PC5 : IMPACT OF ROTATION ON THE KOLMOGOROV CASCADE

Consider a turbulent fluid, initially homogeneous and isotropic. The fluid is in solid body rotation with rotation vector $\underline{\Omega}$. In the rotating frame, the Navier-Stokes equations become:

$$\begin{cases} div\, \underline{u} = 0 \\ \dfrac{d\underline{u}}{dt} + 2\underline{\Omega} \times \underline{u} = -\underline{grad}\, \Pi + \nu\, \Delta \underline{u} \end{cases}$$

where $\underline{u}$ denotes velocity in the rotating frame, and $\Pi = \frac{p}{\rho} - \frac{1}{2}\left(\underline{\Omega} \times \underline{x}\right)^2$ defines an "equivalent" pressure.

We introduce the non-dimensional variables : $\underline{x} = l\overline{\underline{x}}, \underline{u} = V\overline{\underline{u}}, t = \frac{l}{V}\bar{t}, \underline{\Omega} = \Omega\overline{\underline{\Omega}}, \Pi = P\overline{\Pi}$, where l, V are length and velocity scales characteristic of the turbulent motion, and P is the scale for pressure.

(a) Write the equations of motion in non-dimensionalized form. You will introduce the two non-dimensional numbers, the Rossby number $Ro = \frac{V}{2\Omega l}$ and the Ekman number $E = \frac{\nu}{2\Omega l^2}$. What do those numbers represent physically?

(b) Show that in the limit $Ro \to 0, E \to 0$, the motion is invariant along the rotation axis, hence two-dimensional (recall the vector identity : $\underline{rot}\left(\underline{A} \times \underline{B}\right) = \nabla \underline{A}.\underline{B} - \underline{A}\, div \underline{B} \; - \nabla \underline{B}.\underline{A} + \underline{B}\, div\, \underline{A}$).

(c) Therefore, in the limit of strong rotation, the motion becomes two-dimensional. Give at least one argument explaining why, in this limit, the Kolomogorov theory can not apply.

(d) Consider now the case of weak or moderate rotation. Find the "limiting" scale l_Ω, with associated wavenumber κ_Ω, in the inertial regime, which separates scales at which the rotation is felt and scales at which rotation is negligible. (**Hint :** give the expression of the Rossby number at each scale in the Kolmogorov cascade).

(e) Deduce qualitatively how the energy spectrum $E(\kappa)$ of a turbulent flow under moderate rotation evolves, assuming that its "limiting" wavenumber κ_Ω is in the middle of the inertial range.