PC7 - The planar jet

In an incompressible homogeneous fluid, initially at rest, consider a two-dimensional jet resulting from a thin hole infinite in the Oz direction.

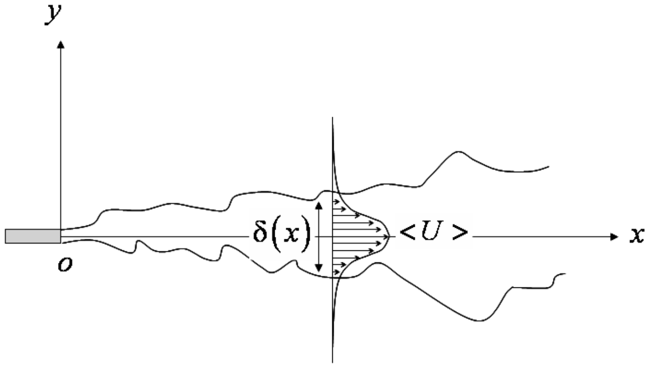

Fig.1 – Turbulent jet

We assume that the flow is turbulent, and that it is in statistical steady state. The velocity $\underline{U}(x,y,t)$ of the fluid is decomposed into a mean field $<\underline{U}>(x,y) = (<U>,<V>,0)(x,y)$, where $<.>$ denotes an ensemble average, and a three-dimensional fluctuation $\underline{u}(x,y,z,t)=(u,v,w)(x,y,z,t)$.

We further assume that the flow is confined in a thin layer, in other words gradients in the Oz and Ox directions are much smaller than gradients in the Oy direction:

$$\frac{\partial}{\partial z} \sim \frac{\partial}{\partial x} \sim \frac{1}{L}, \frac{\partial}{\partial y} \sim \frac{1}{\delta} \tag{1a}$$

$$\frac{\delta}{L} = \varepsilon << 1 \tag{1b}$$

where δ denotes a characteristic thickness of the jet. The mean pressure field $<P>(x,y)$ satisfies:

$$\frac{\partial <P>}{\partial x} = 0 \tag{2}$$

1. Denoting U the characteristic scale of $<U>$ and V that of $<V>$, show from the continuity equation that:

$$V \sim \varepsilon U \tag{3}$$

2. We now assume that the Reynolds number $Re = \frac{UL}{\nu}$ is very large, such that:

$$Re = \frac{UL}{\nu} >> \frac{1}{\varepsilon^2} \tag{4}$$

and we introduce the scale for the turbulent velocity u_0 satisfying:

$$<u^2> \sim <v^2> \sim <w^2> \sim <uv> \sim u_0^2 \tag{5}$$

(i.e. all the second order moments have the same order of magnitude u_0^2).

Write the Reynolds equations in the Ox direction ($<U>$ equation). Looking at the leading order terms of this equation, from (1a), (1b), (2), (3), (4) et (5) show that the turbulent scale u_0 must satisfy :

$$u_0 \sim \sqrt{\varepsilon} \times U , \tag{6}$$

and that the equation becomes:

$$<U> \frac{\partial <U>}{\partial x} + <V> \frac{\partial <U>}{\partial y} = -\frac{\partial <uv>}{\partial y} \tag{7}$$

3. Rewrite this equation in conservative form. Accounting for the fact that the flow at infinity is at rest, show that:

$$M = \int_{-\infty}^{\infty} \rho < U >^2 \, dy = Cte \qquad (8)$$

This equation translates the conservation of momentum per unit length in z.

4. Experiments show that $\delta(x)$ which characterizes the width of the jet, is linear $\delta \sim x$. Furthermore, the mean field is observed to depend on the similarity variable:

$$< U > (x, y) = U_0(x) f(\eta), \qquad (9)$$

where

$$\eta = \frac{y}{\delta(x)}, \qquad (10)$$

and $U_0(x)$ denote the maximum mean velocity on the jet axis

$$U_0(x) = < U > (x, y = 0), \qquad (11)$$

From the results of question **3**, use (8), (9) and (10) to determine the evolution along the jet axis of the three following quantities:

- the velocity on the axis $U_0(x)$; (12)
- the kinetic energy flux $\frac{1}{2} < U^2 >$: $E(x) = \frac{1}{2}\int_{-\infty}^{\infty} \rho < U >^3 \, dy$; (13)
- the mass flux: $m(x) = \int_{-\infty}^{\infty} \rho < U > dy$. (14)

Comment on the physical interpretation of the two last results (variation of the energy and mass flux).

5. What do those 3 laws become in the case of an axisymmetric jet, in which case:

$$M = \int_{-\infty}^{\infty} \rho < U >^2 \, r \, dr = Cte, \qquad (15)$$

assuming that we also observe a linear behavior of $\delta(x)$, as in (9), and a self-similar distribution of $< U >$, as in (10) ?

6. What happens if a wall is placed near the jet (planar or axisymmetric), as in figure 2(a)? What happens if another jet is placed in the neighborhood of the first jet, as in figure 2(b)? Modify (qualitatively) each panel in the figure below to account for those results.

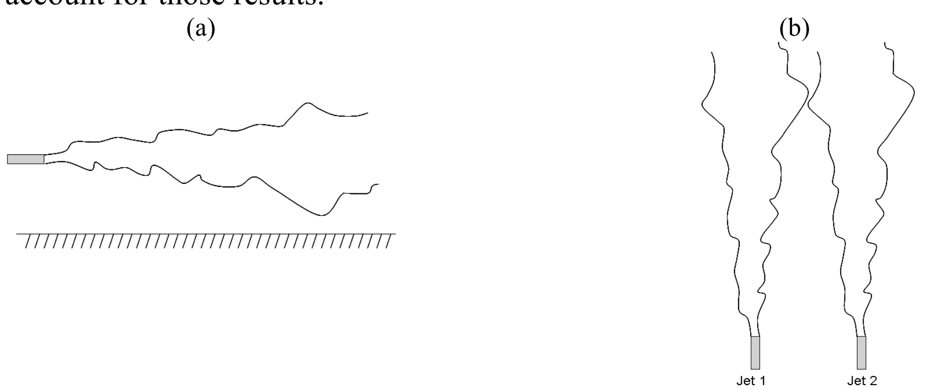

Fig. 2 – (a) : jet near a wall, (b) : two parallel jets