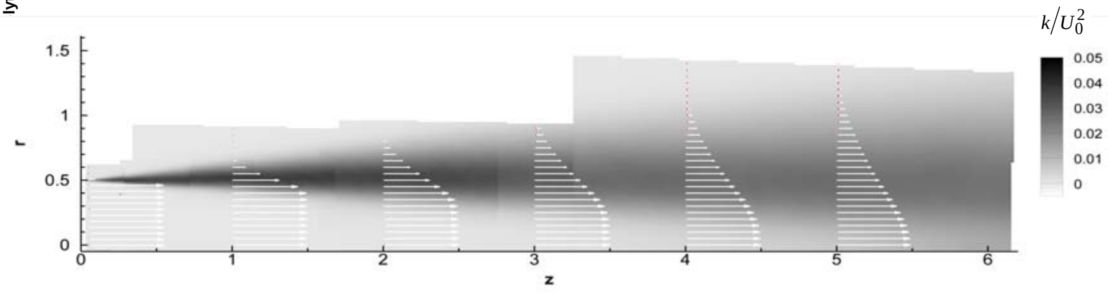


Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation (…)**

not projected



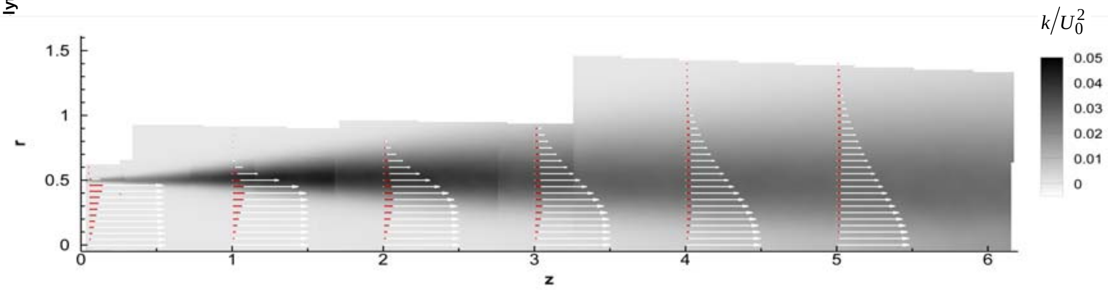
S=0

Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation (…)**

not projected



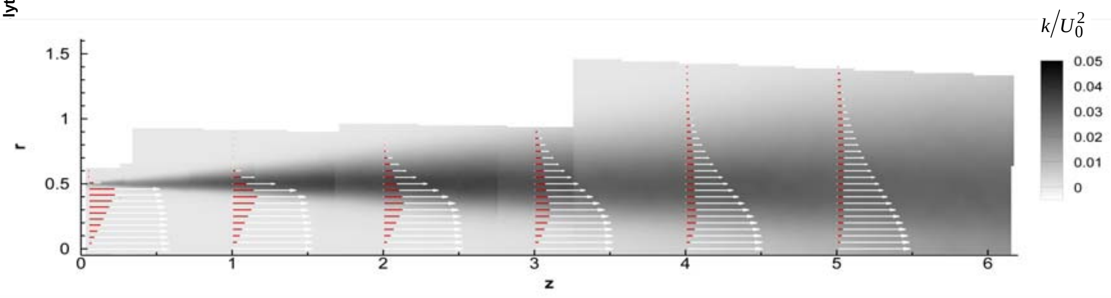
S=0.2

Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- jet : turbulent shear flows under rotation (…)

not projected



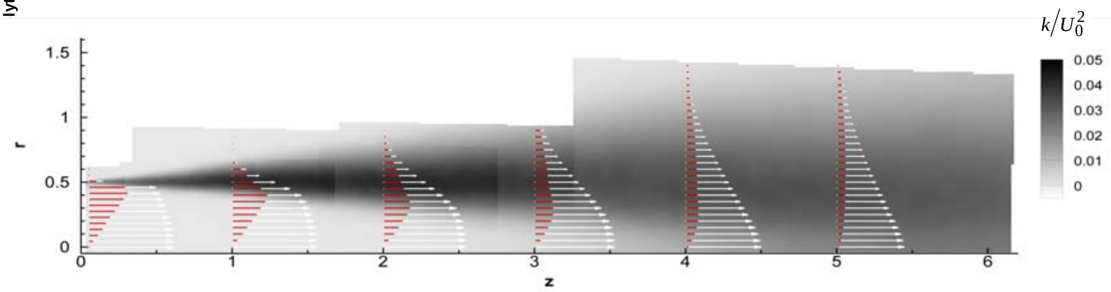
S=0.4

Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- jet : turbulent shear flows under rotation (…)

not projected



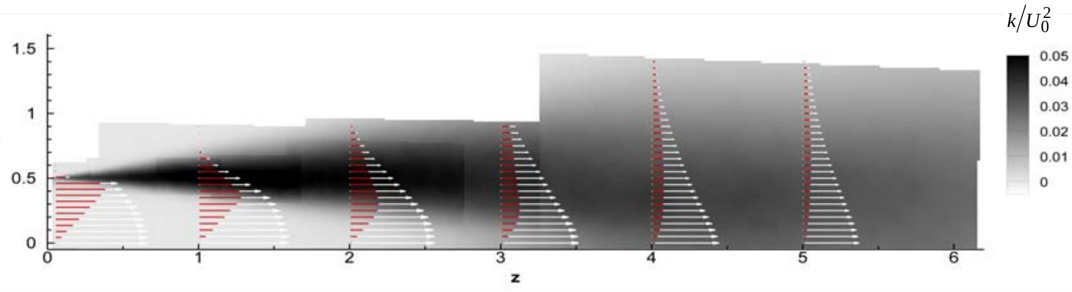
S=0.6

Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under rotation (…)**

not projected



S=0.8

Davoust et al. (2012)

16.8 turbulent shear flows : research briefs (…)

- **jet : turbulent shear flows under acoustic forcing**

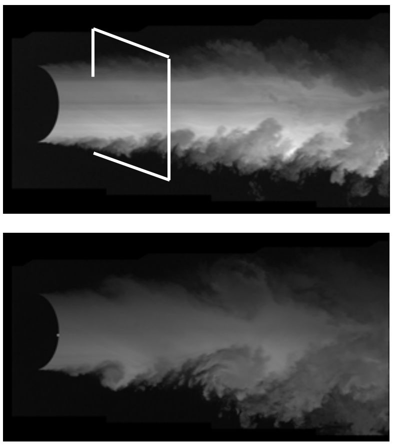
not projected

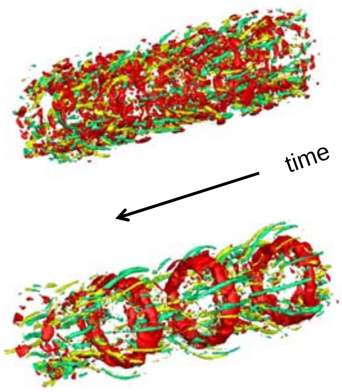
smoke visualisation

time resolved 2D-PIV at $x / D = 2$

✓ no forcing

✓ strong forcing





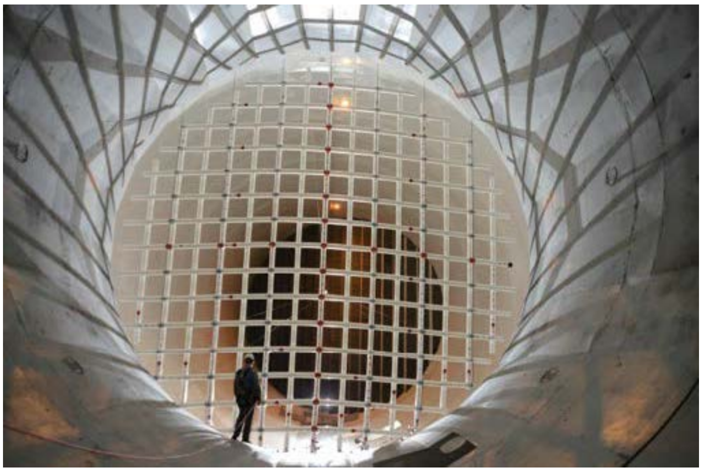
Courtier et al., in preparation

16.8 turbulent shear flows : research briefs (…)

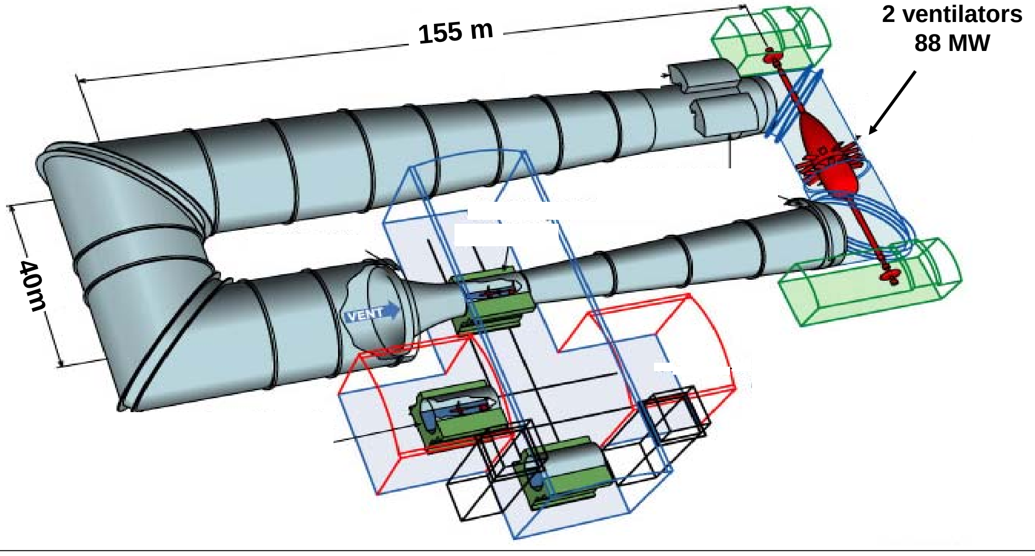
- **grid turbulence : not a turbulent shear flow (but a research brief on turbulence)**

✓ the "biggest" grid experiment on grid turbulence in the world (July 2014) !

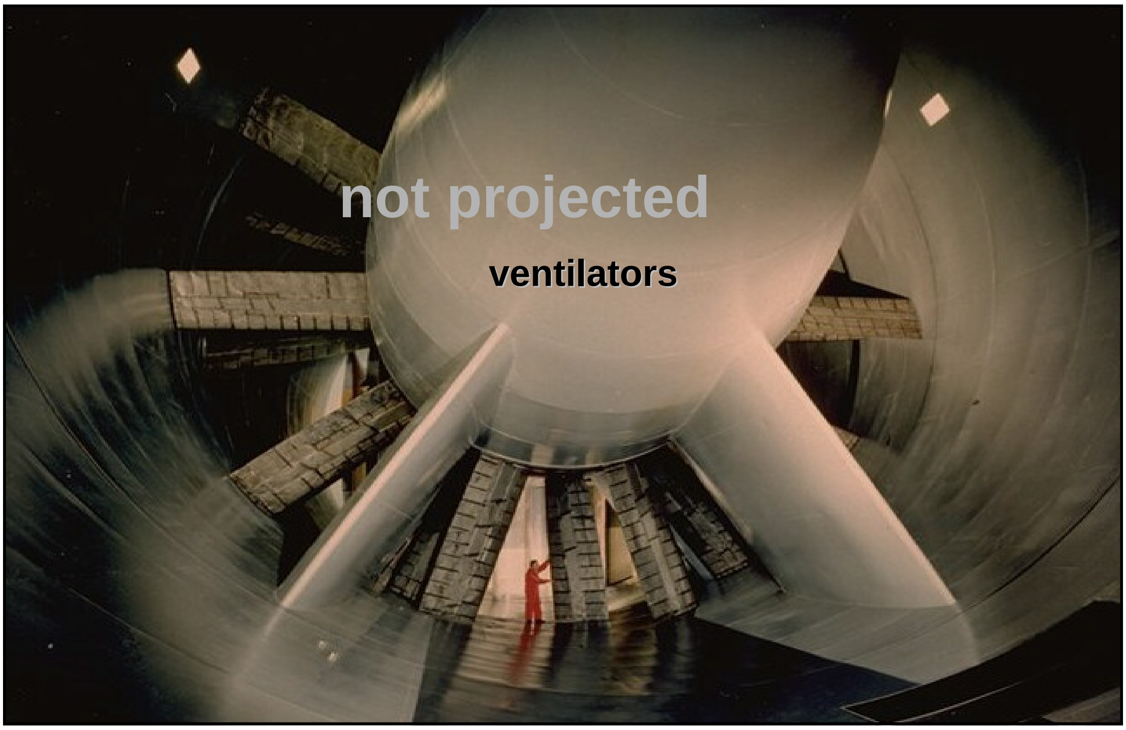
not projected

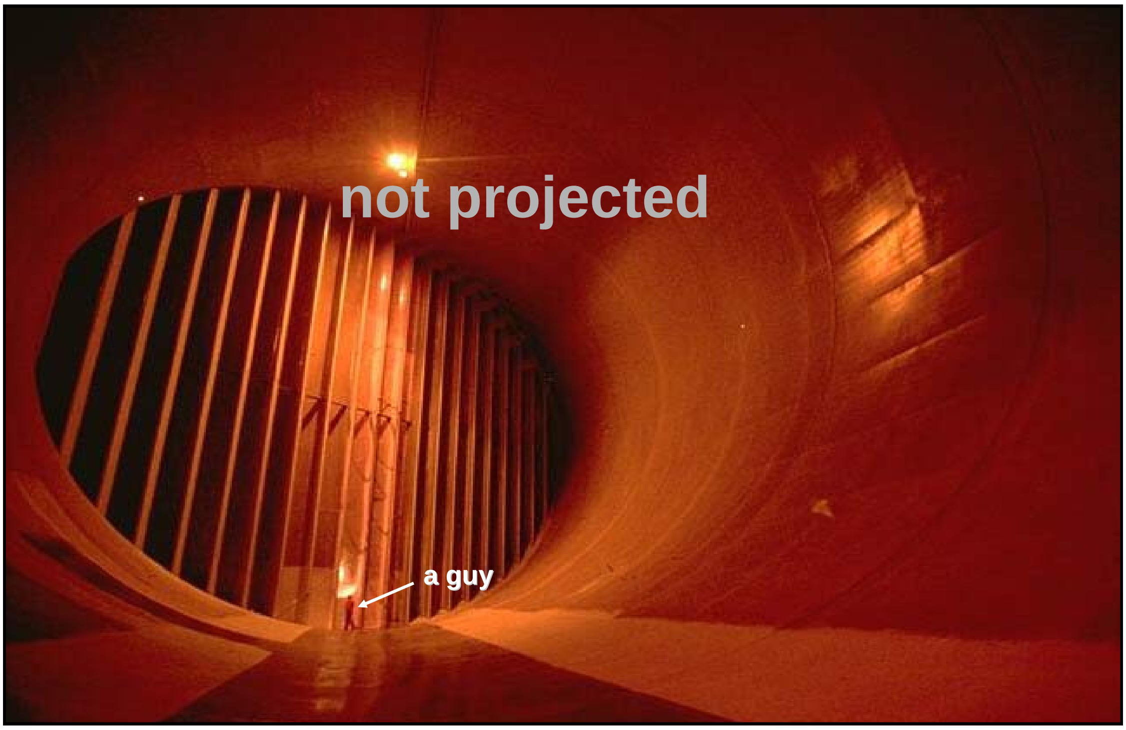


not projected



Transonic wind tunnel ONERA - S1 - Modane


not projected
ventilators


not projected
a guy

END