Statistical downscaling of precipitation using extreme value theory

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Outline

Statistical downscaling – general overview

- Aim of statistical downscaling
- Approaches and Methods
- with regard to extremes

Our Problem

- Data
- Probabilistic Downscaling
- Extreme value theory
- Censored quantile regression
- Verification

Results

- Extreme precipitation events
- Skill of downscaling

Conclusions

Weather Forecasting

DYNAMICS:

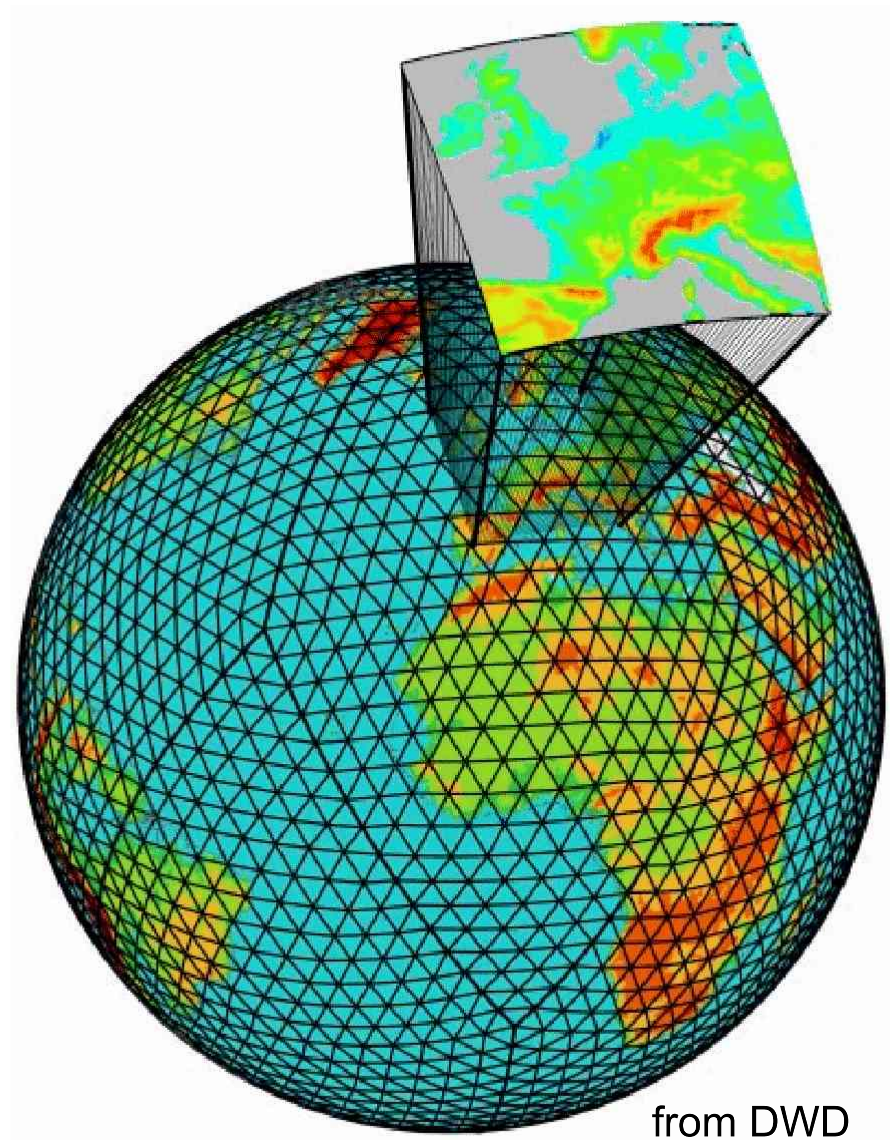
In numerical weather forecasting the **Navier Stokes equations** on a rotation sphere and the 1st principle of thermodynamics are numerically intergrated in time

PHYSICS:

sub-scale processes are parametrized
- cloud micro-physics
- radiative transfer
- exchange processes with land surface...
- turbulence
- **convection** ...

Extreme weather phenomena are (mostly) on small scales

–> post-processing

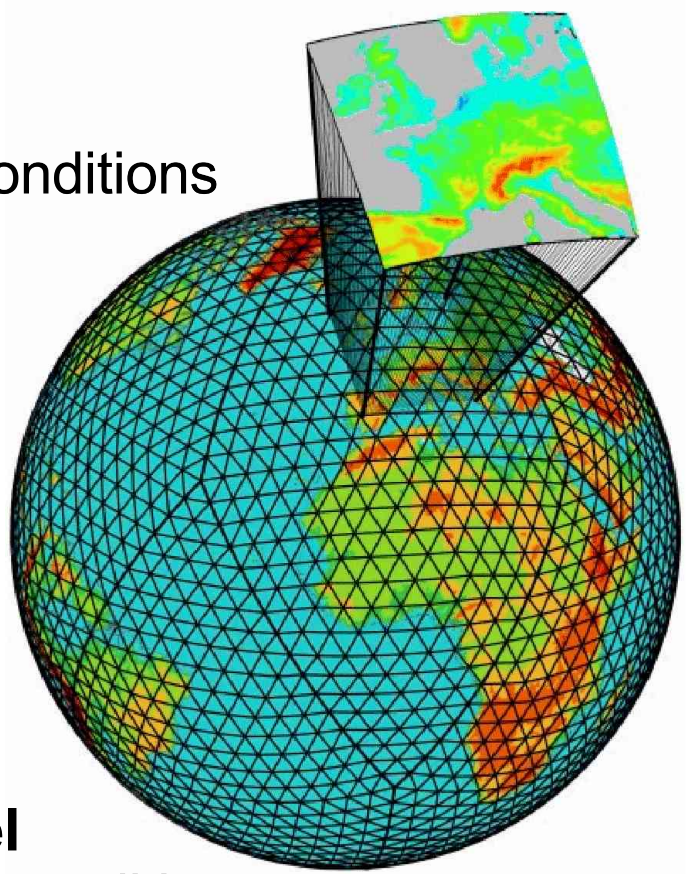


from DWD

Downscaling

Aim of downscaling:

Represent and forecast local weather or climate conditions on scales not resolved by the model.
Here: downscaling in space!



Dynamical and **statistical** downscaling

Dynamical downscaling uses a **regional, high resolution model**, where the boundary conditions are determined by the global model.

Statistical downscaling derives a **statistical model** between the large scale model state and the local conditions.

Downscaling

Applications:

Short range weather prediction
• regional models to account for local conditions (orography) and small scale processes (convection)
• statistical downscaling (MOS) for local (weather station) forecasts
• assessing probabilities e.g. of extreme events

Climate change projections
• estimate possible man induced changes on the local scale

Weather generator
• generate realistic precipitation patterns for hydrological modeling

Methods of Stat. Downscaling

Linear transfer functions

- multiple linear regression
- canonical correlation analysis

Non-linear transfer functions

- neural networks

Stochastic models

- multivariate autoregressive models
- conditional weather generator

Weather typing methods

- conditional resampling
- analogue-based methods

Probabilistic downscaling

- estimate conditional distribution function

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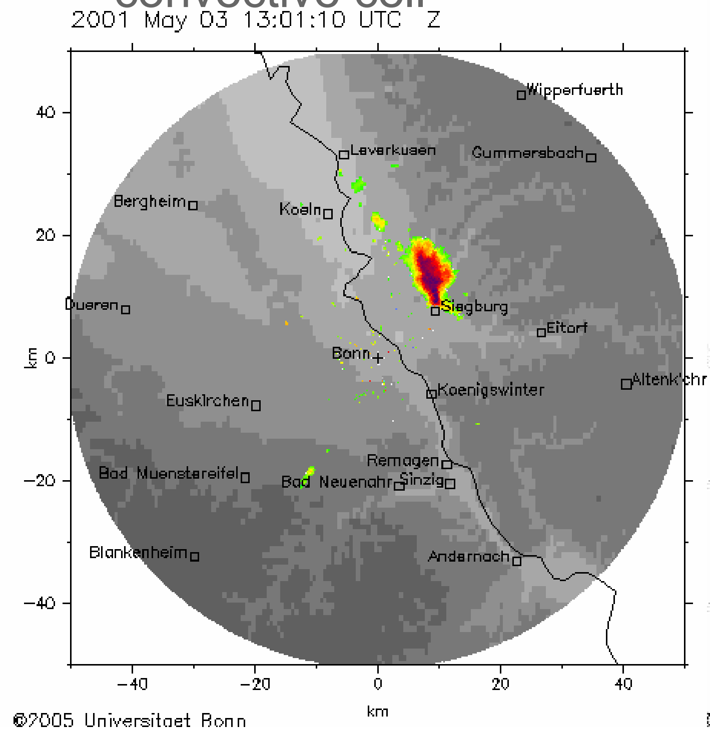
Extreme Weather Phenomena

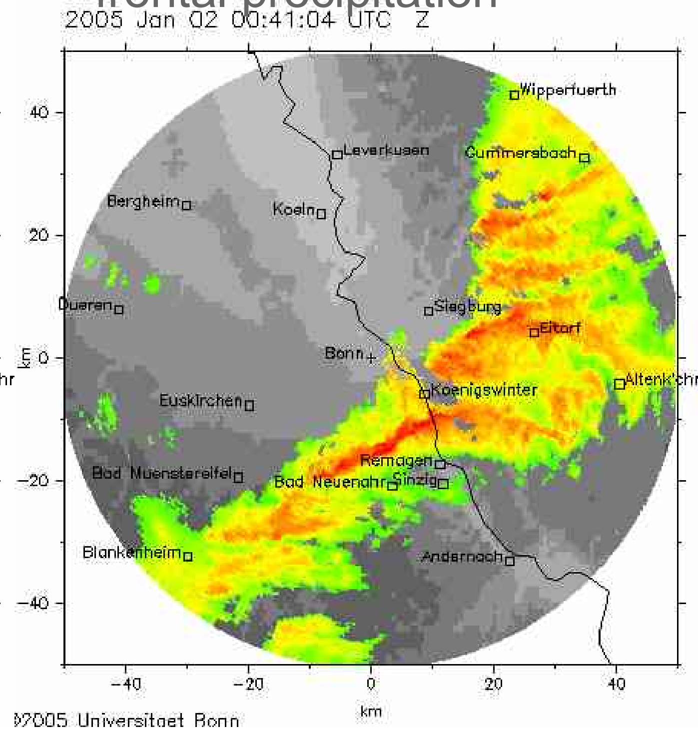
universität bonn

heavy precipitation events:
reasonable observational network
spacial dependence – unknown, depends on weather

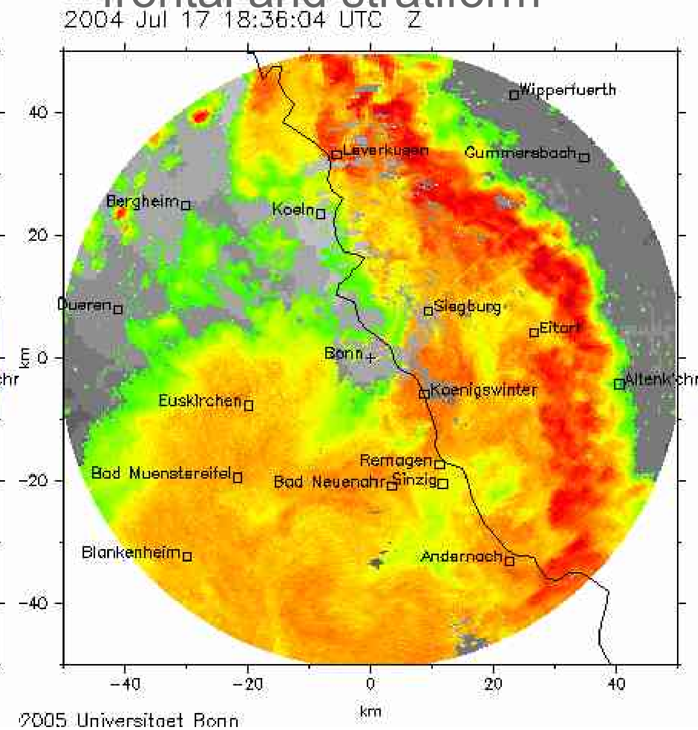
2001 May 03
convective cell



2005 Jan 02
frontal precipitation



2004 Jul 17
frontal and stratiform



wind gusts:
must less observations
processes not well known
spacial dependence - unknown

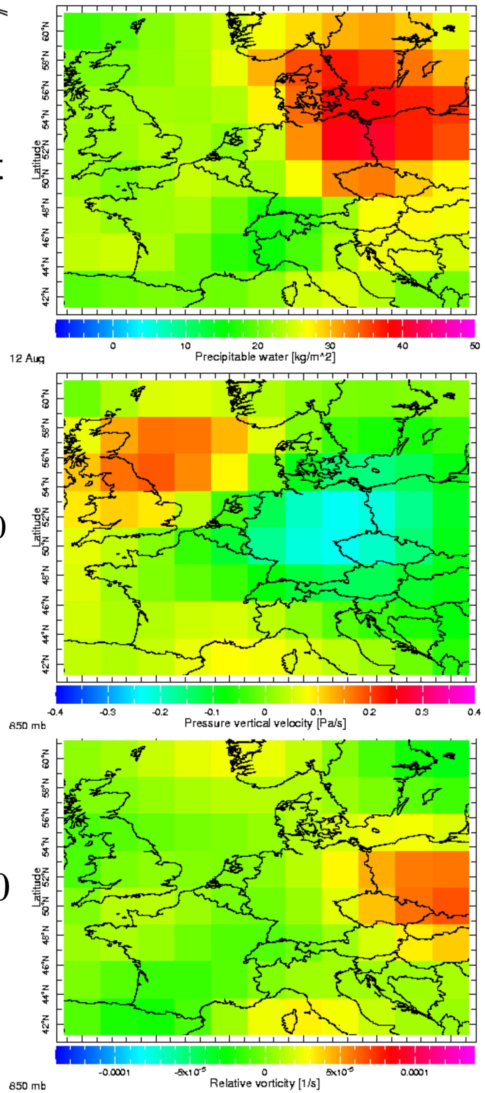
heat waves:
good observations
but long memory

Data

PWat

ω_{850}

ζ_{850}



NCEP reanalysis

Daily mean fields of ω_{850}, ζ_{850} and PWat

Period from:

1948 to 2004

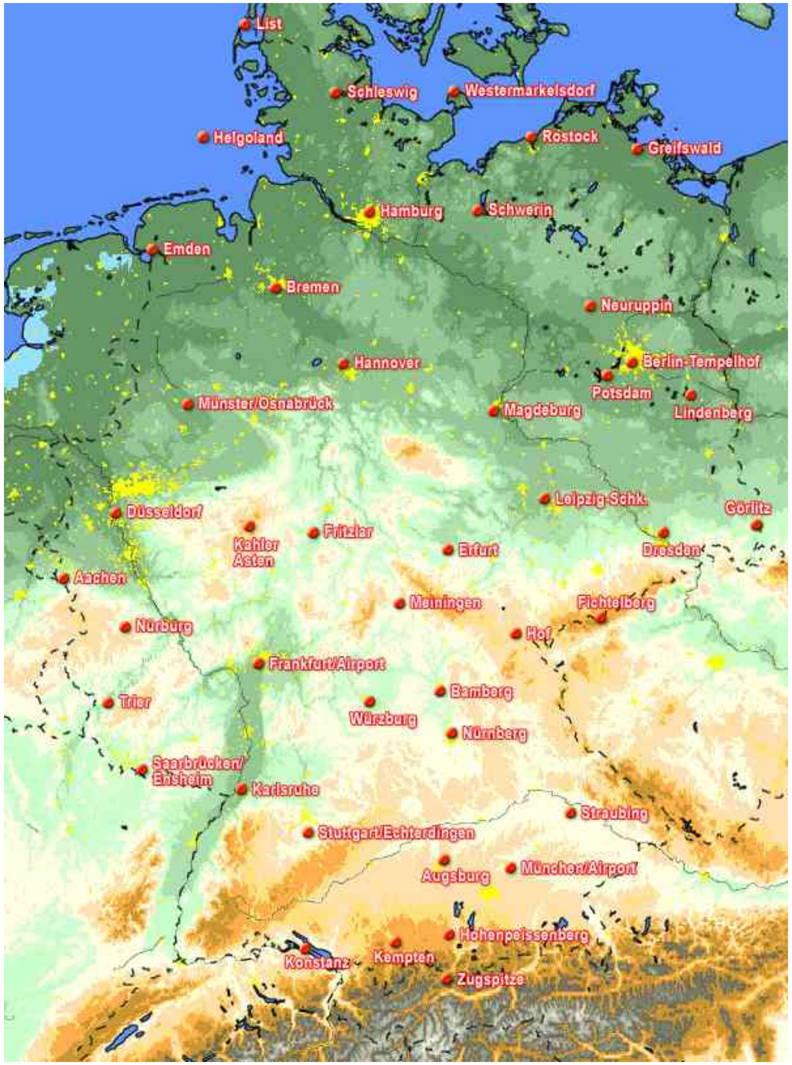
Separate:

cold (NDJFM) and

warm seasons (MJJAS)

About:

8600 daily totals



DWD (German Weather Service)

~ 2000 stations in Germany

Daily precipitation totals

Downscaling

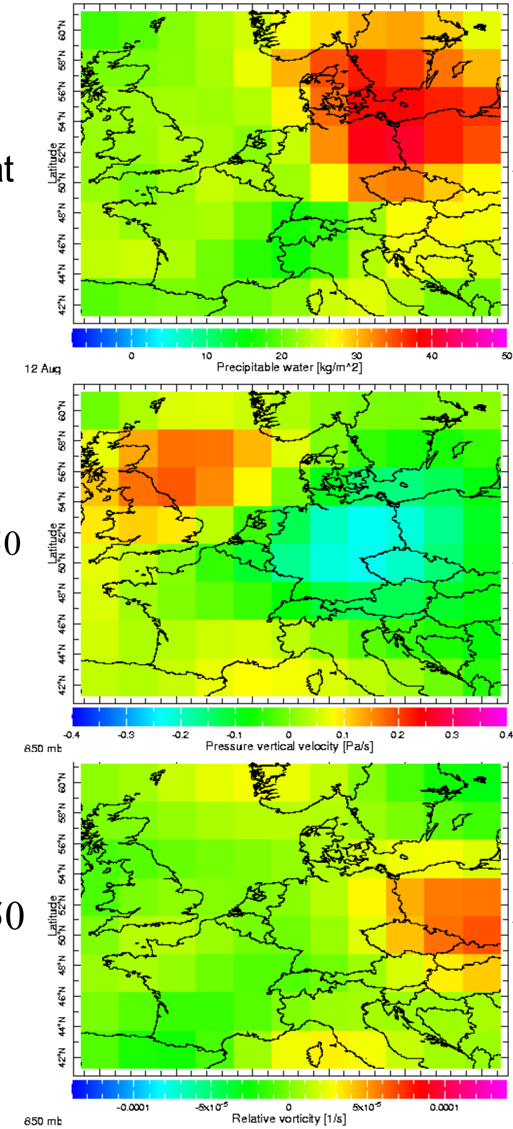
Downscaling seeks for:
a **statistical model** between the **covariate X** and a **variable** ***y***

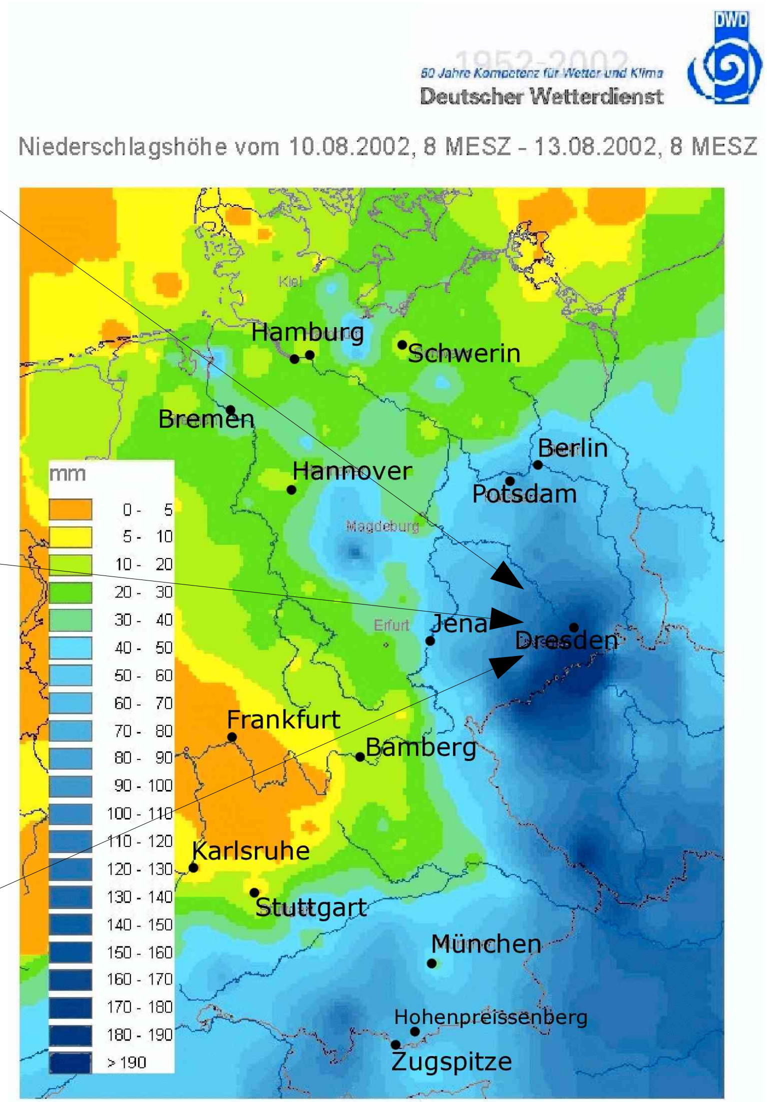
7 EOF

PWat

ω_{850}

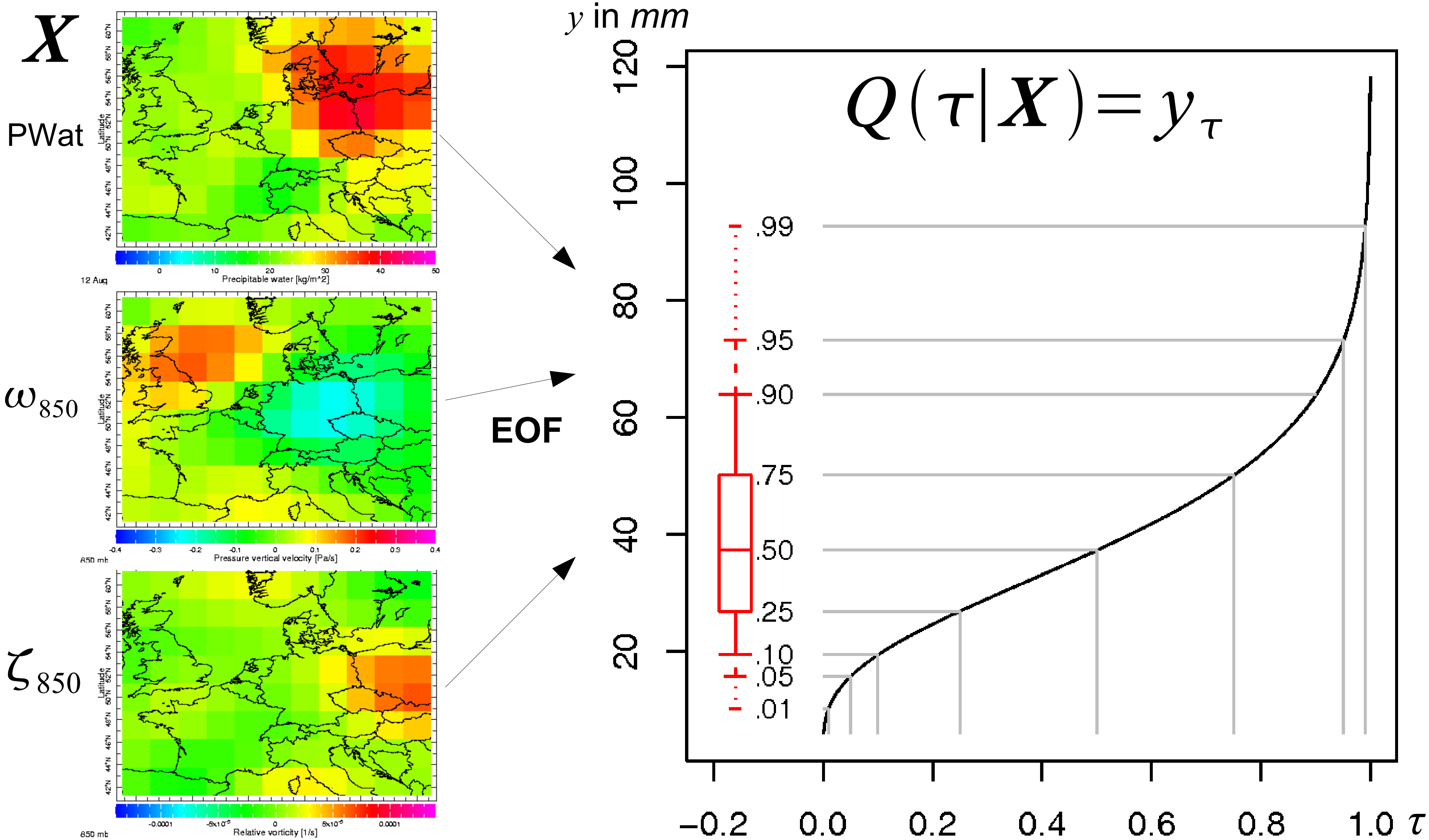
ζ_{850}





Probabilistic Downscaling...

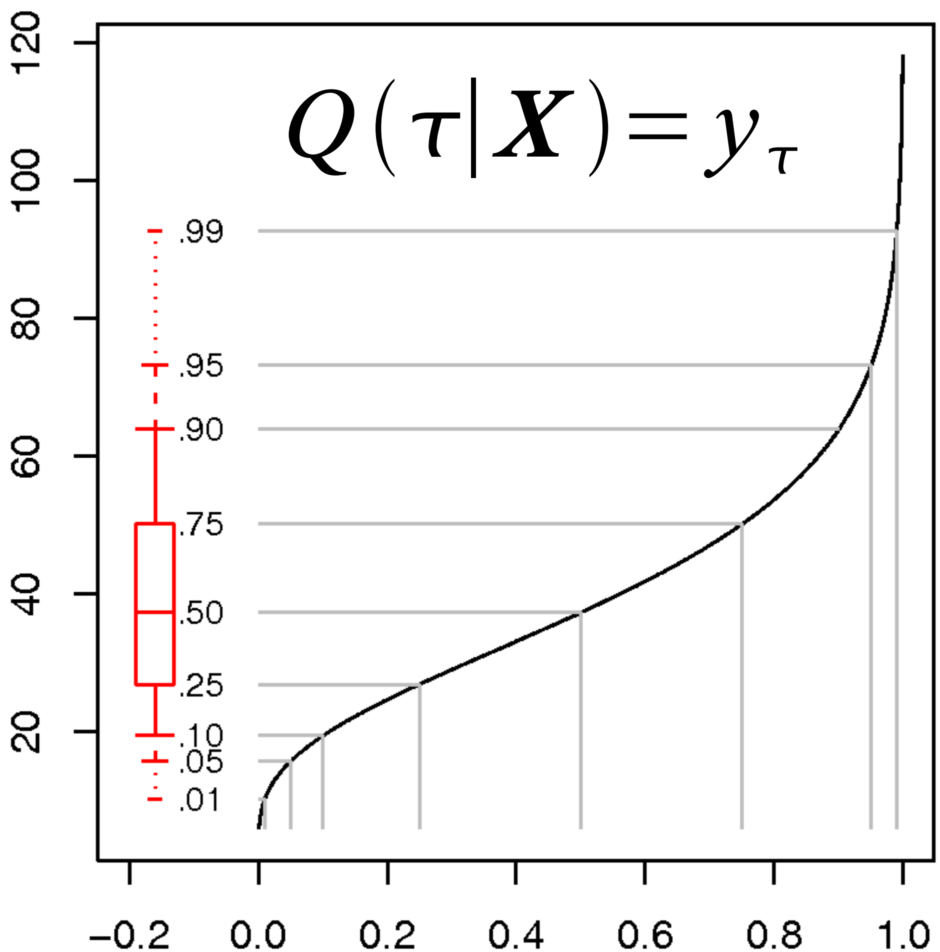
... seeks for a **statistical model** between the **covariate X** and a **probabilistic measure of** *y* - with special emphasis on



Probabilistic Downscaling

How to estimate a probabilistic measure of *y* given **X**?

- Distribution function: $F(y|\mathbf{X}) = p$
- Quantile function: $Q(\tau|\mathbf{X}) = y_\tau$



Semi-parametric:

Fix threshold *y*,
estimate 1-*p* probability of threshold exceedance using logistic regression

**Fix probability τ=*p*,
estimate y quantile using quantile regression**

Parametric:

Distributional assumption:
estimate parameter vector given **X**

Poisson point – GPD process
estimate parameter μ, σ, ξ given **X**

POT approach with u = u(**X**)
with parameters σ_u, ξ given **X**

Censored Quantile Regression

The **censored** linear model

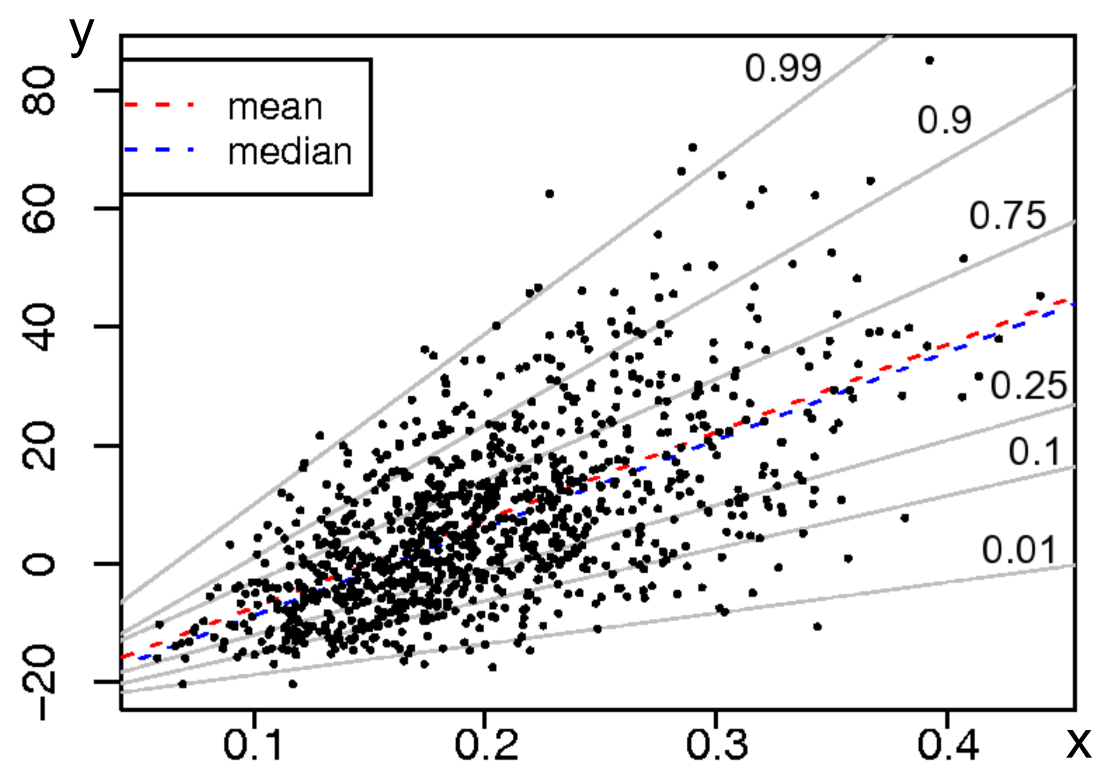
$$Y|X = max(0, \boldsymbol{\beta}^T X + \gamma^T X u) \qquad u \sim i.i.d.$$

the conditional quantile function

$$Q_y(\tau|X) = max(0, \hat{\boldsymbol{\beta}}_{\tau}^T X)$$

Example
no censoring:

$$Q_{\tilde{y}}(\tau|X) = \hat{\boldsymbol{\beta}}_{\tau}^T X$$



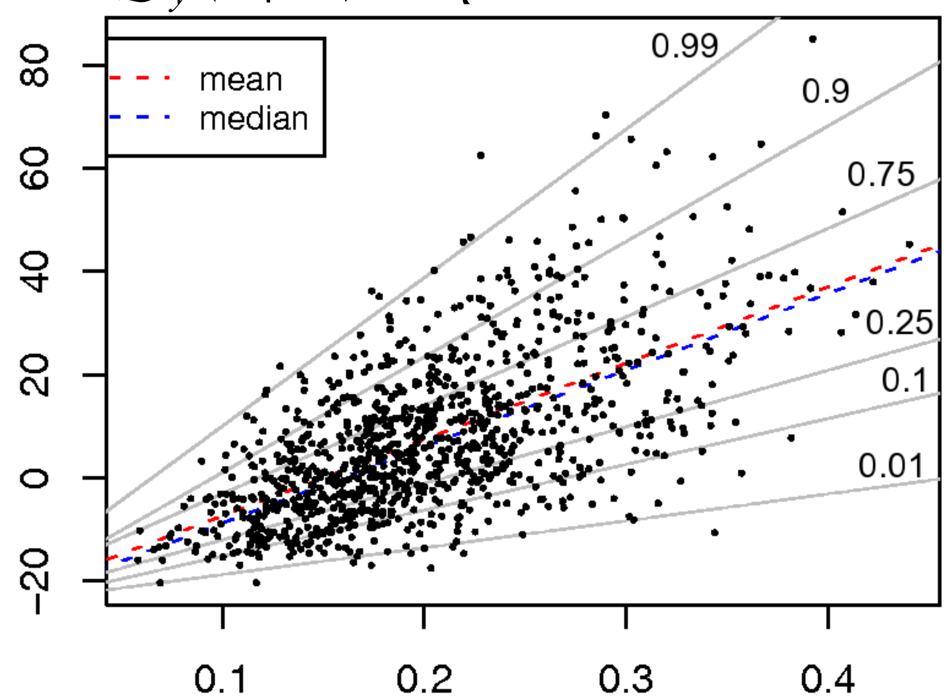
Censoring

Conditional quantiles are **equivariant to monotone transformations** $h(.)$

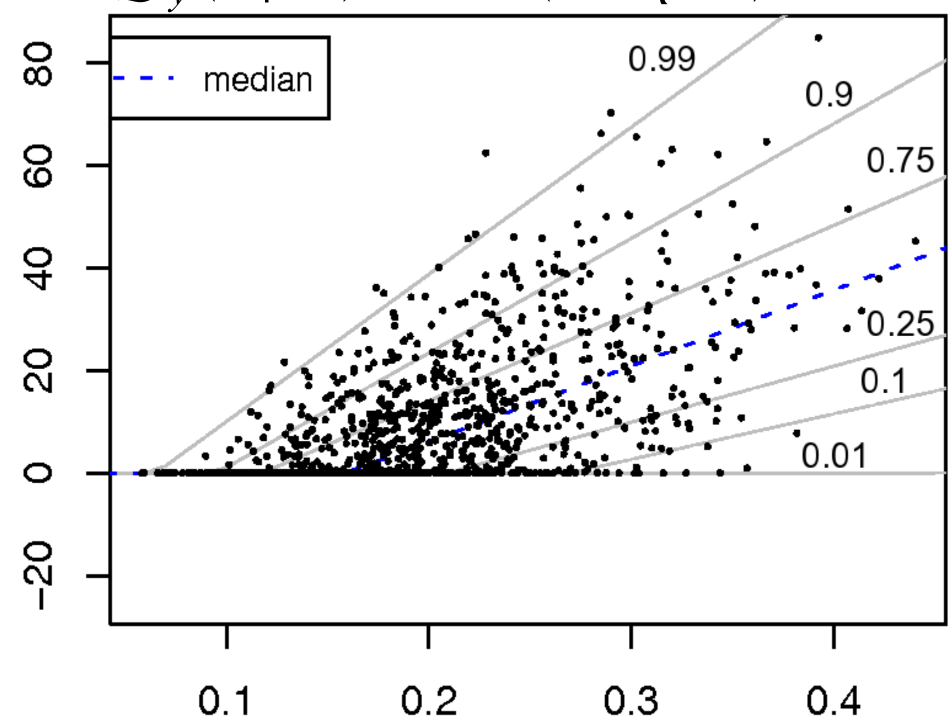
$$Q_{h(\tilde{Y})}(\tau) = h(Q_{\tilde{Y}}(\tau))$$

in out case $y = h(\tilde{y}) = max(0, y)$

$Q_{\tilde{y}}(\tau|X) = \hat{\beta}_\tau^T X$



$Q_y(\tau|X) = max(0, \hat{\beta}_\tau^T X)$



Censored Quantile Regression

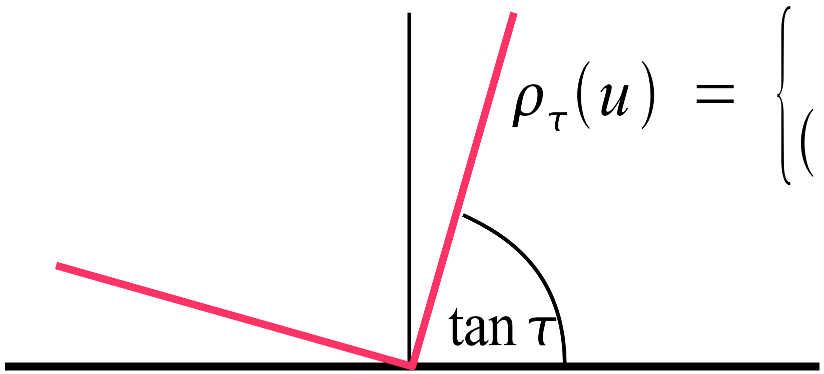
The **censored** linear model

$$Y|\boldsymbol{X} = max(0, \boldsymbol{\beta}^T \boldsymbol{X} + \gamma^T \boldsymbol{X} u) \qquad u \sim i.i.d.$$

the conditional quantile function $\hat{y}_\tau = Q_y(\tau|\boldsymbol{X}) = max(0, \hat{\boldsymbol{\beta}}_{\boldsymbol{\tau}}^{\boldsymbol{T}} \boldsymbol{X})$

Censored quantile regression minimizes the **absolute deviation function**

$$\hat{\boldsymbol{\beta}}_{\boldsymbol{\tau}} = arg\, min_{\boldsymbol{\beta}_\tau} \sum_n \rho_\tau [y_n - max(0, \boldsymbol{\beta}_\tau^T \boldsymbol{x}_n)]$$



$$\rho_\tau(u) = \begin{cases} \tau u & if\ u \geq 0 \\ (\tau - 1) u & if\ u < 0 \end{cases}$$

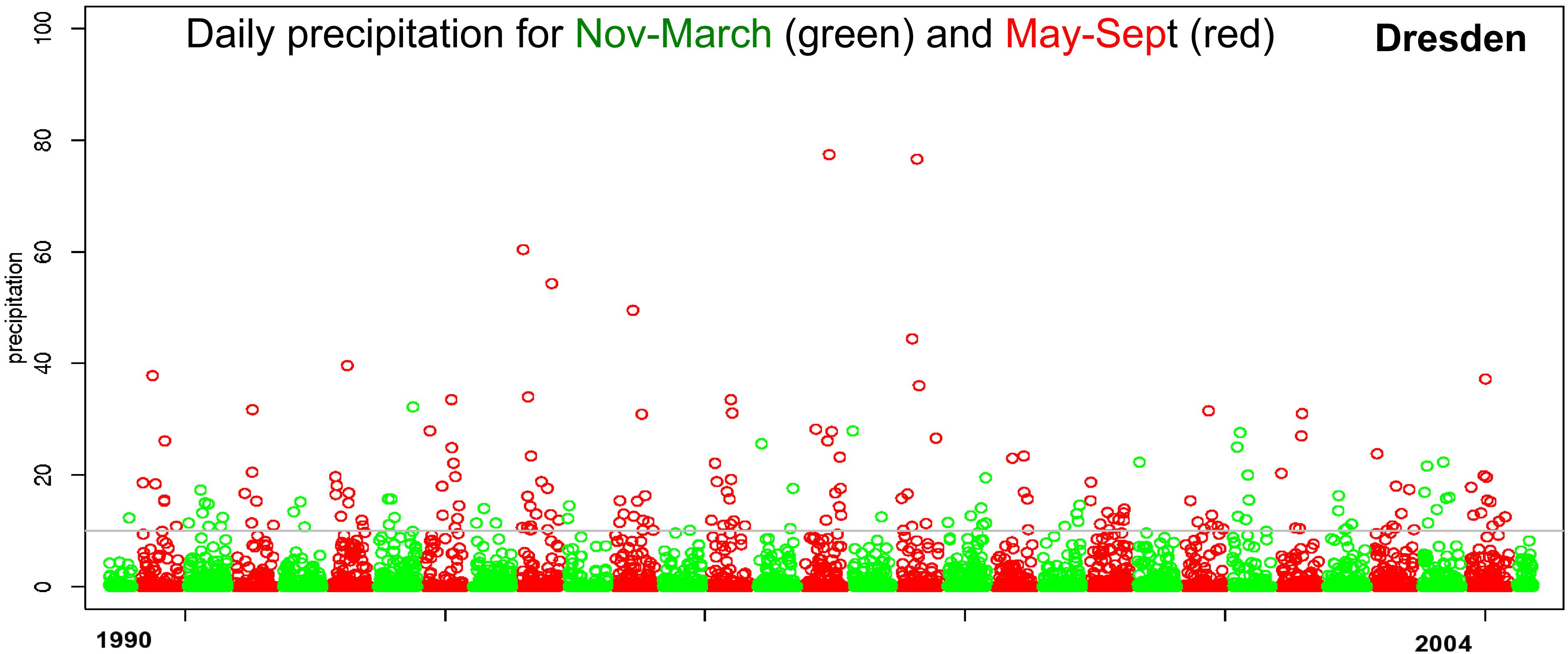
Peak Over Threshold

Peak over Threshold (POT)

$$\{ Y_i - u | Y_i > u \}$$

very large threshold u

follow a **Generalized Pareto Distribution (GPD)**



Peak Over Threshold

Peak over Threshold (POT)

The distribution of $Z_i := Y_i - u | Y_i > u$ exceedances over large threshold u are asymptotically distributed following a

Generalized Pareto Distribution (GPD)

$$H(z|Y_i > u) = \begin{cases} 1 - \left(1 + \xi \dfrac{z}{\sigma_u}\right)^{-1/\xi} & 1 + \xi \dfrac{z}{\sigma_u} > 0 \quad \xi > 0 \\ 1 - \exp\left(\dfrac{-z}{\sigma_u}\right) & z > 0 \quad \xi = 0 \end{cases}$$

two parameters

- σ_u scale parameter
- ξ shape parameter

POT with variable threshold

For the model we have 3 parameters

threshold u is definded as conditional τ-quantile

$$\hat{u}_{\tau} = Q_y(\tau_u | \boldsymbol{X}) = \hat{\boldsymbol{\beta}}^T \boldsymbol{X}$$

scale and **shape parameter** of GPD

$$\hat{\sigma} = \boldsymbol{\sigma}^T \boldsymbol{x}_n \qquad \hat{\xi} = \boldsymbol{\xi}^T \boldsymbol{x}_n$$

Estimation of threshold uses censored quantile regression, conditional GPD parameters of the exceedances are estimated using maximum likelihood method.

POT with variable threshold

The conditional distribution of precipitation is

$$F(y|\boldsymbol{X}) = H(y-u_x|y>u_x,\boldsymbol{X})\,Prob(y>u_x|\boldsymbol{X}) + Prob(y\leqslant u_x|\boldsymbol{X})$$

$$= H(u-u_x|y>u_x,\boldsymbol{X})(1-\tau_u)+\tau_u \;=\; \tau>\tau_u$$

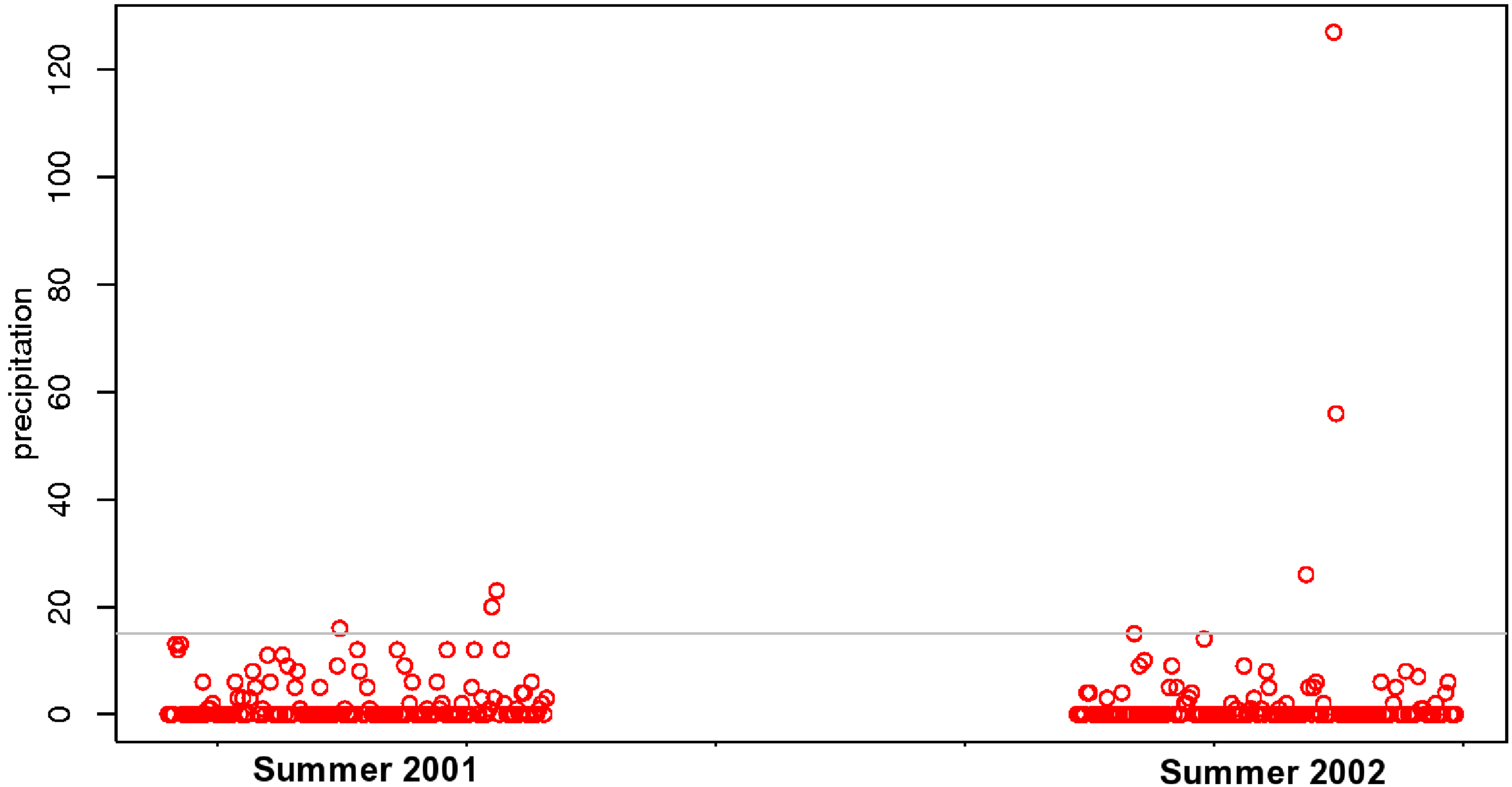
The respective τ-is estimated as

$$\hat{y}_\tau = H^{-1}(\tau|\boldsymbol{X}) = \begin{cases} u_x + \dfrac{\hat{\sigma}_u}{\hat{\xi}}\left[(1-\tilde{\tau})^{-\hat{\xi}}\right] & \xi>0 \\ u_x + \hat{\sigma}_u \log(1-\tilde{\tau}) & \hat{\xi}=0 \end{cases}$$

Poisson Point - GPD

Poisson point – GPD process with intensity

$$\Lambda(A) = (t_2 - t_1)\left[1 + \xi\left(\frac{y-u}{\sigma_u}\right)\right]^{-1/\xi} \quad \text{on} \quad A = (t_1, t_2) \times (y, \infty)$$



Poisson Point - GPD

Poisson point – GPD process with intensity

$$\Lambda(A)=(t_2-t_1)[1+\xi(\frac{y-u}{\sigma_u})]^{-1/\xi} \quad \text{on} \quad A=(t_{1,}t_2)\times(y,\infty)$$

Non-stationarity:
assume parameters to depend linearly on covariate **X**

$$\hat{\mu}_n=\boldsymbol{\mu}^T\boldsymbol{x}_n \qquad \hat{\sigma}_n=\boldsymbol{\sigma}^T\boldsymbol{x}_n \qquad (\hat{\xi}_n=\boldsymbol{\xi}^T\boldsymbol{x}_n)$$

Estimate parameters using **maximum likelihood**

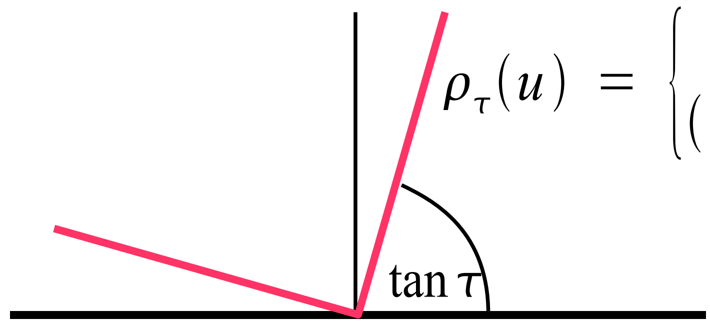
Get the **conditional extremal quantiles**

$$\hat{y}_\tau=u-\frac{(\hat{\sigma}+\hat{\xi}(u-\hat{\mu}))}{\hat{\xi}}(1-(\frac{-\log\tau}{\hat{\lambda}})^{-\hat{\xi}})$$

Quantile verification score

Quantile regression minimizes
the **absolute deviation function**

$$QVS = \sum_n \rho_\tau \left[y_n - \hat{y}_{n,\tau} \right]$$



$$\rho_\tau(u) = \begin{cases} \tau u & \text{if } u \geqslant 0 \\ (\tau - 1) u & \text{if } u < 0 \end{cases}$$

proper scoring rule (Gneiting and Raftery 2005)

censored quantile verification skill score

$$CQVSS(\tau) = 1 - \frac{CQVS(\tau, \hat{\boldsymbol{y}})}{CQVS(\tau, \boldsymbol{y}_{ref})}$$

Forecast verification

A **Scoring rule** $S(P,y)_{y\sim Q}$

measures the quality or utility of a probabilistic forecast

here in terms of a cost function

Let $$E[S(P,y)]_Q := S(P,Q)$$

Let Q be the forecasters best judgement
S is a proper scoring rule if for all P and Q

$$S(P,Q) \geqslant S(Q,Q)$$

strictly proper if

$$S(P,Q) = S(Q,Q) \quad iff \quad P = Q$$

Gneiting T, A E Raftery (2007): Strictly Proper Scoring Rules, Prediction, and Estimation. Journal of the American Statistical Association 102,359-378

Forecast verification

Compare QR, POT and PP approach
extend censoring to threshold u

$$S_\tau(P_{QR}, \boldsymbol{y}) = \sum_n \rho_\tau[y_n - max(0, \hat{\boldsymbol{\beta}}_\tau^T \boldsymbol{x}_n)]$$

$$S_\tau(P_{POT/PP}, \boldsymbol{y}) = \sum_n \rho_\tau[y_n - \hat{y}_{n,\tau}]$$

$$S_\tau(P_{ref}, \boldsymbol{y}) = \sum_n \rho_\tau[y_n - Q_y(\tau)]$$

define a quantile verification skill score

$$QVSS_\tau = 1 - \frac{S_\tau(P_{QR/POT/PP}, \boldsymbol{y})}{S_\tau(P_{ref}, \boldsymbol{y})}$$

cross-validation

Cross validation

Derive forecast using **cross-validation**
through separation in **training** period and **target** season

To obtain set of forecasts $\{\hat{y}_n\}$ (target season)

we derive estimates of coefficients $\hat{\boldsymbol{\beta}}_\tau, \quad \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}, \hat{\boldsymbol{\xi}}, \quad u_x, \hat{\boldsymbol{\sigma}}_u, \hat{\boldsymbol{\xi}}$

from set of observations $\{\ldots, y_{n'\neq n}, \ldots\}$

(training period)

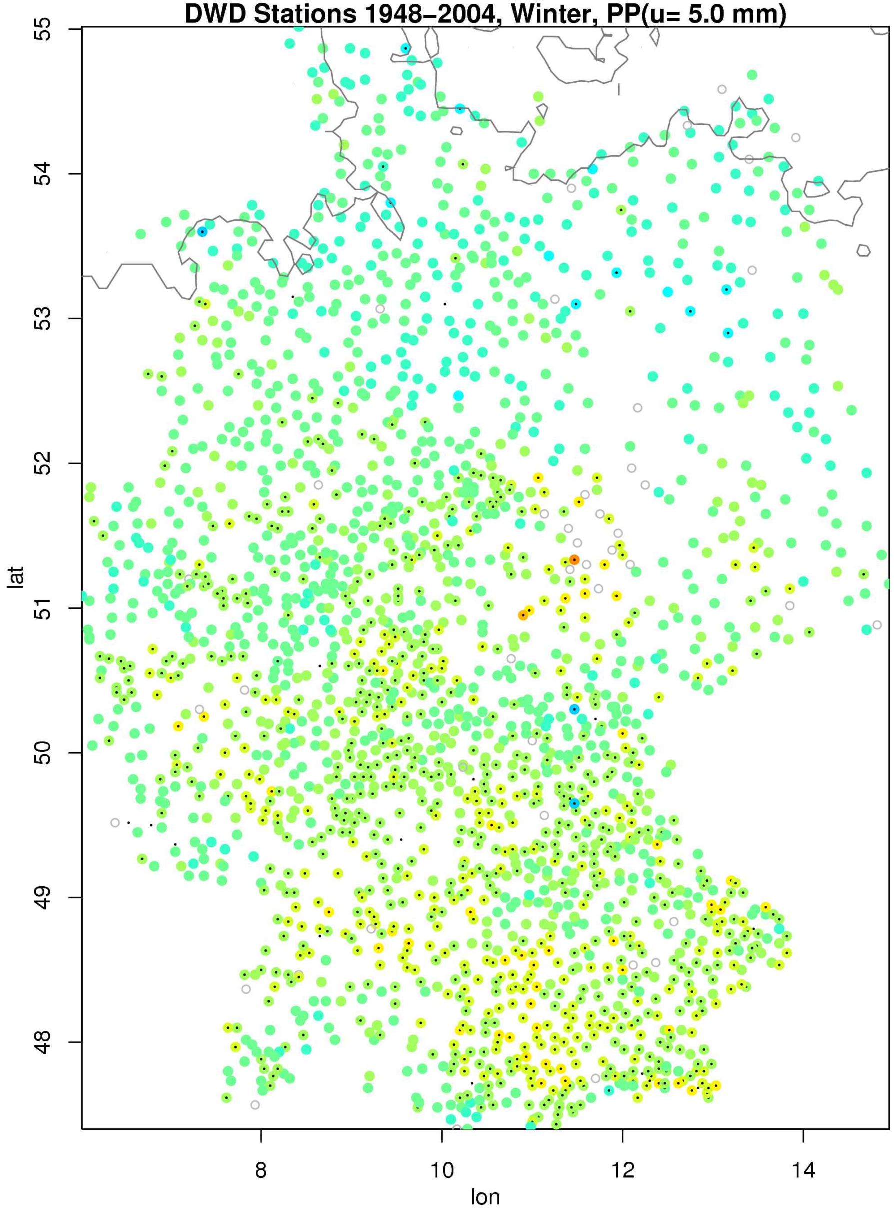
and covariates $\{\ldots, \boldsymbol{x}_{n'\neq n}, \ldots\}$

The forecast is derived for each target season in the time sequence

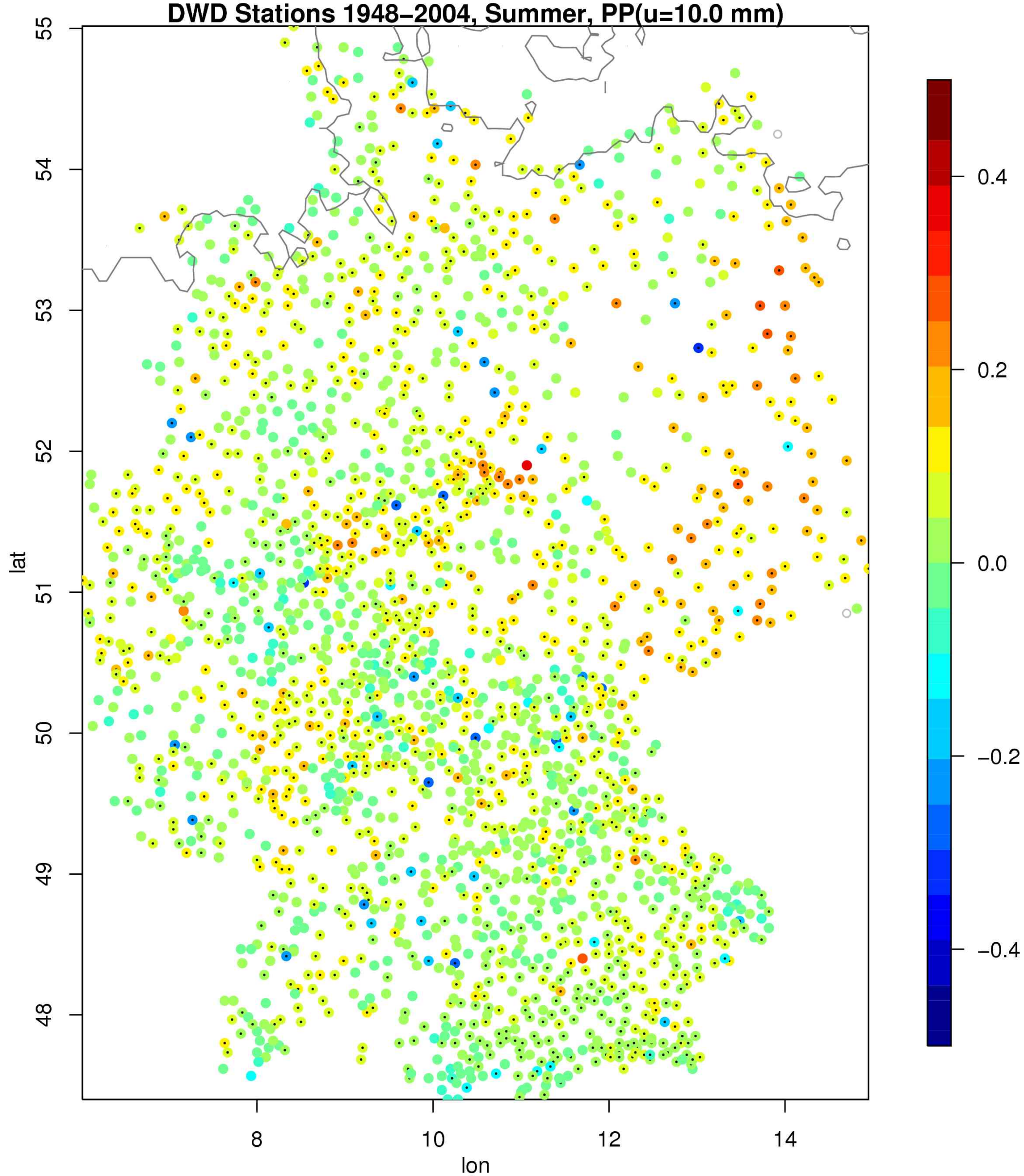
$$\hat{y}_{n,\tau} = max(0, \hat{\boldsymbol{\beta}}_\tau^T \boldsymbol{x}_n) \qquad \hat{y}_{n,\tau} = Q_{y,POT/PP}(\tau|\boldsymbol{x}_n)$$

EVD: Shape Parameter

Winter, u = 5mm



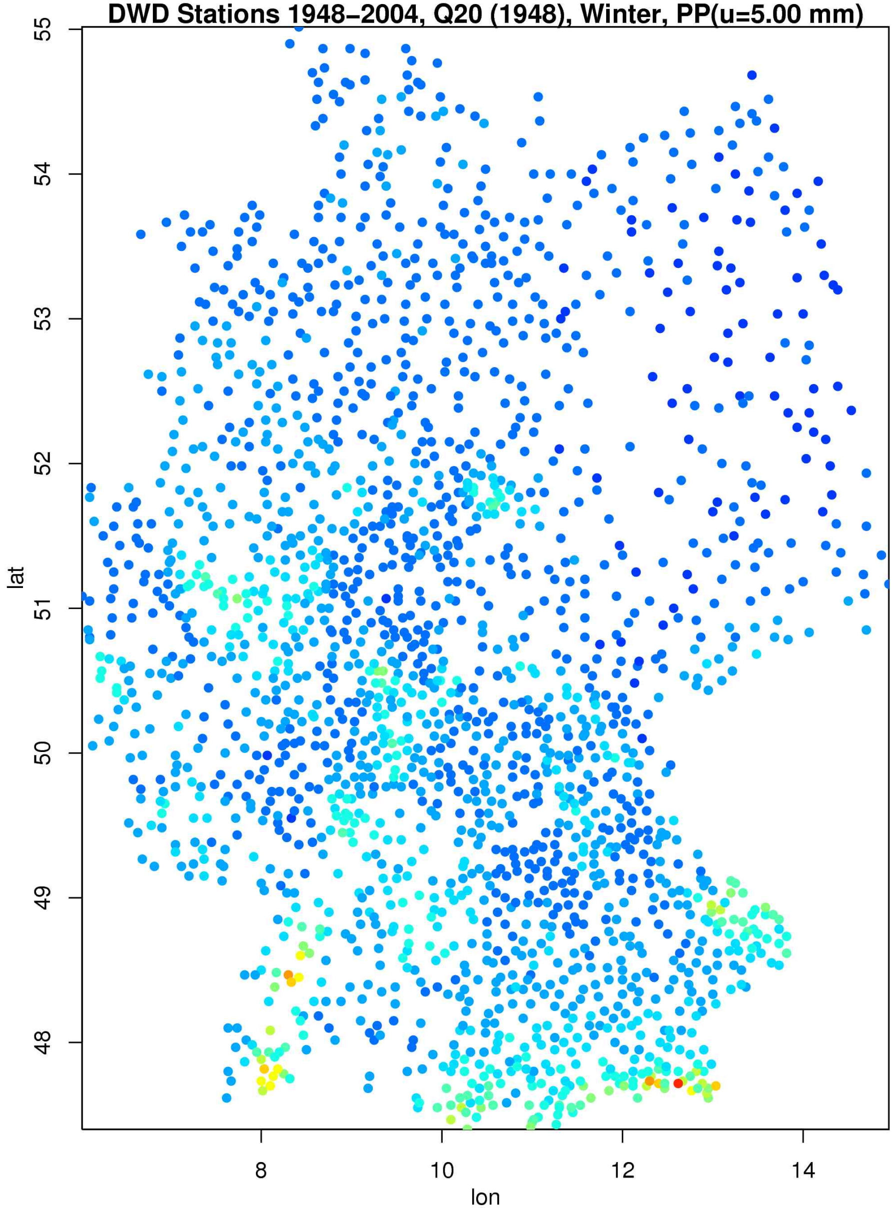
Summer, u = 10mm

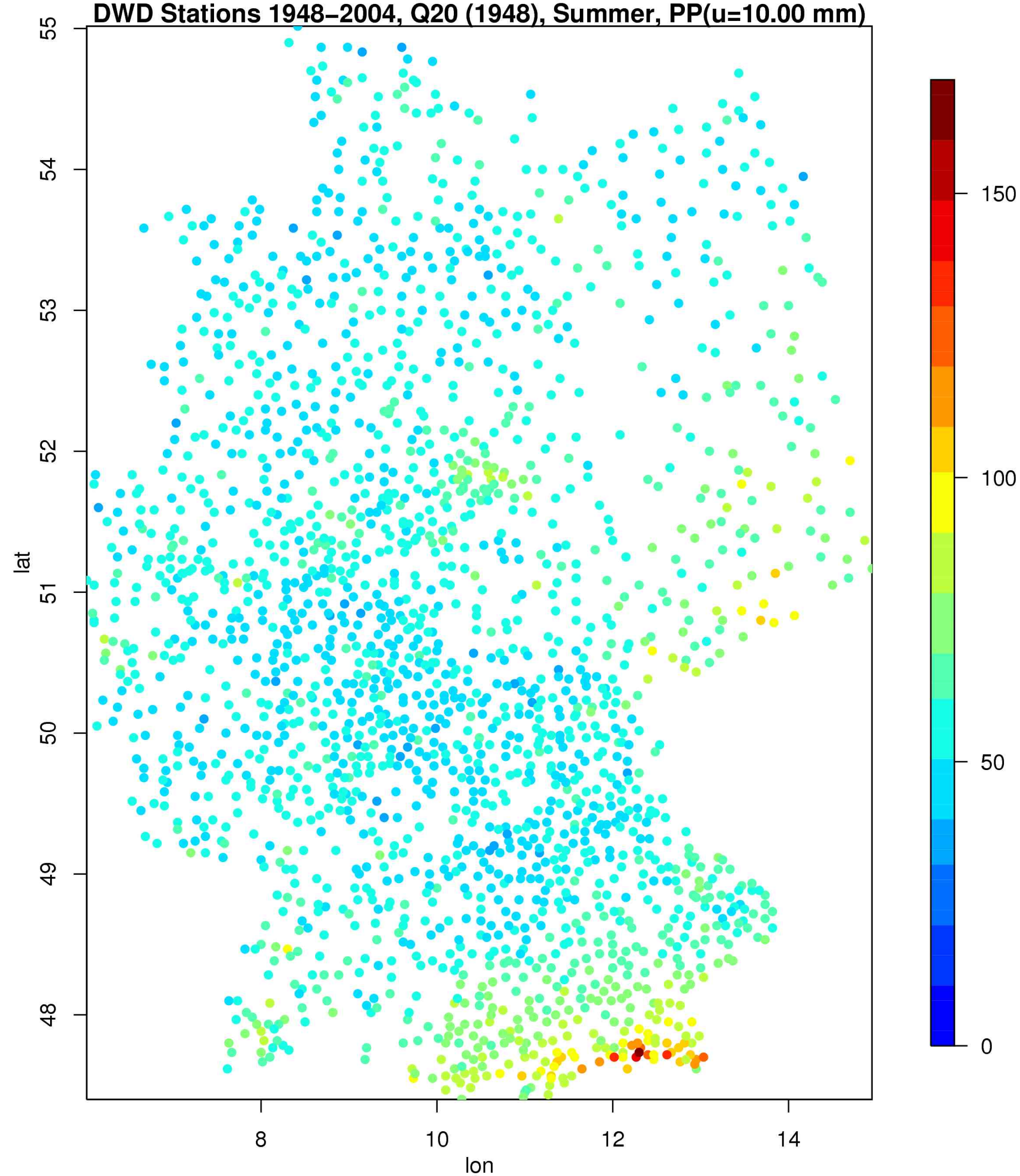


EVD: 20y return values

Winter, u = 5mm

Summer, u = 10mm

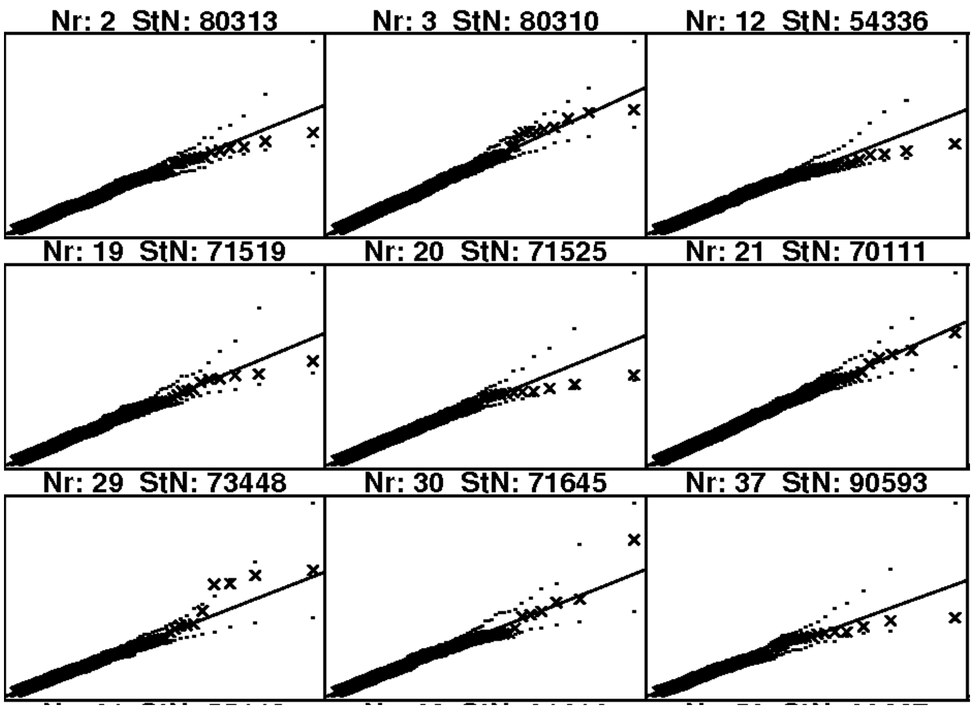




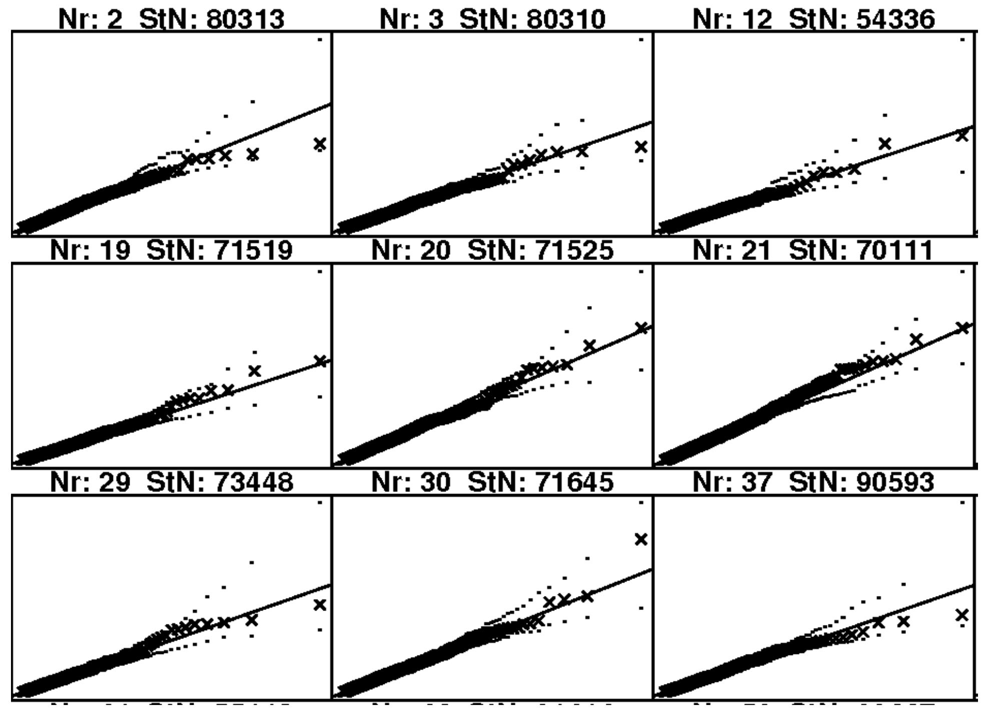
Goodness of Fit

u = 5mm

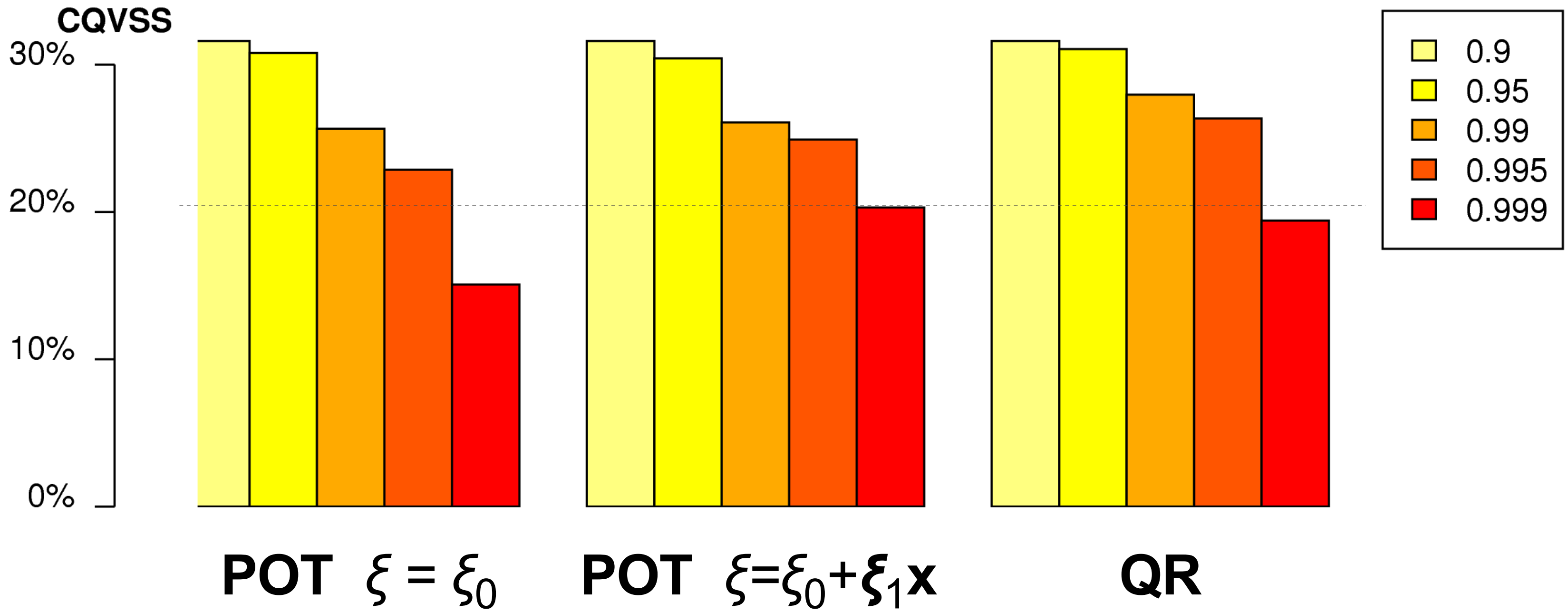
Winter



Summer



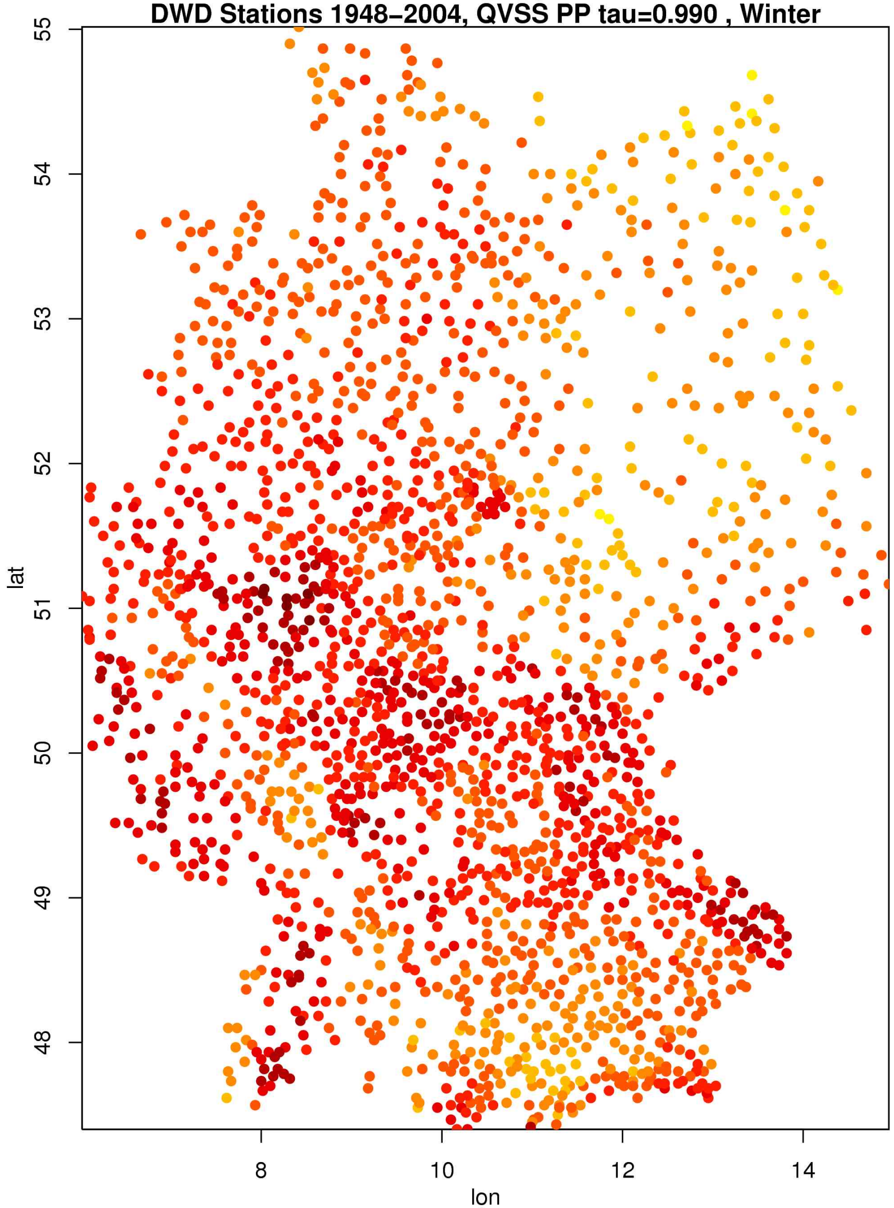
QVSS - Winter



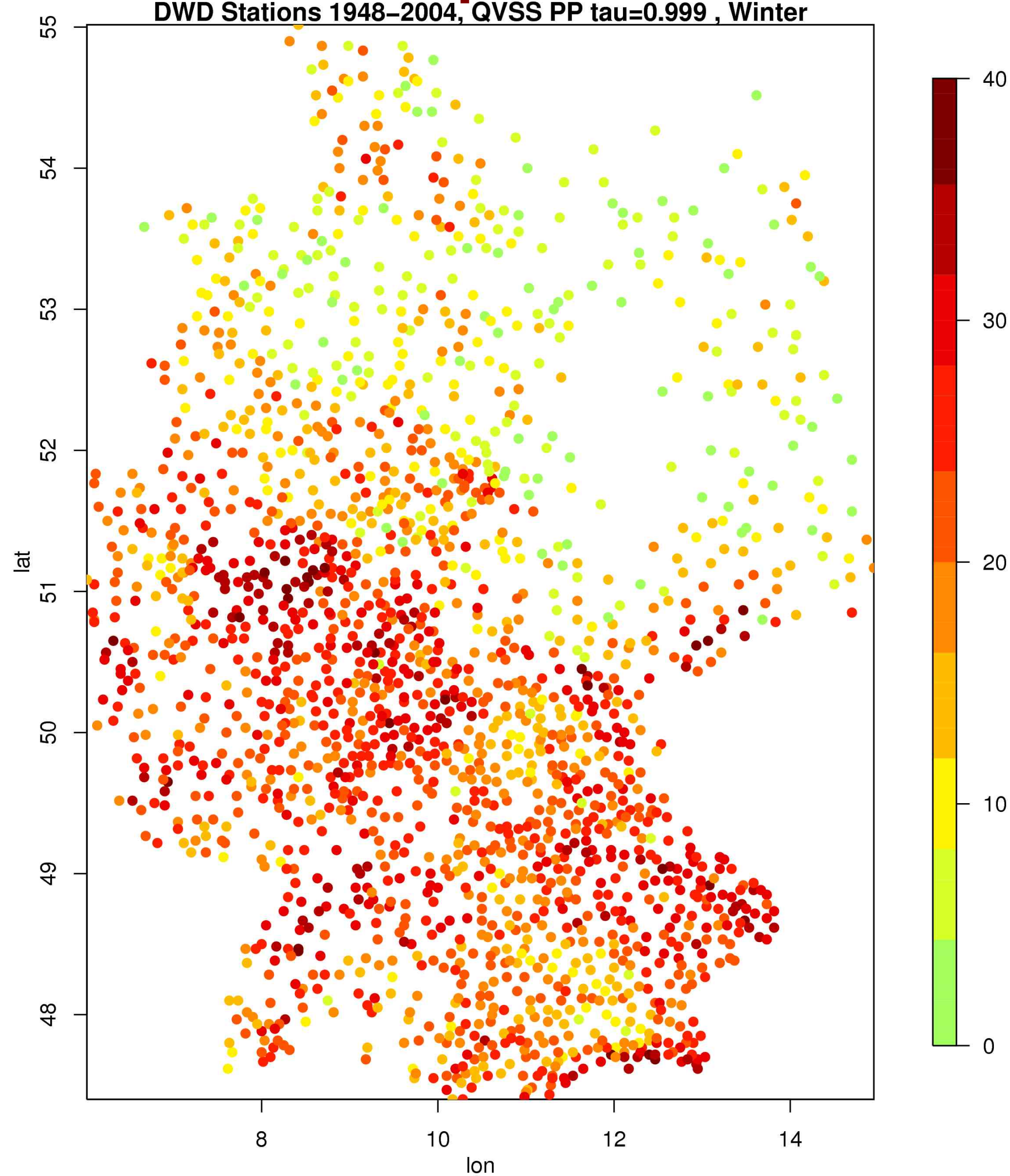
$$\tau_u = 0.9$$

QVSS - Winter all Stations

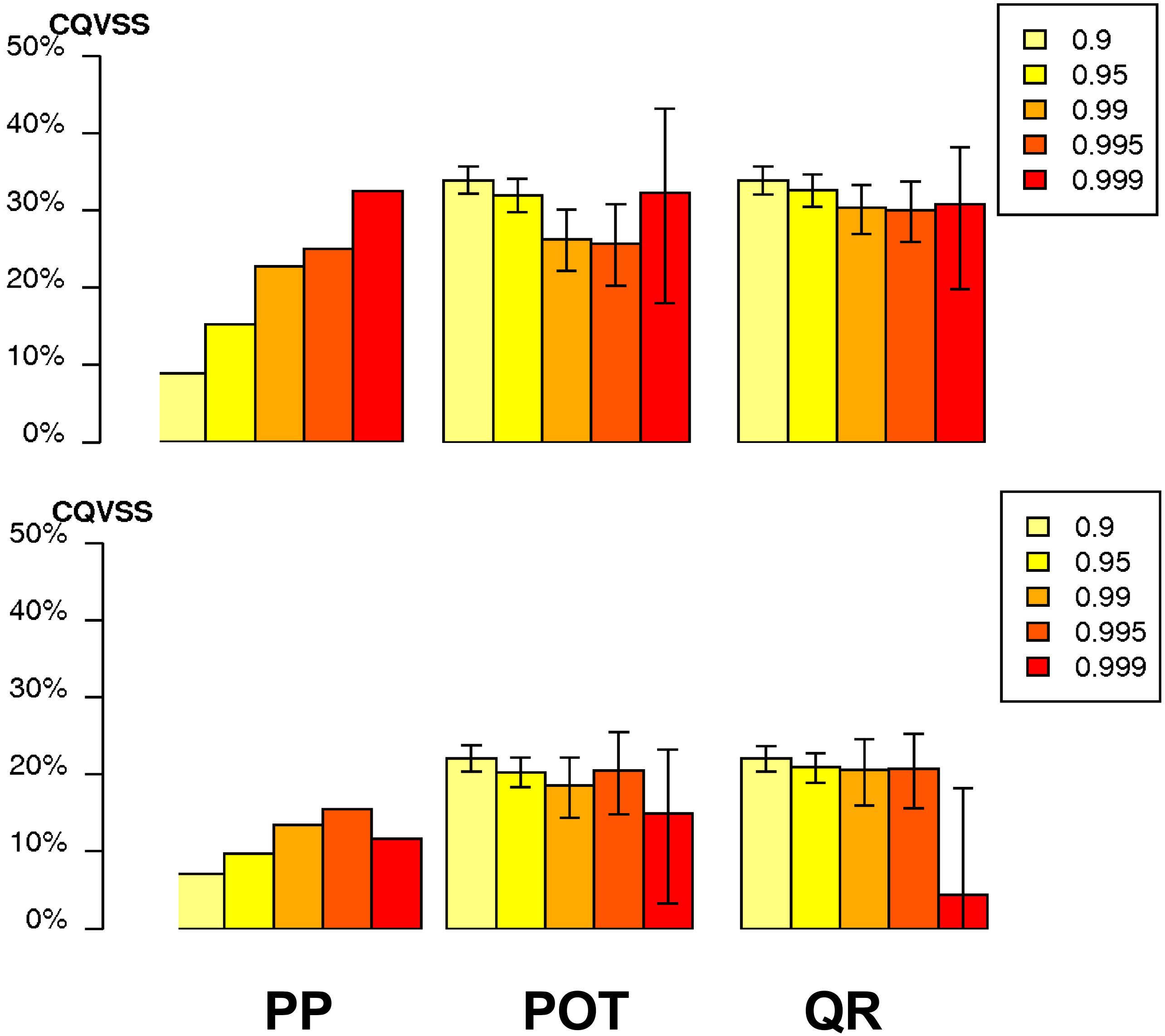
0.99 quantile



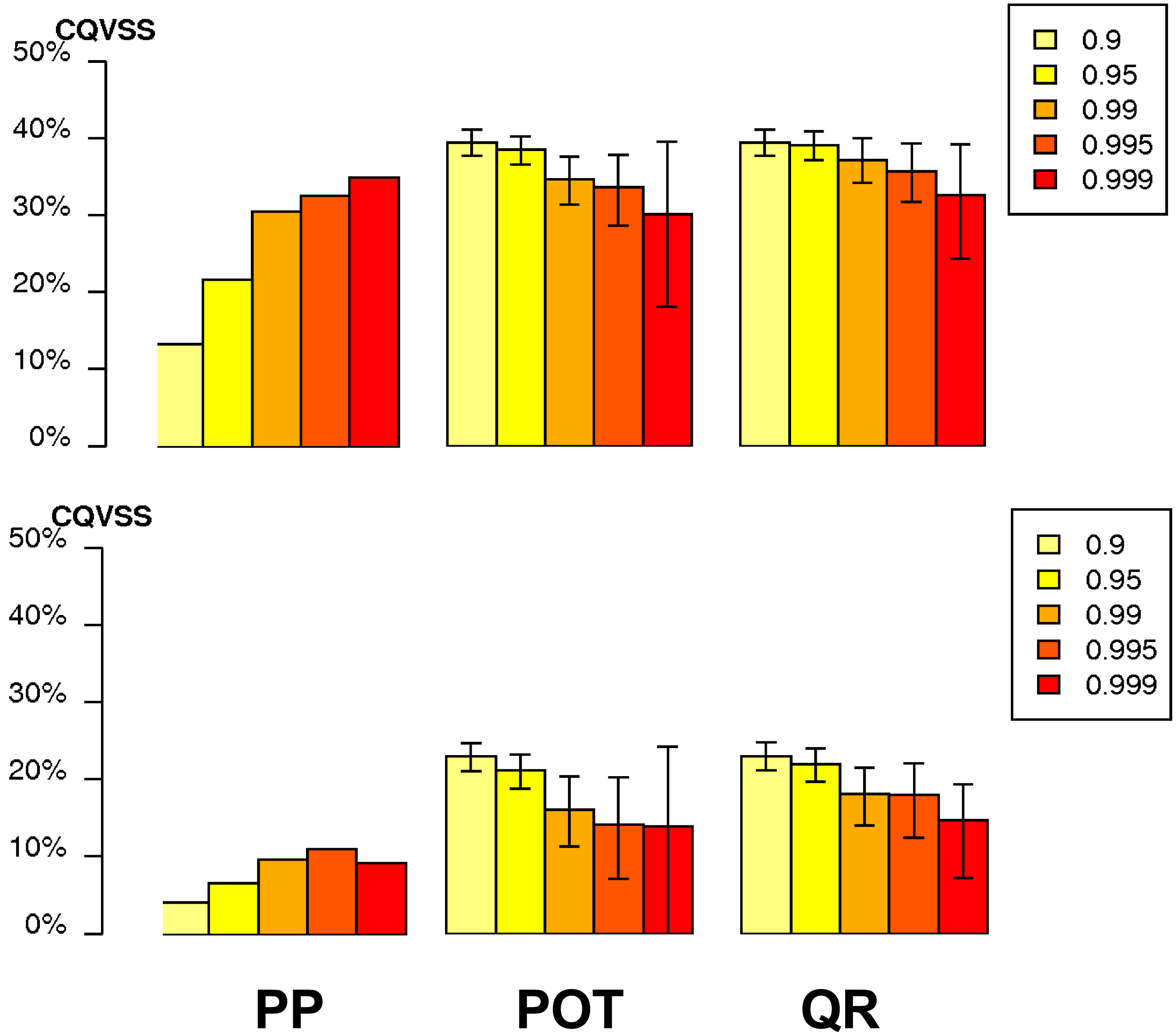
0.999 quantile



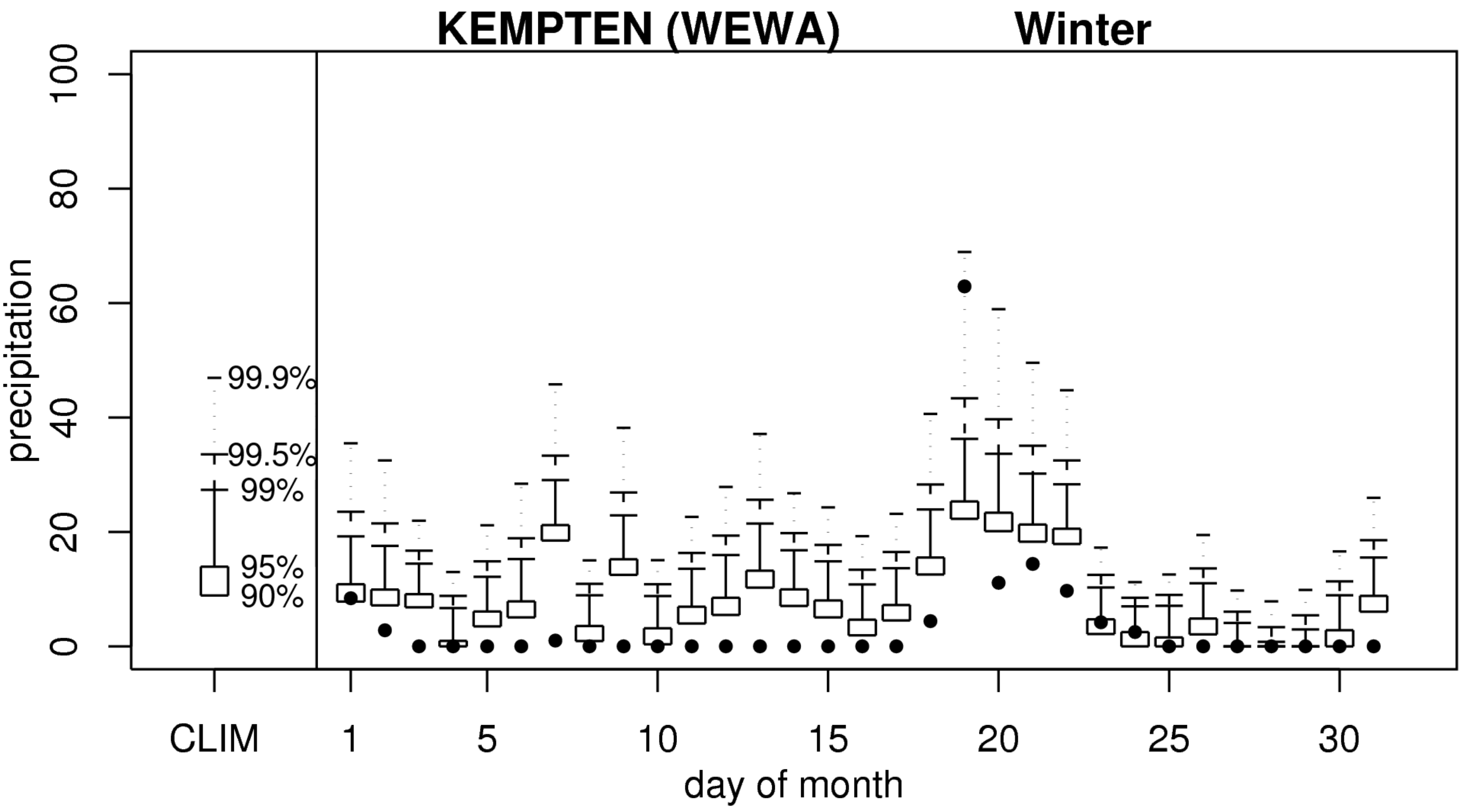
QVSS - Kempten



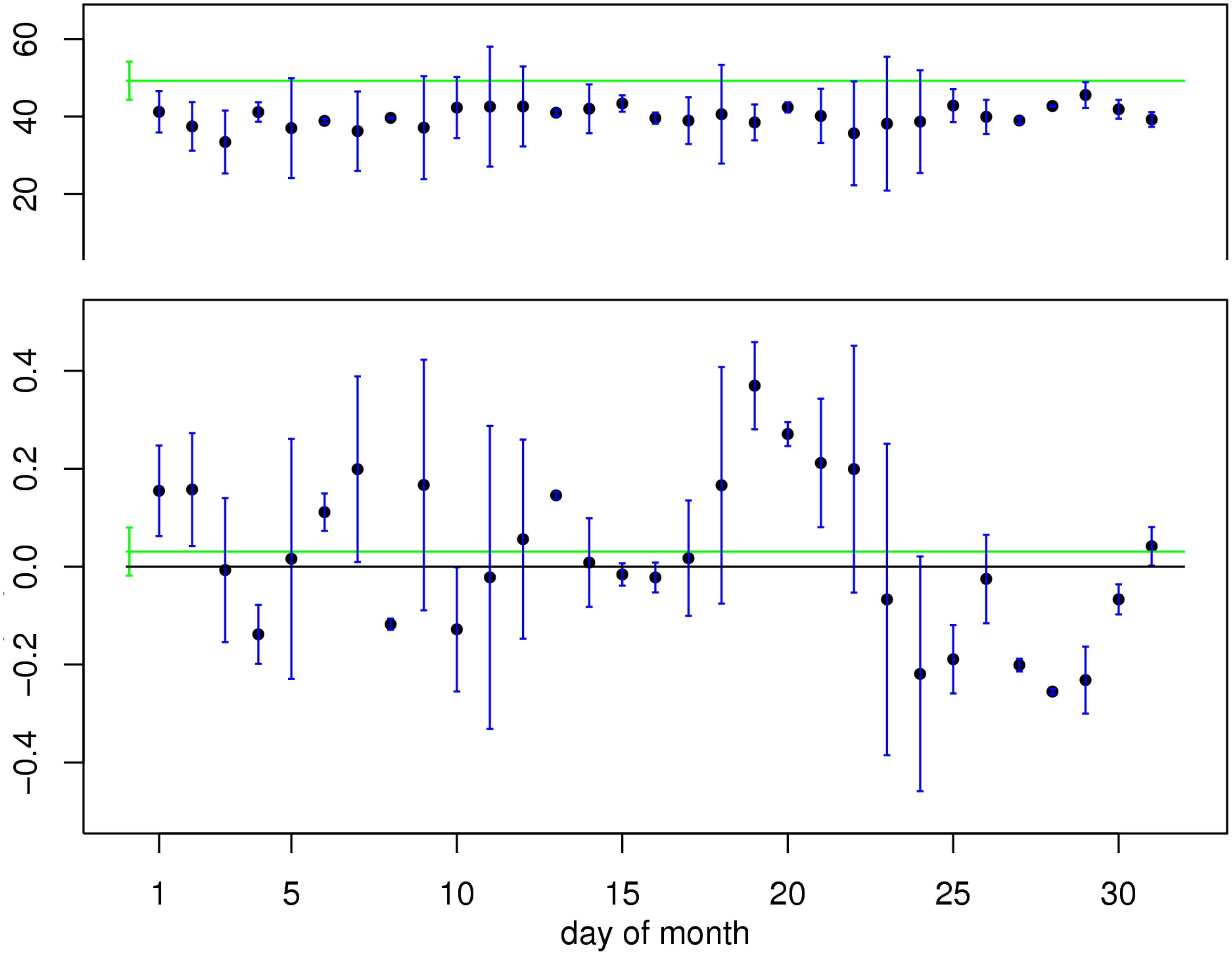
QVSS - Fichtelberg



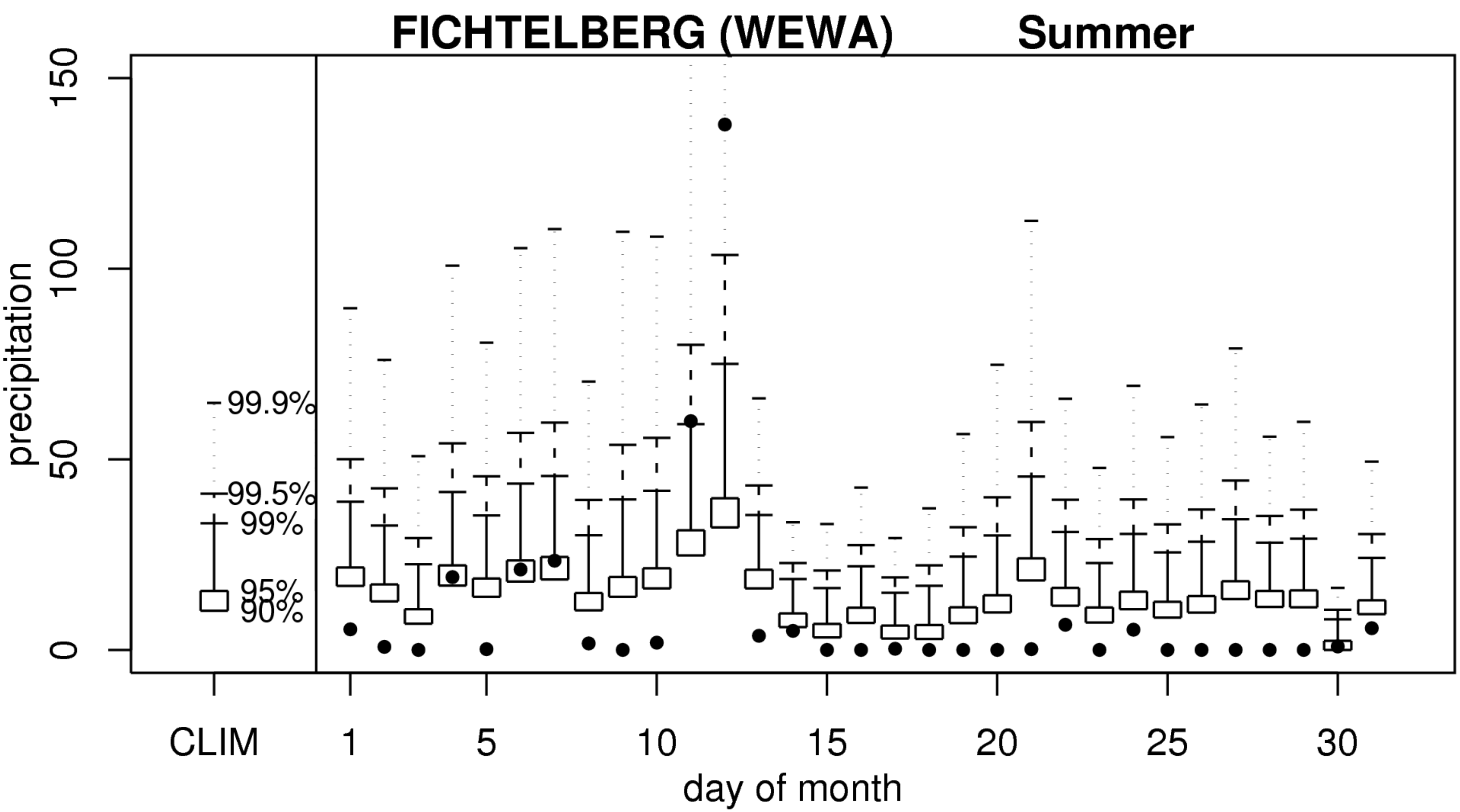
Kempten March 2002



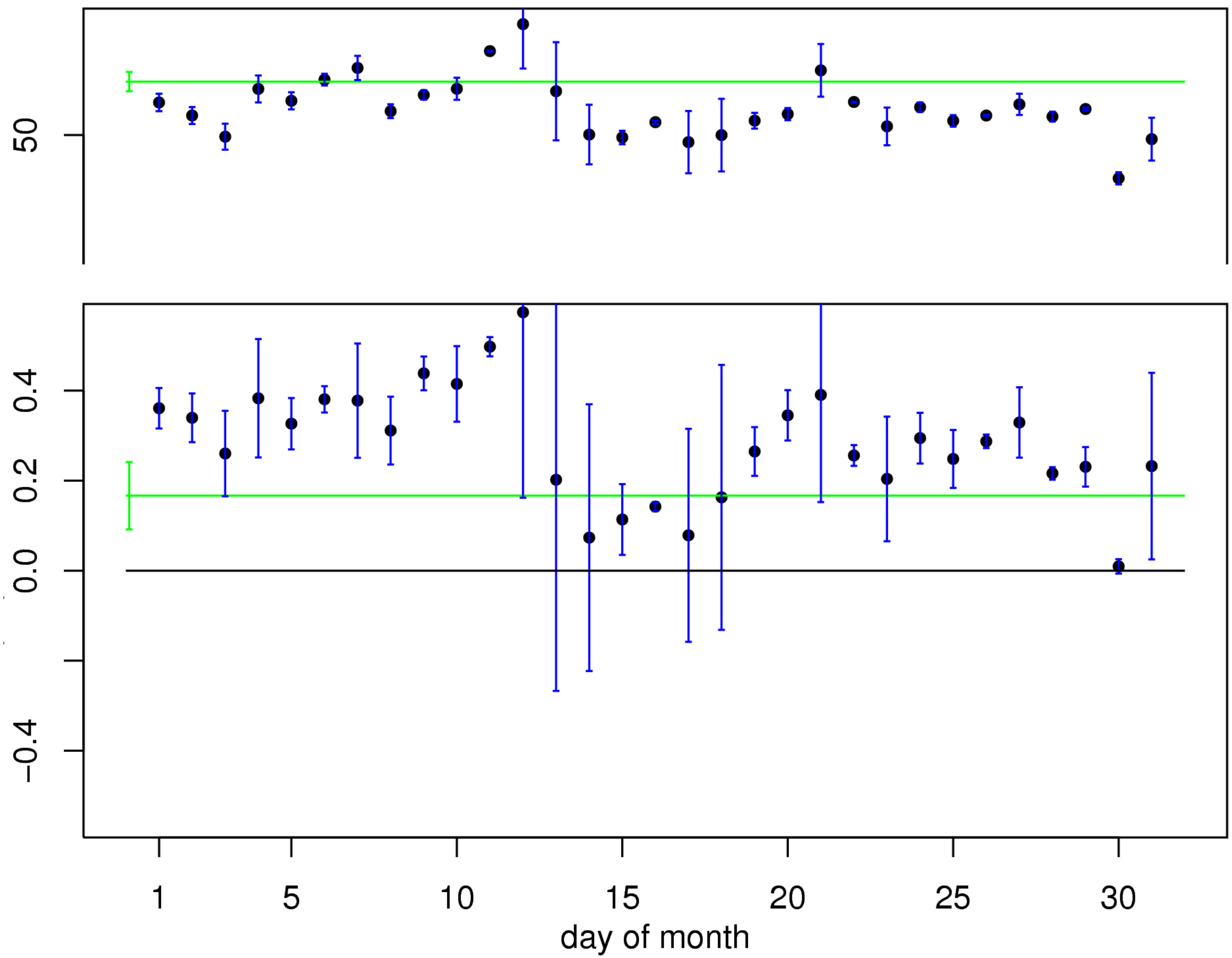
Kempten March 2002



Fichtelberg August 2002



Fichtelberg August 2002



Conclusions

Skill

more skill for winter than summer precipitation

(between20% to 40% in winter, and 10% to 30% in summer)

largest skill obtained for high quantiles

Threshold

largest skill obtained with threshold of about 2*mm*

EV parameter

shape parameter is positive in winter and summer

allowing regression for shape parameter induces large error in parameter estimates
-> statistical downscaling model becomes instable

regression for shape parameter increases skill in winter

Outlook

separate convective and stratriform precipitation

estimate quantiles taking into account spacial dependence

References

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