Spectral Analysis of Stochastic Processes

Theory

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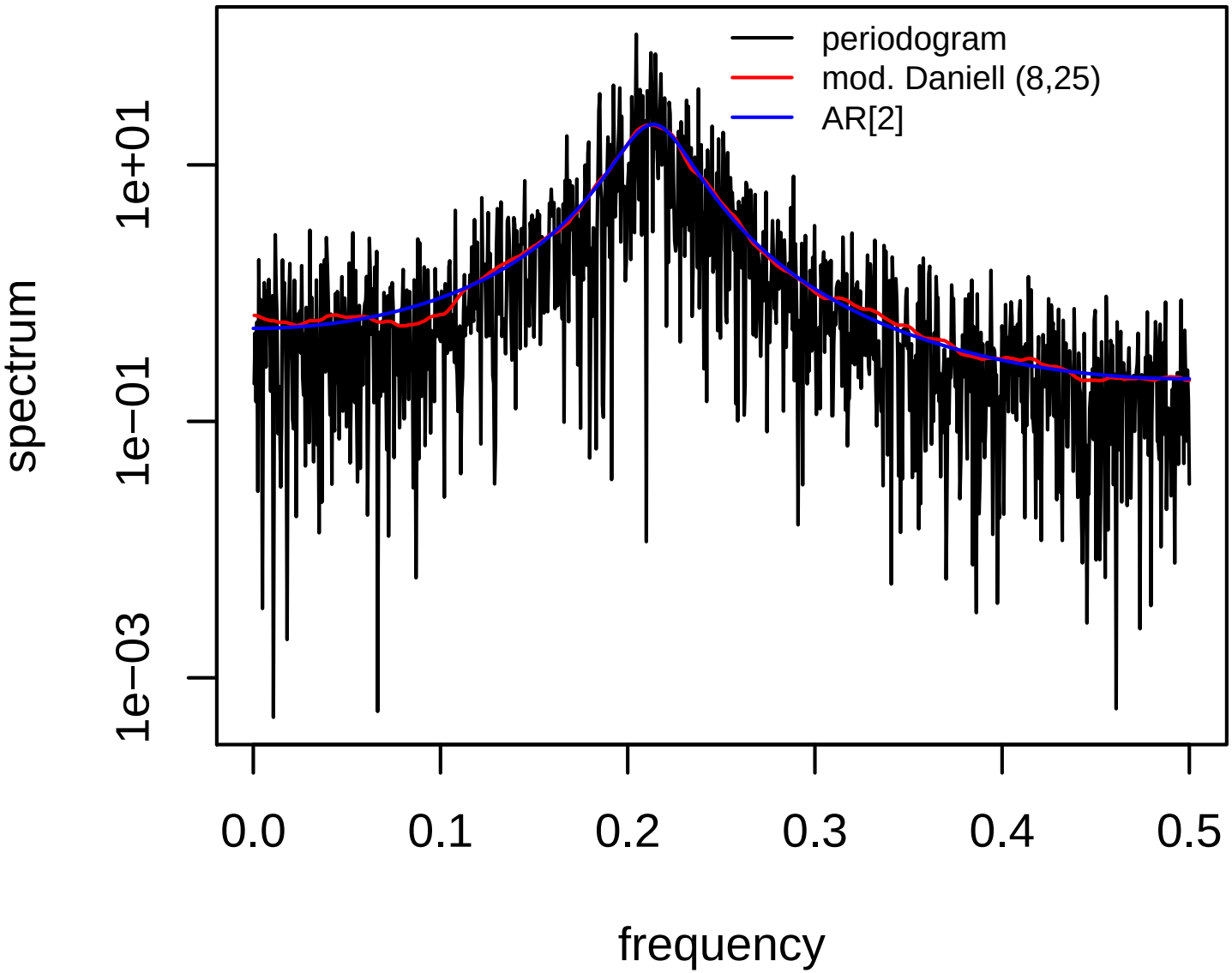
E2C2 Summer School, Comorova, September 2007

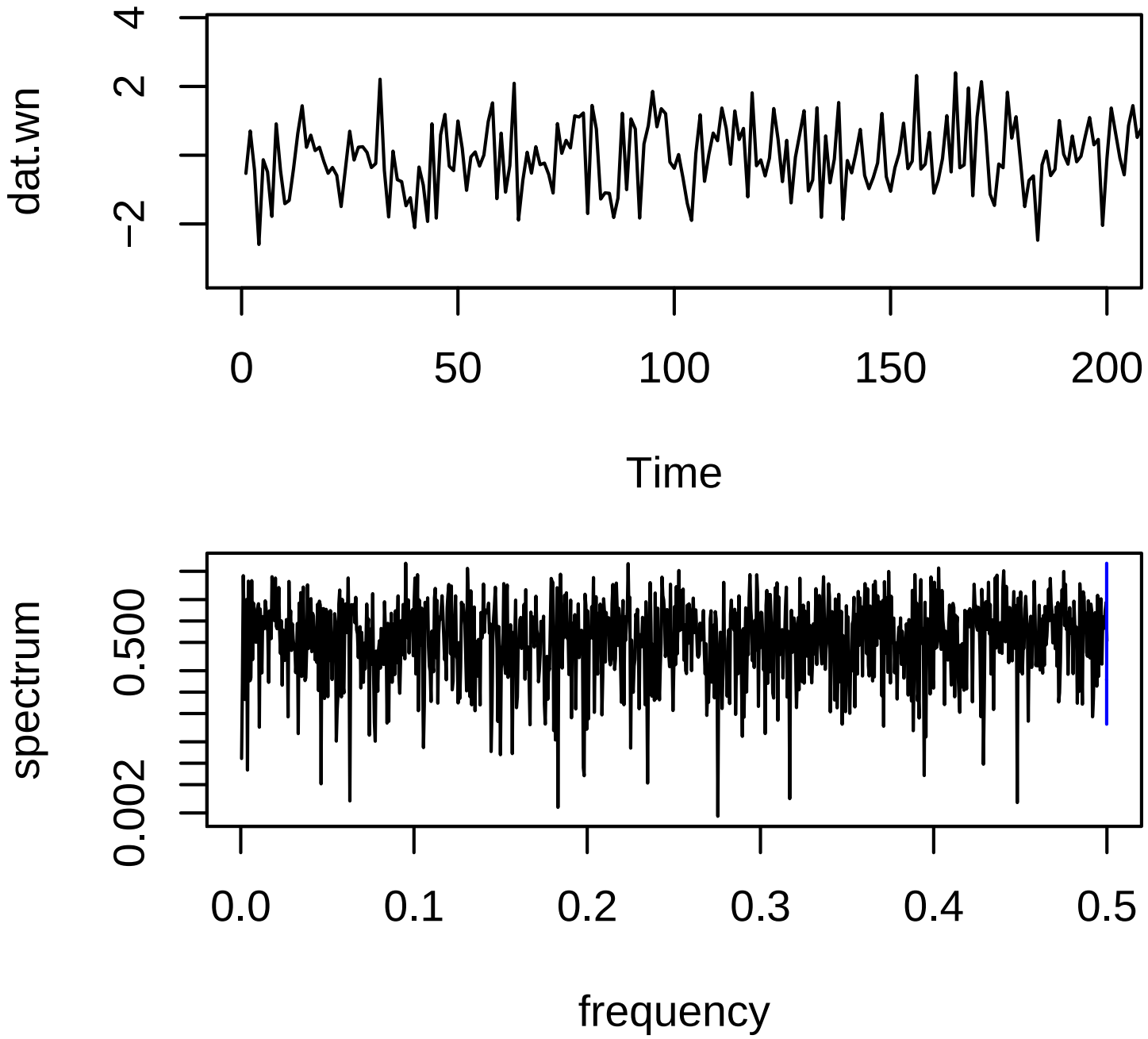
Outline

Lecture notes in shared folder "Spectral Analysis"

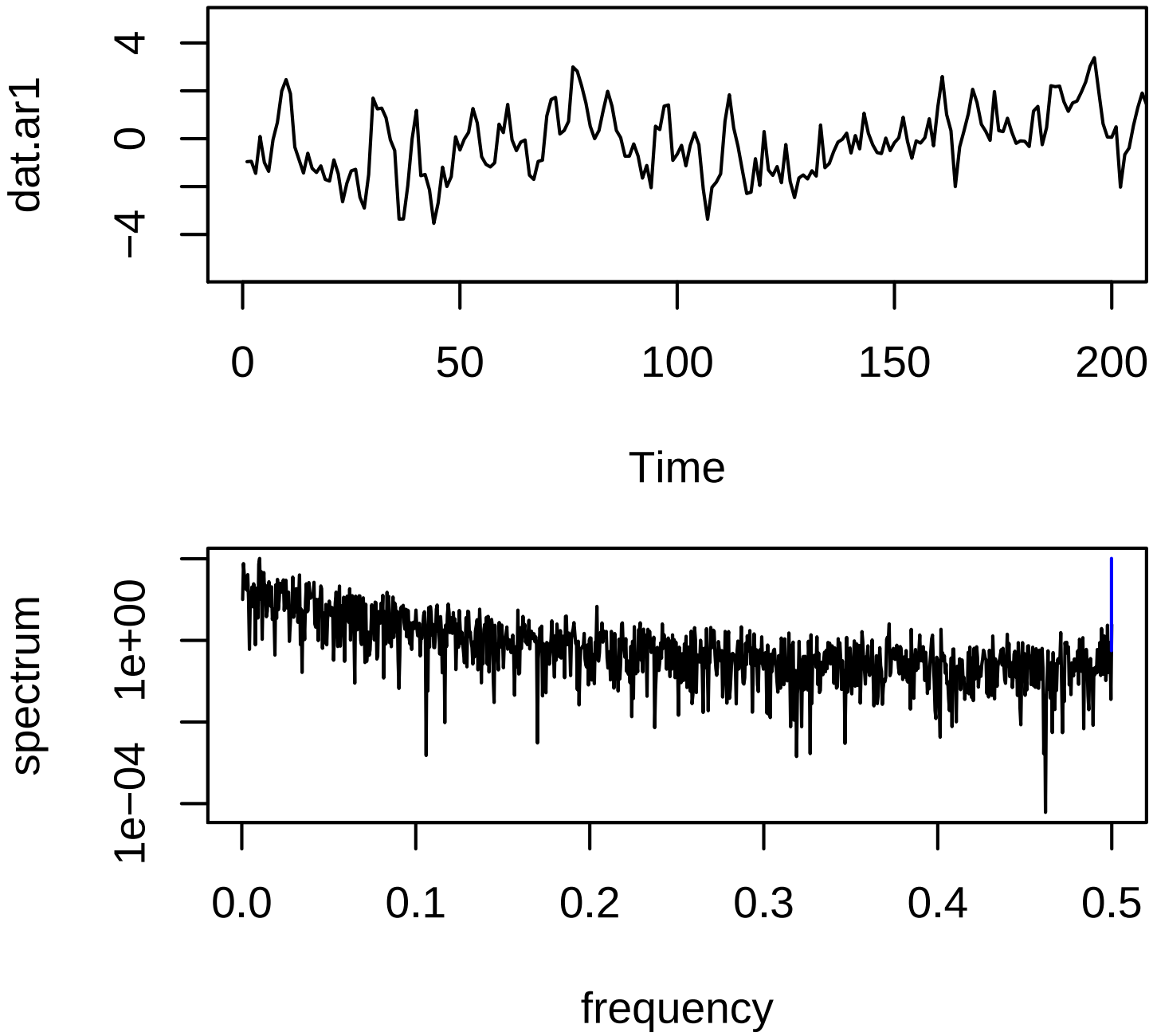
Spectral Analysis



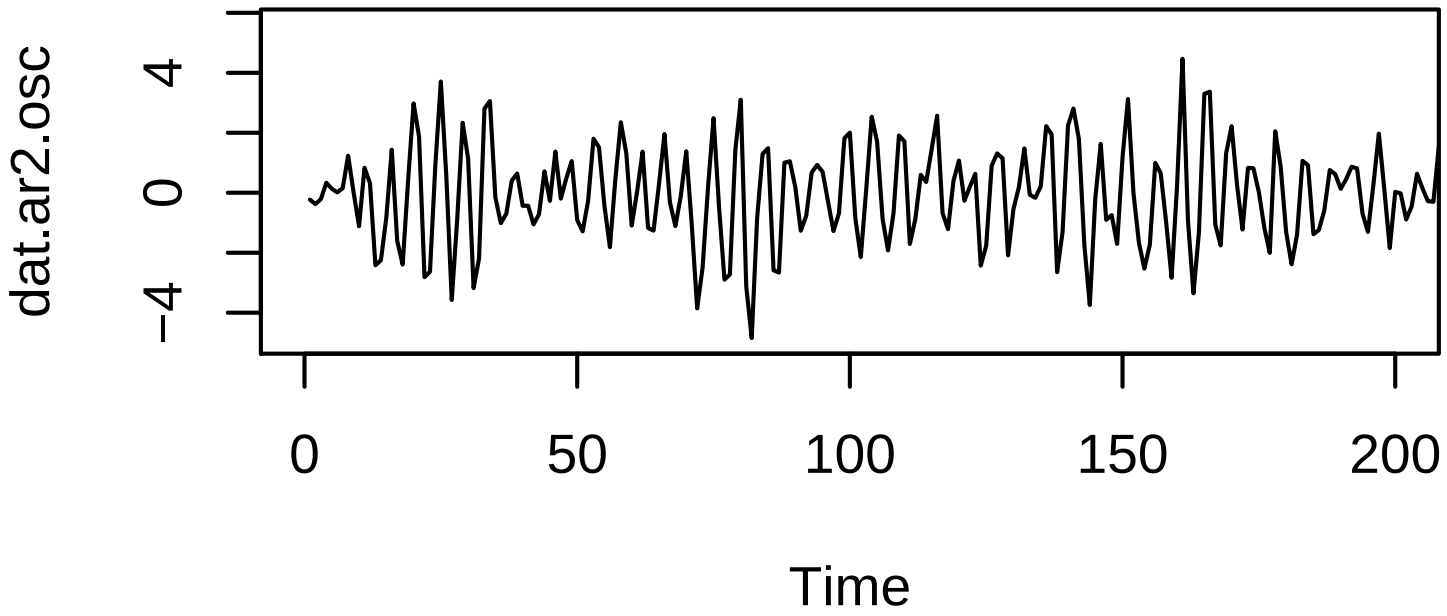
White Noise

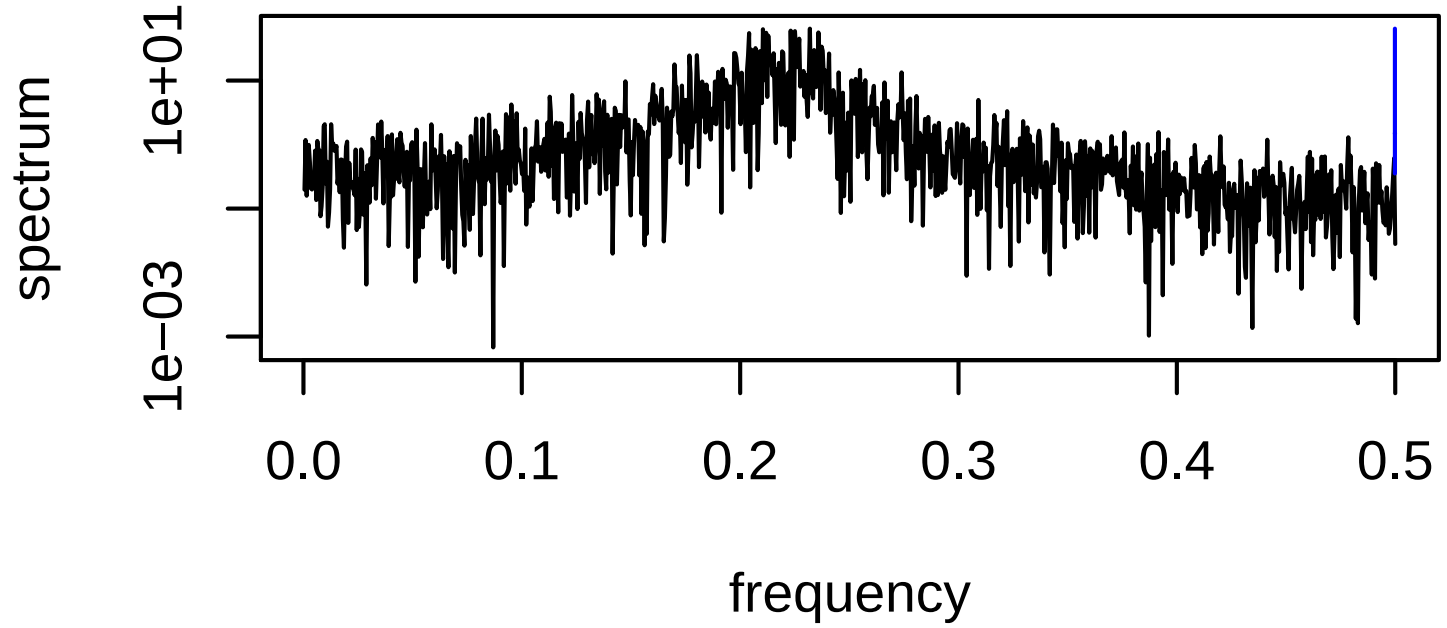


AR[1]

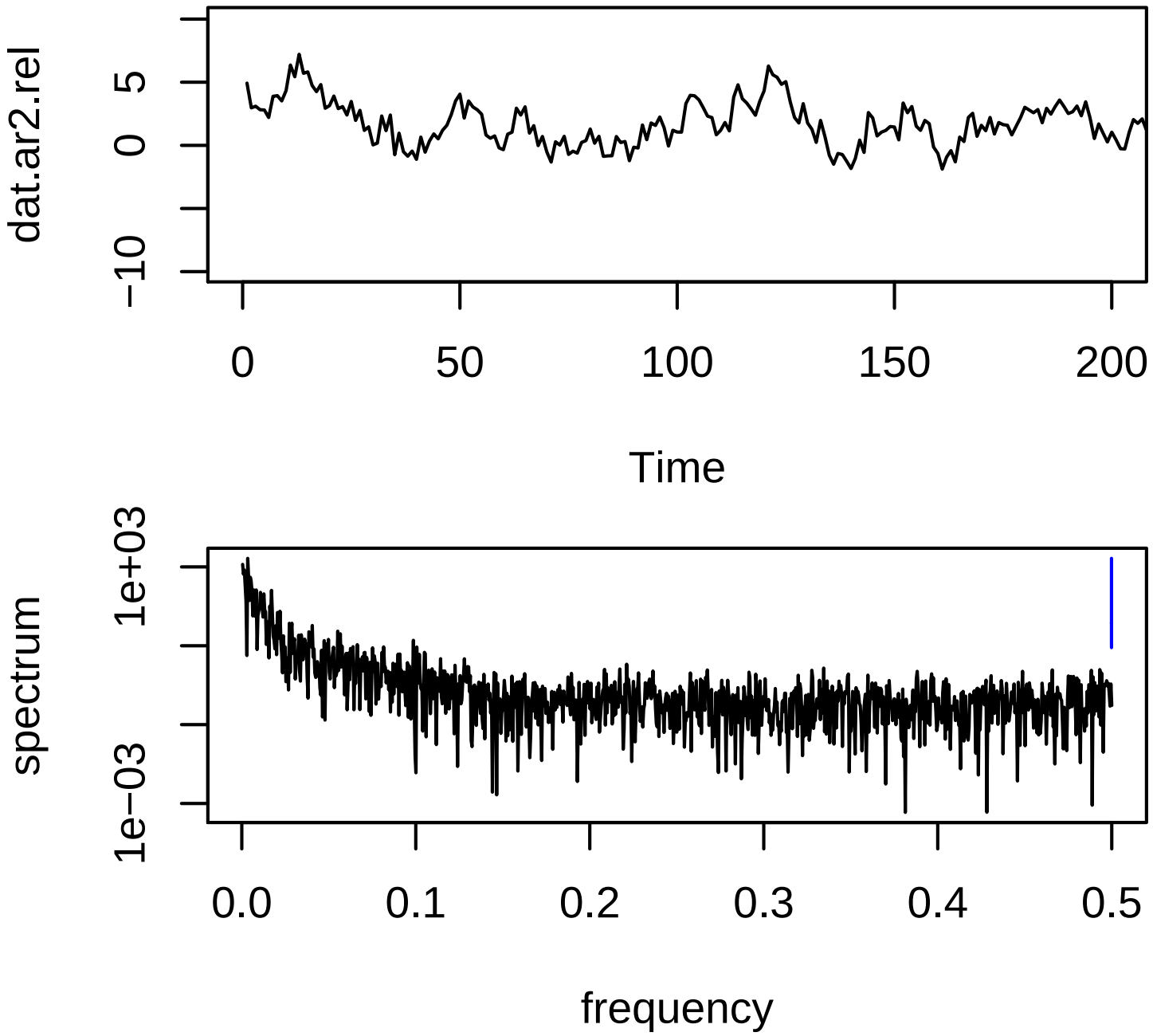


AR[2], Oscillating

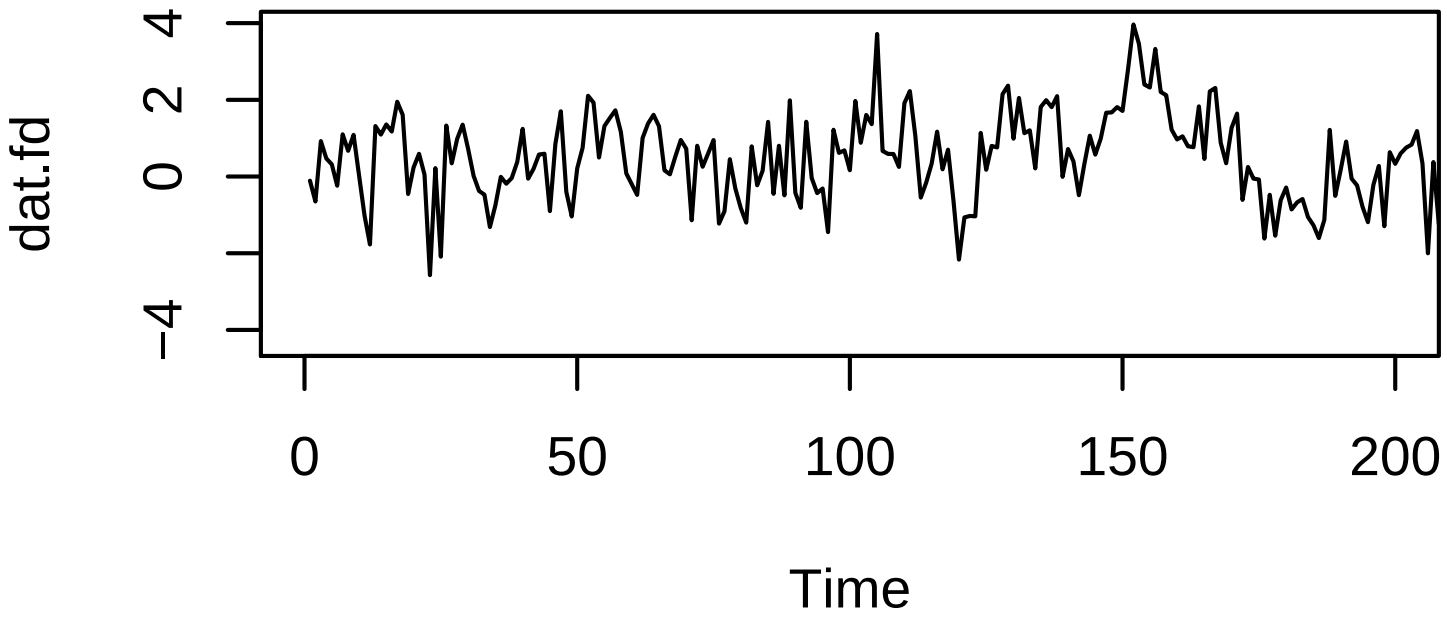


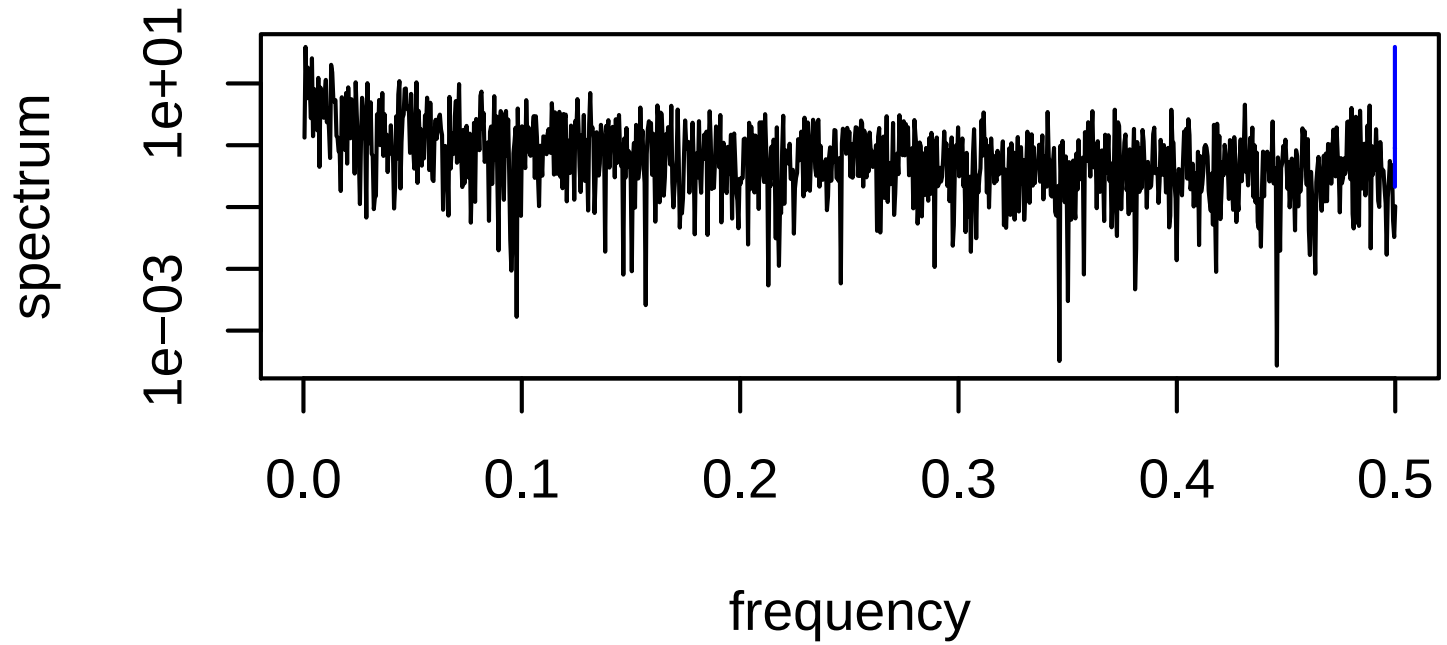


AR[2], Relaxating



Fractional Differenced (FD)





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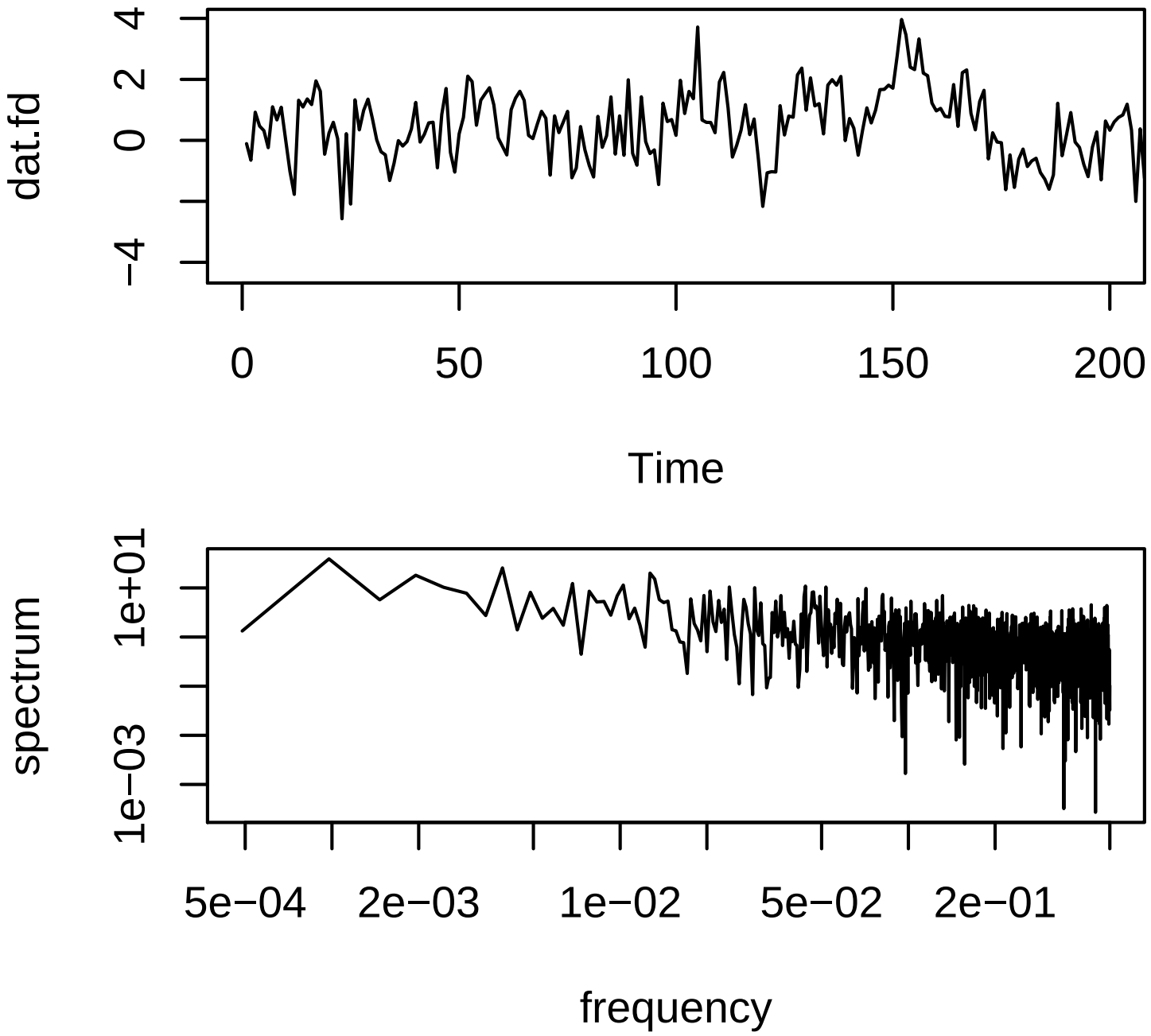
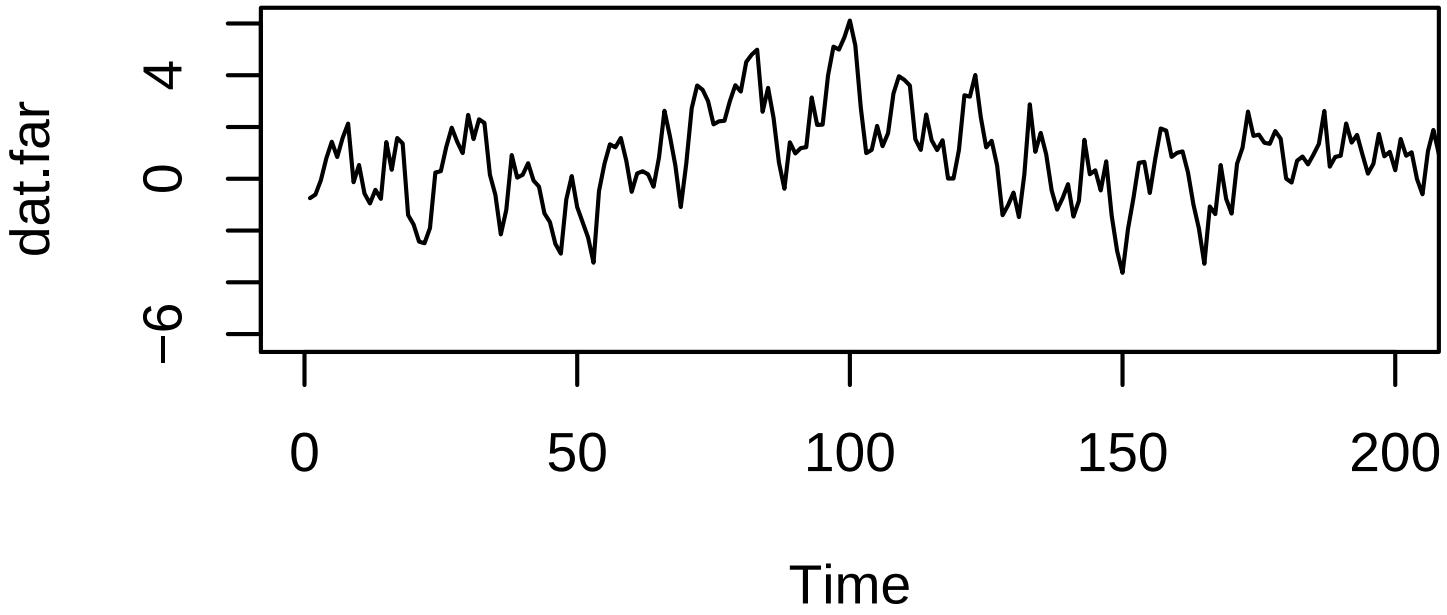
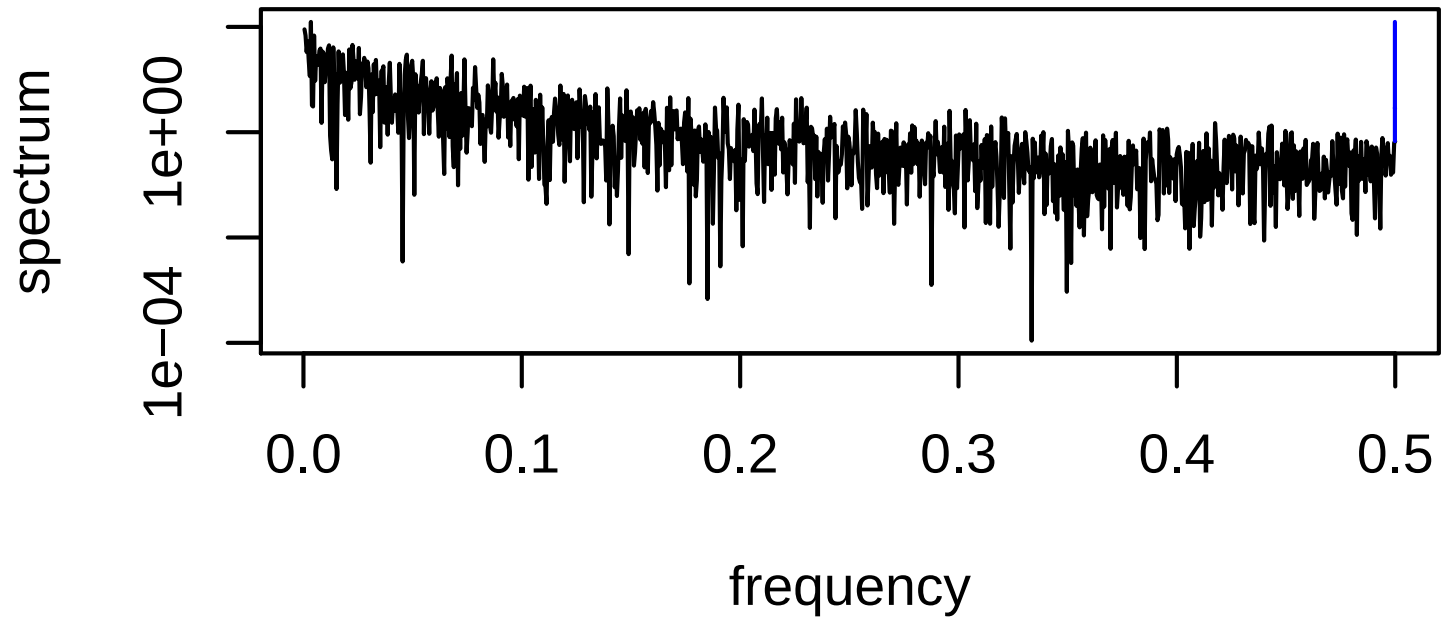


FARIMA[1,d,0]





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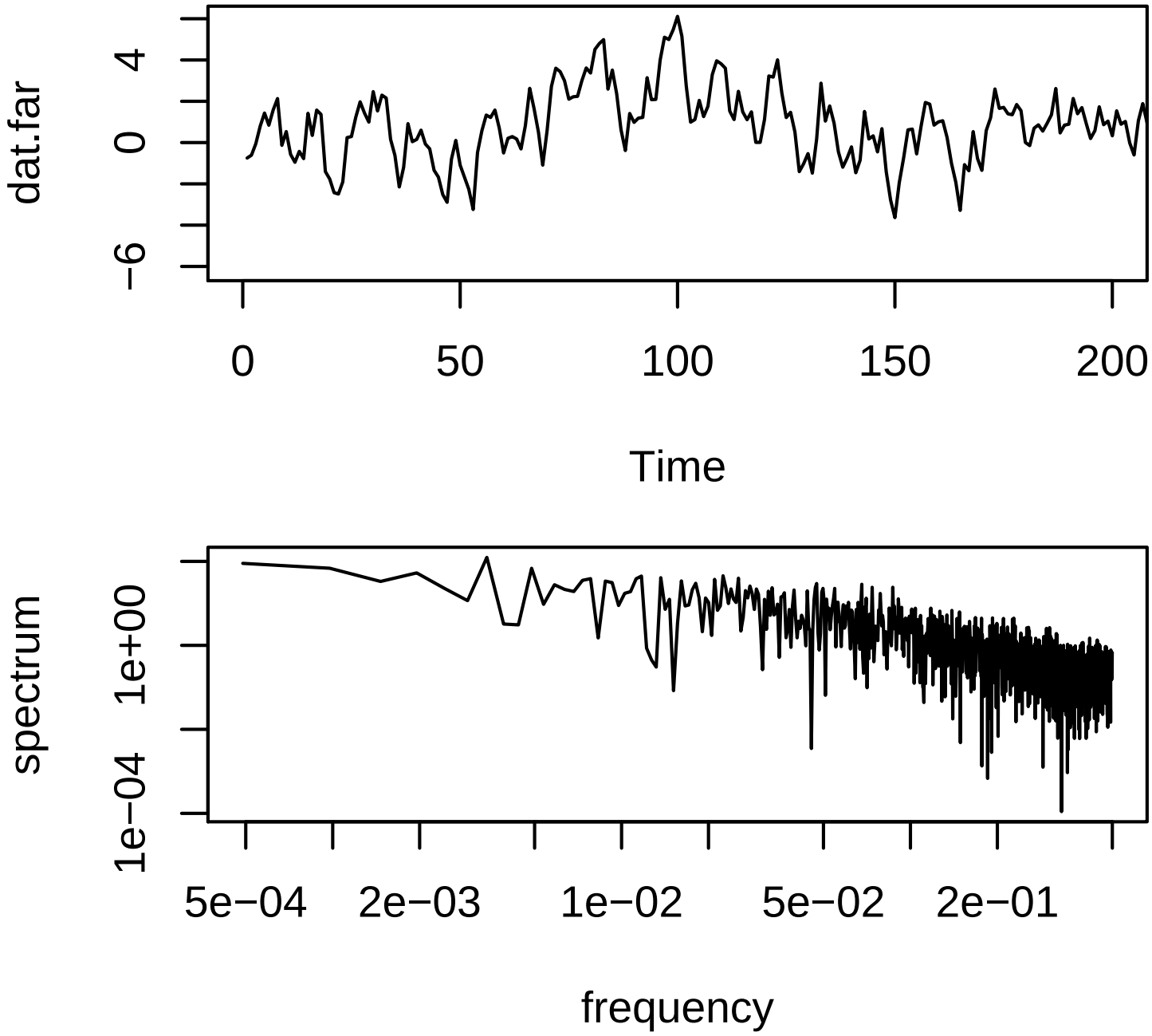
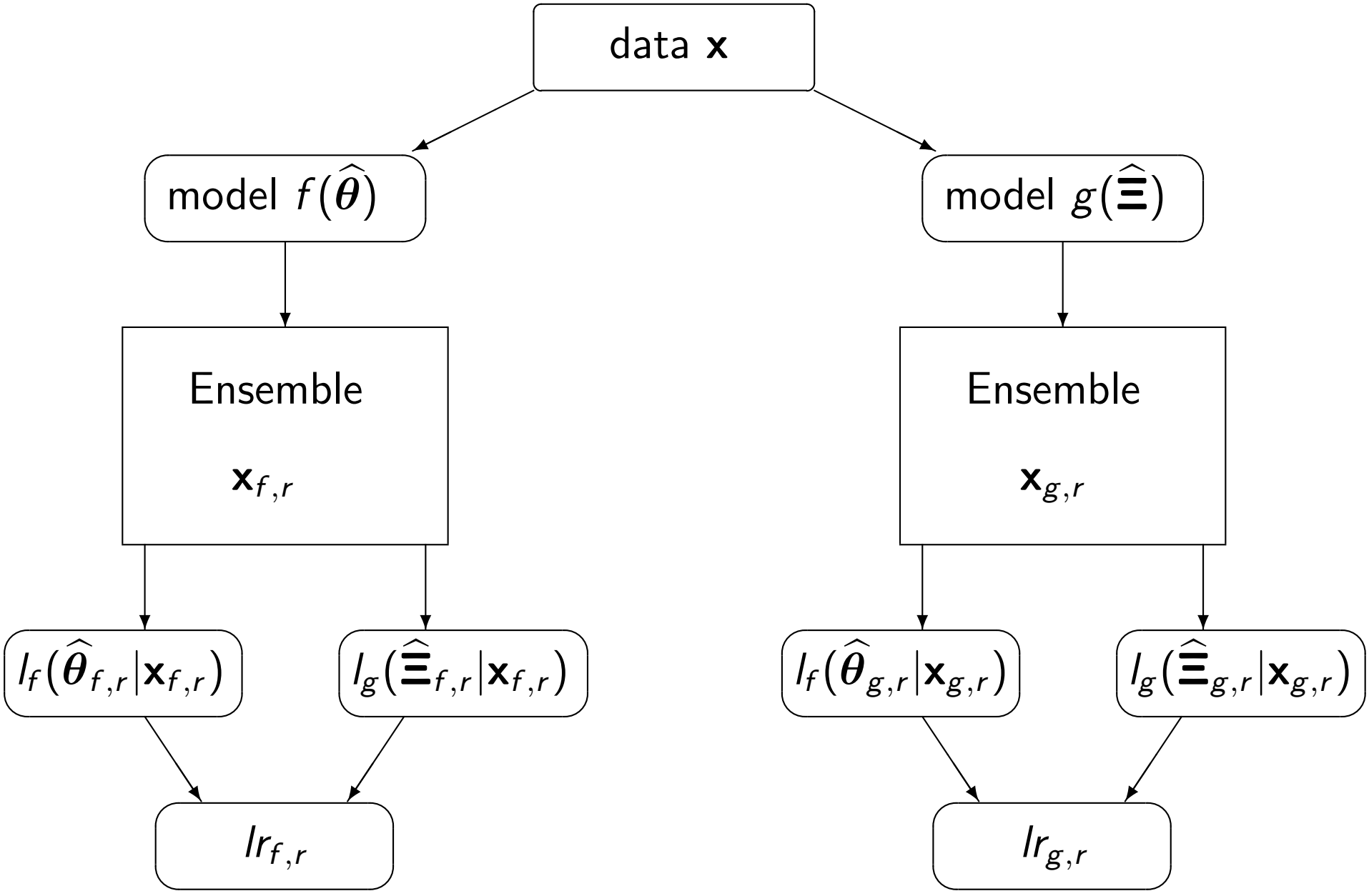
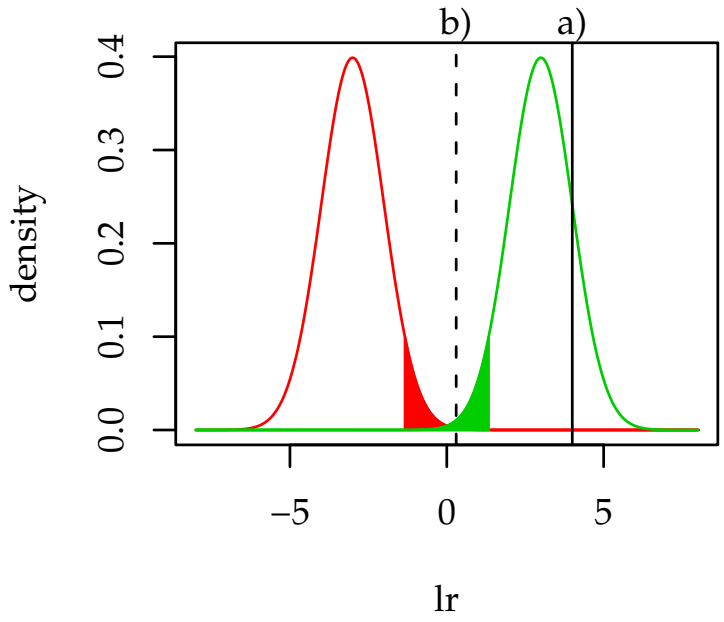
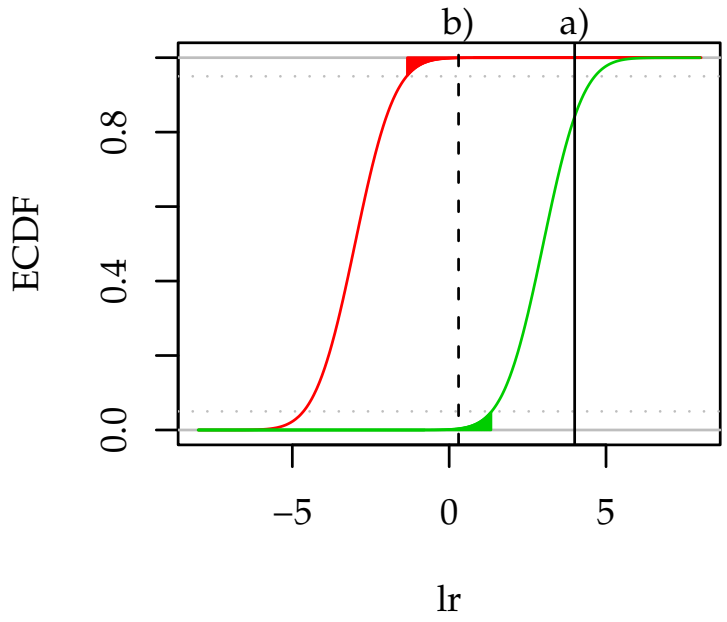


Non-Nested Model Selection, Scheme

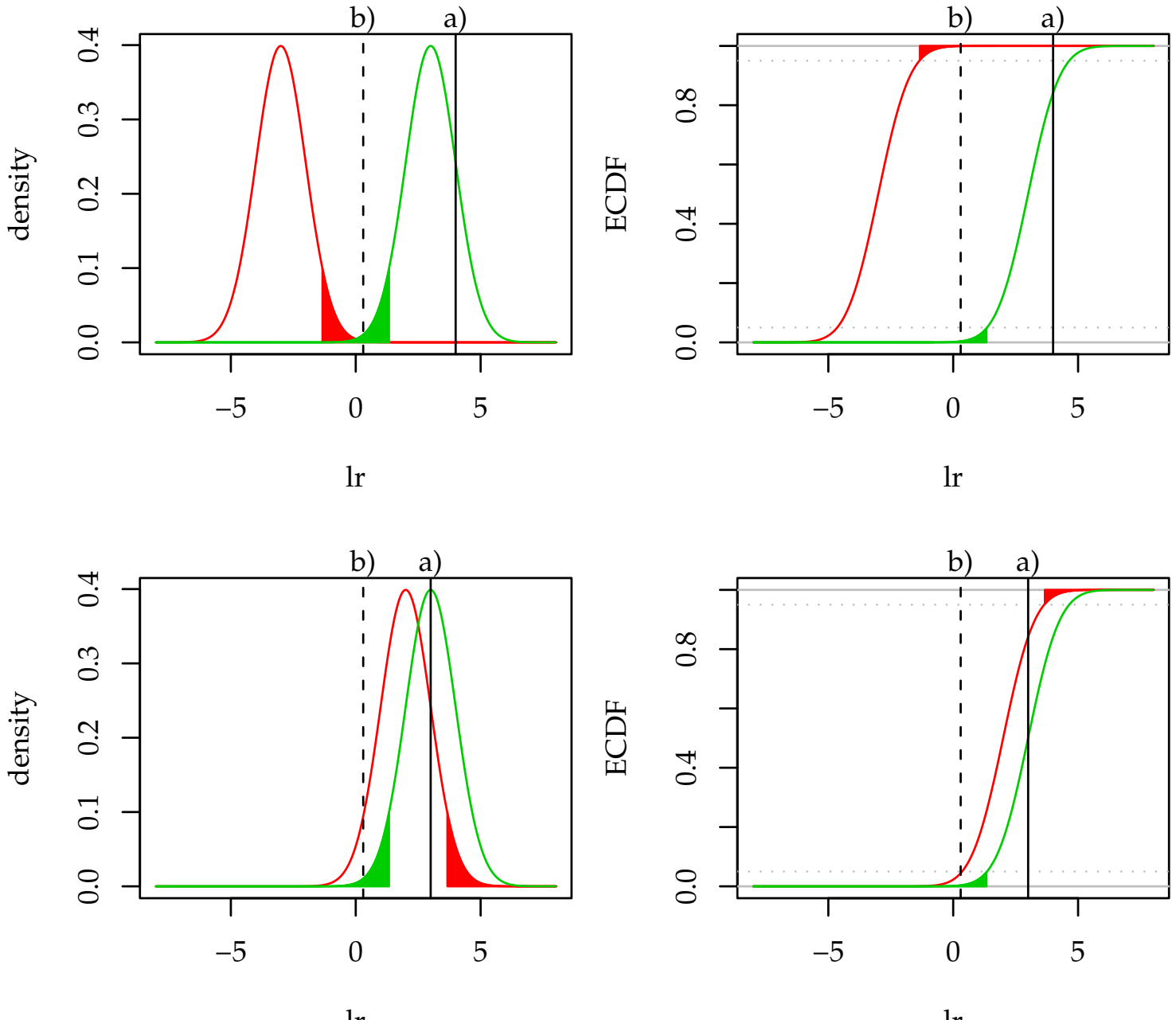


Non-Nested Model Selection, Outcome





Non-Nested Model Selection, Outcome



E2C2 Summer Cup – Round II

Prices

- one good answer – one good drink [1]

Questions

- What is the difference between the spectral density and the periodogram?
- What is the difference between SRD and LRD?

[1] can be substituted with organic bittersweet chocolate

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Appendix

Long-Range Dependence

Definition

A stationary stochastic process X_t is called long-range dependent (LRD) if its autocorrelation function $\rho(\tau)$ is not summable, i.e.

$$\sum_{-\infty}^{\infty} \rho(\tau) = \infty.$$

Examples

- algebraically decaying ACF for large lags τ
  $\lim_{\tau \to \infty} \rho(\tau) = c_\rho \tau^{-\gamma}$
- pole in the spectrum at $\omega = 0$
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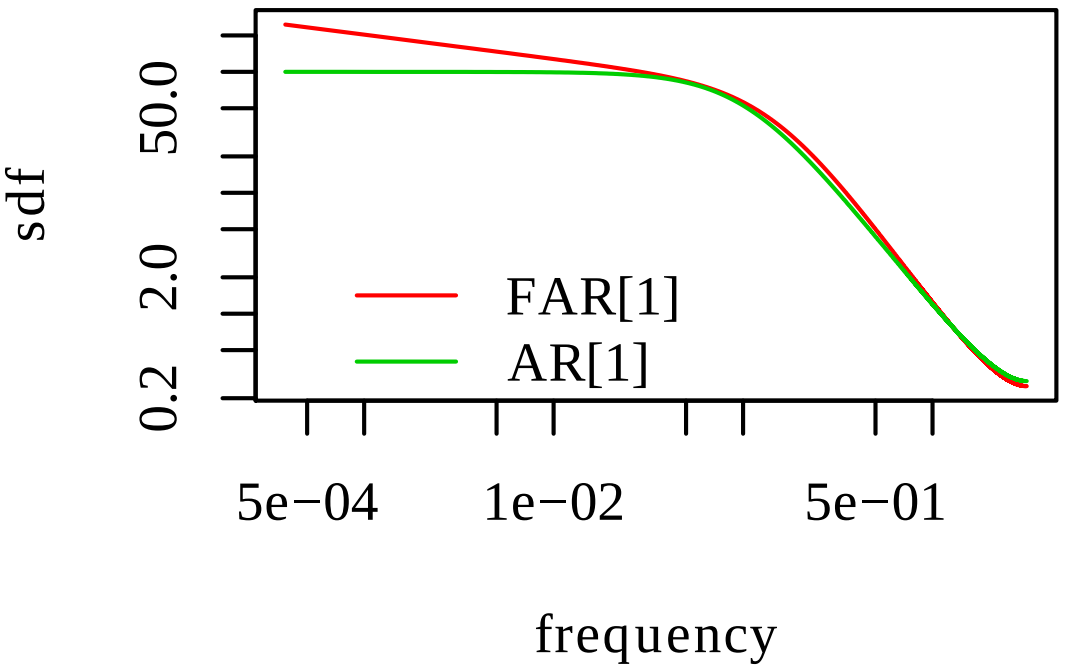
FARIMA Processes

Example



Spectral Density

$$S(\omega) = \frac{\sigma_\eta^2}{2\pi} \frac{|\Psi(e^{i\omega})|^2}{|\Phi(e^{i\omega})|^2} |1 - e^{i\omega}|^{-2d} \quad \overset{\omega \to 0}{\longrightarrow} \quad c_S |\omega|^{-2d}$$