PC 1 : DISSIPATION, ENERGETICS, AND TURBULENT CASCADE

Goals:

- To manipulate the equations, the concept of dissipation and associated energetics; To introduce dimensional analysis of turbulent regimes.

1. Dissipation rate

Consider an incompressible viscous fluid contained in a volume Ω (bounded or not). We will denote its velocity and its pressure $\underline{u}(\underline{x},t)$, $p(\underline{x},t)$.

(a) The local form of the kinetik energy equation is obtained by multiplying the Navier-Stokes equations by the velocity $\underline{u}$. The viscous term of the Navier-Stokes equations for an incompressible fluid being $div\left(2\nu\, \underline{\underline{d}}\right) = \nu\, \Delta \underline{u}$, show that the viscous term of the kinetik energy equation satisfies the following relationship:

$$D = \nu\, \underline{u}.\Delta\underline{u} = 2\nu\, div\left(\underline{\underline{d}}.\underline{u}\right) - \varepsilon \qquad (1.1)$$

where

$$\varepsilon = 2\nu\, \underline{\underline{d}} : \underline{\underline{d}} \qquad (1.2)$$

is the dissipation rate per unit mass, positive definite, and $\underline{\underline{d}} = \frac{1}{2}\left(\nabla\underline{u} + {}^t\nabla\underline{u}\right)$ is the tensor of deformation rates.

(b) Compute the two terms of D in the case of a simple steady shear flow $\underline{u} = (\alpha\, y, 0, 0)$ with shear $\underline{u} = (\alpha\, y, 0, 0)$. Comment.

(c) Show the following identity :

$$\varepsilon = 2\nu\, div\left(\nabla\underline{u}.\, \underline{u}\right) + \nu\, \underline{\omega}^2 \qquad (1.3)$$

where $\underline{\omega} = \underline{rot\, u}$.

(d) Determine the equation governing the conservation of kinetik energy in the volume Ω. Show that it can be written as:

$$\frac{\partial}{\partial t}\int_\Omega \frac{u^2}{2} d\Omega = \int_{\partial\Omega}\left[\left(-\frac{p}{\rho}\underline{\underline{1}} + 2\nu\, \underline{\underline{d}}\right).\underline{u} - \frac{u^2}{2}\underline{u}\right].\underline{n}\, da - \int_\Omega \varepsilon\, d\Omega \qquad (1.4)$$

where $\underline{n}$ denotes the unit vector normal to $\partial\Omega$.

2. Pressure spectrum

Let $E_\Pi(k)$ denote the spectrum of pressure fluctuations $\Pi = \frac{p'}{\rho}$ of a homogeneous isotropic turbulent fluid. Using the Kolmogorov theory, determine the scaling law of this spectrum in the inertial range.

3. Size of bubbles

Consider air bubbles in a homogeneous turbulent flow of water, without mean velocity. The interfacial tension between air and water is denoted γ, with dimension $kg\, s^{-2}$.

Using the Kolmogorov theory, deduce as a function of dissipation ε, of the liquid density (water) ρ_L and of γ, the characteristic radius R of the bubbles at equilibrium in this turbulent flow.