Extreme Value Theory
(or how to go beyond of the range data)

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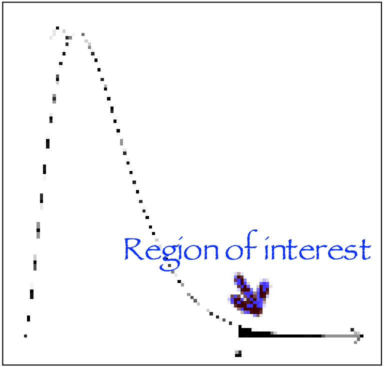
Katz et al., Statistics of extremes in hydrology,
Advances in Water Resources 25 (2002) 1287–1304

E2C2 Extreme Events, Causes and Consequences

Extreme quotes

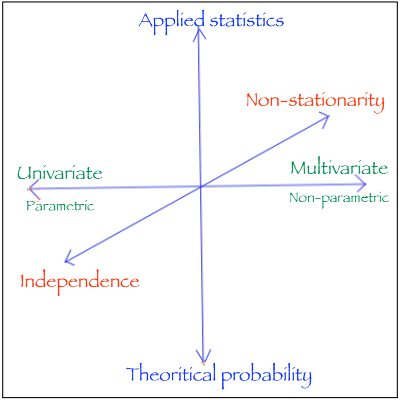
1. *"Man can believe the impossible, but man can never believe the improbable"* Oscar Wilde (Intentions, 1891)
2. *"Il est impossible que l'improbable n'arrive jamais"* Emil Julius Gumbel (1891-1966)

Extreme events ? ... a probabilistic concept linked to the **tail** behavior : low frequency of occurrence, large uncertainty and sometimes strong amplitude.



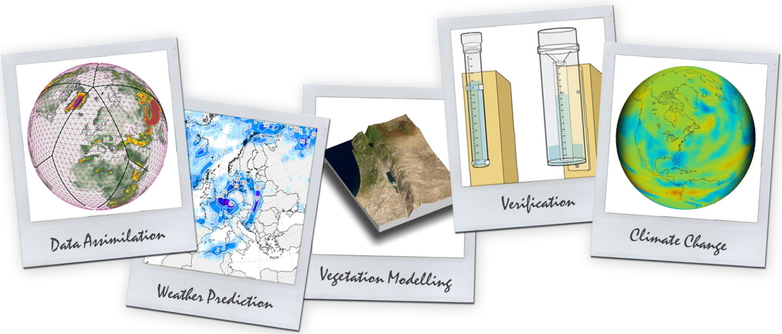
Important issues in Extreme Value Theory

- An asymptotic probabilistic concept
- A statistical modeling approach
- Identifying clearly assumptions
- Assessing uncertainties
- Goodness of fit and model selection



Outline

Why heavy rainfalls are important in geosciences ?



"It is very likely that hot extremes, heat waves, and heavy precipitation events will continue to become more frequent" and that "precipitation is highly variable spatially and temporally"

The policymakers summary of the 2007 Intergovernmental Panel on Climate Change

Random variable types

- Climate : maxima or mimina (daily, monthly, annually), dry spells, etc
- Hydrology : return levels.

  a quantile estimation pb : how to find z_p such that $\mathbb{P}(Z > z_p) = p$

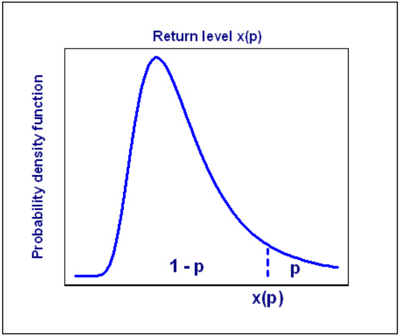
$\Longrightarrow$ Exceedances

Return levels and return periods

A **return level** with a **return period** of $T = 1/p$ years is a high threshold z_p whose probability of exceedance is p.
E.g., p = 0.01 ⇒ T = 100 years.

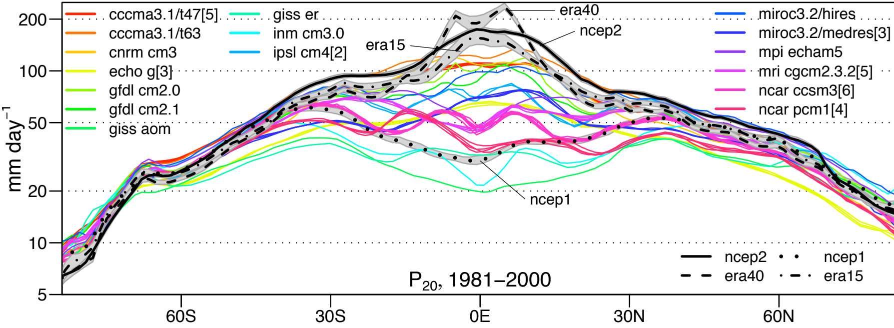
Return level interpretations

- Waiting time : Average waiting time until next occurrence of event is T years
- Number of events : Average number of events occurring within a T-year time period is one



Our main random variable of interest : precipitation

1. Relevant parameter in meteorology and climatology
2. Highly stochastic nature compared to other meteorological parameters



Kharin and Zwiers, Journal of Climate 2007, P_{20} (1981-2000)

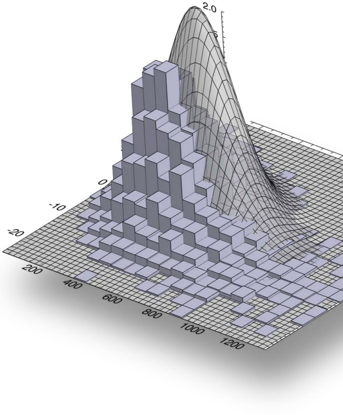
Heavy rainfall distributions

The problems at hand

- Classical distributions (Gamma, Weibull, Stretched-exponential, . . .) not satisfying for extremes
- EVT not adequate low and medium precipitation

Our main question

- How to go beyond the **univariate site-per-site** modeling and to take into account the spatial pairwise dependence among sites ?



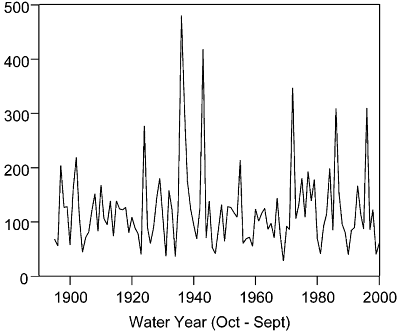
Three applications

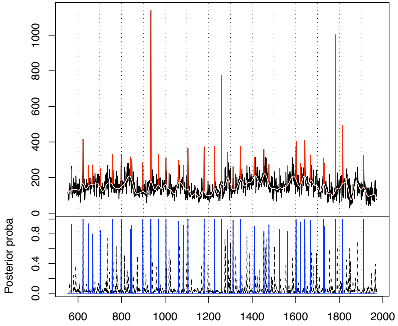
- **Measuring the spatial dependence among maxima** (Max-stable processes) :
  Vannitsem & Naveau (2007), Schlather & Tawn (2003), de Haan & Pereira (2005)
- **Spatial Interpolation of return levels in Colorado** (Hierarchical Bayesian models) :
  Cooley, Nychka and Naveau (2007), Coles & Tawn (1996).
- **Downscaling extremes over Illinois** (latent processes) :
  Vrac and Naveau (2007)

Two toy examples

Annual maximum peak flow
Potomac river (cfs)

Crete ice core
Greenland (ecm)



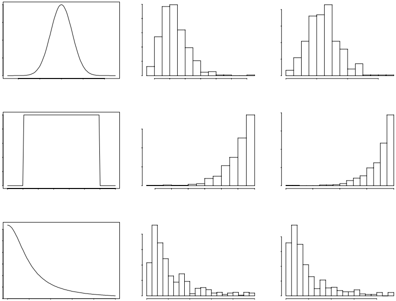


Maxima Distribution

Normal density ⇒ ⇐ Gumbel density

Uniform density ⇒ ⇐ Weibull density

Cauchy density ⇒ ⇐ Fréchet density



$n = 50$ $n = 100$

Max-stability

Let $M_n = \max(X_1, \ldots, X_n)$ with X_i iid with distribution F.

Problem : find a_n and b_n for a given F such that

$$\mathbb{P}\left(\frac{M_n - a_n}{b_n} < x\right) = F^n(a_n x + b_n) = F(x)$$

Home work A

Unit-Frèchet $F(x) = \exp(-1/x)$ for $x > 0$. Then $a_n = 1$ & $b_n = 0$

Gumbel $F(x) = \exp(-\exp(-x))$ for all real x. Then $a_n = 1$ & $b_n = -\log n$

Weibull $F(x) = \exp(-|x|^{\alpha})$ for $x < 0$ (1 otherwise). Then $a_n = n^{1/\alpha}$ & $b_n = 0$

Max-stability

Let $M_n = \max(X_1, \ldots, X_n)$ with X_i iid with distribution F.

Problem : find a_n and b_n for a given F such that

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{M_n - a_n}{b_n} < x\right) = \lim_{n\to\infty} F^n(a_n x + b_n) = F(x)$$

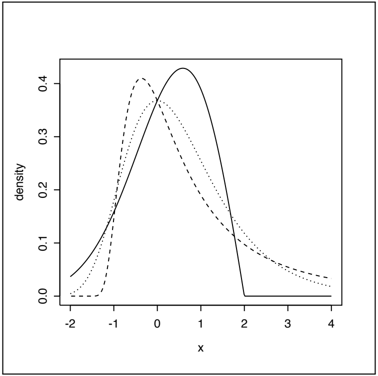
Home work B

Exponential $F(x) = 1 - \exp(-x)$ for $x > 0$. Then $a_n = 1$ & $b_n = \log n$

Uniform $F(x) = x$ for $0 < x < 1$. Then $a_n = 1/n$ & $b_n = 1$

Generalized Extreme Value (GEV) distribution

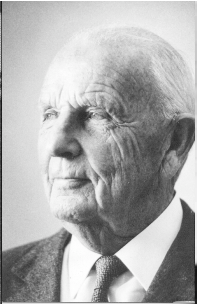
$$\mathbb{P}\left(\frac{M_n - a_n}{b_n} < x\right) \sim \mathrm{GEV}(x) = \exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\}$$



Home work C : show that a GEV is max-stable

Historical perspective

Gumbel (1891-1966)



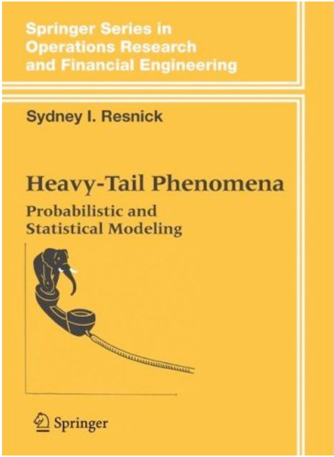
Weibull (1887-1979)

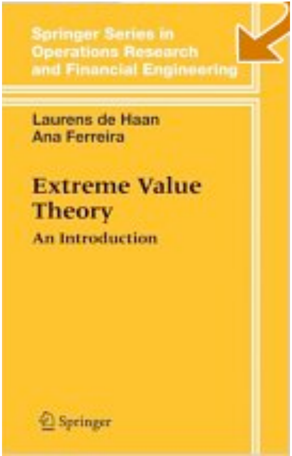
Fréchet (1878-1973)

- Emil Gumbel was born and trained as a statistician in Germany, forced to move to France and then the U.S. because of his pacifist and socialist views. He was a pioneer in the application of extreme value theory, particularly to climate and hydrology.
- Waloddi Weibull was a Swedish engineer famous for his pioneering work on reliability, providing a statistical treatment of fatigue, strength, and lifetime.
- Maurice Frechet was a French mathematician who made major contributions to pure mathematics as well as probability and statistics. He also collected empirical examples of heavy-tailed distributions.

Other important names : Fisher and Tippet (1928), Gnedenko (1943), etc

An active statistical and probabilistic field





GEV and return levels

$$\mathrm{GEV}(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\}$$

Computing the return level z_p such that GEV$(z_p) = 1 - p$

$$z_p = \mathrm{GEV}^{-1}(1-p)$$

Hence, $\boxed{z_p = \mu + \frac{\sigma}{\xi}\left([-\ln(1-p)]^{-\xi} - 1\right)}$

GEV and return levels estimation

$$z_p = \mu + \frac{\sigma}{\xi}\left([-\ln(1-p)]^{-\xi} - 1]\right)$$

Estimating the return level z_p

$$\boxed{\hat{z}_p = \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left([-\ln(1-p)]^{-\hat{\xi}} - 1]\right)}$$

Estimating the GEV parameters estimates $(\hat{\mu}, \hat{\sigma}, \hat{\xi})$

- Maximum likelihood estimation
- Methods of moments type (PWM and GPWM)
- Exhaustive tail-index approaches

GEV and return levels estimation

$$\hat{z}_p = \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left([-\ln(1-p)]^{-\hat{\xi}} - 1]\right)$$

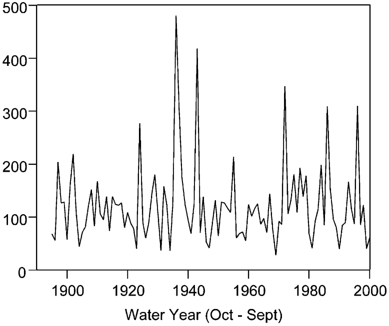
Maximum likelihood estimates of $(\hat{\mu}, \hat{\sigma}, \hat{\xi})^t$

Asymptotically distributed as a multivariate Gaussian vector with mean $\theta = (\hat{\mu}, \hat{\sigma}, \hat{\xi})^t$ and covariance matrix that is the inverse of the **expected information matrix** whose elements are equal

$$\mathbb{E}\left(-\frac{\partial^2 \log l(\theta)}{\partial\theta_i \partial\theta_j}\right)$$

where $l(\theta)$ is the likelihood function of the GEV distributed sample

Our first toy example



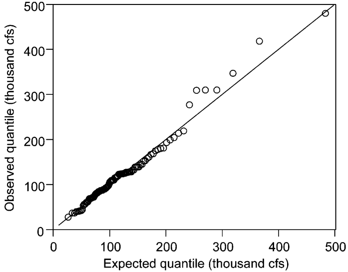
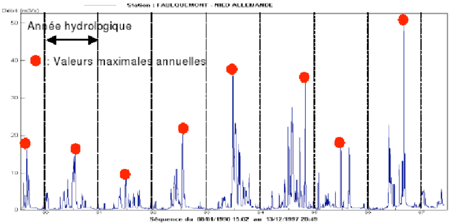
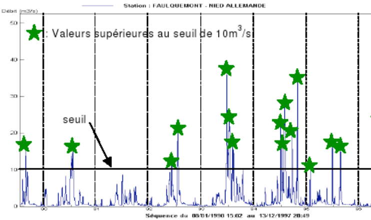


Fig. 4. *Q–Q* plot for fit of GEV distribution to annual peak flow of Potomac River (line of equality indicates perfect fit).

$\hat{\xi} = 0.191$ with a P-value of 0.002 for likelihood ratio test of $\xi = 0$

Peak over Threshold (POT)

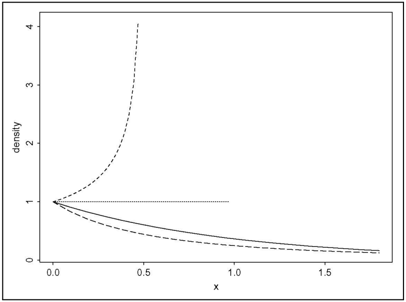




Thresholding : the Generalized Pareto Distribution (GPD)

$$\mathbb{P}\{\mathbf{R}-u > y | \mathbf{R} > u\} = \left(1 + \frac{\xi\, y}{\sigma_u}\right)_+^{-1/\xi}$$

Vilfredo Pareto : 1848-1923



Born in France and trained as an engineer in Italy, he turned to the social sciences and ended his career in Switzerland. He formulated the power-law distribution (or ”Pareto’s Law”), as a model for how income or wealth is distributed across society.

Generalized Pareto Distribution (GPD)

$$\boxed{\mathbb{P}\{\mathbf{R} - u > y | \mathbf{R} > u\} = \left(1 + \frac{\xi\, y}{\sigma_u}\right)^{-1/\xi}}$$

Parameters

- u = predetermined threshold
- σ_u = scale parameter to be estimated
- ξ = shape parameter to be estimated

Advantages & Practical issues

- Flexibility to describe three different types of tail behavior
- More data are kept for the statistical inference
- Problem of threshold selection

GPD

$$\mathbb{P}\{\mathbf{R} - u > y | \mathbf{R} > u\} = \left(1 + \frac{\xi\, y}{\sigma_u}\right)_+^{-1/\xi}$$

Special cases (home work D)

Unit-Frèchet $F(x) = \exp(-1/x)$ for $x > 0$. Then $\sigma_u = 1$ and $\xi = -1/\alpha$

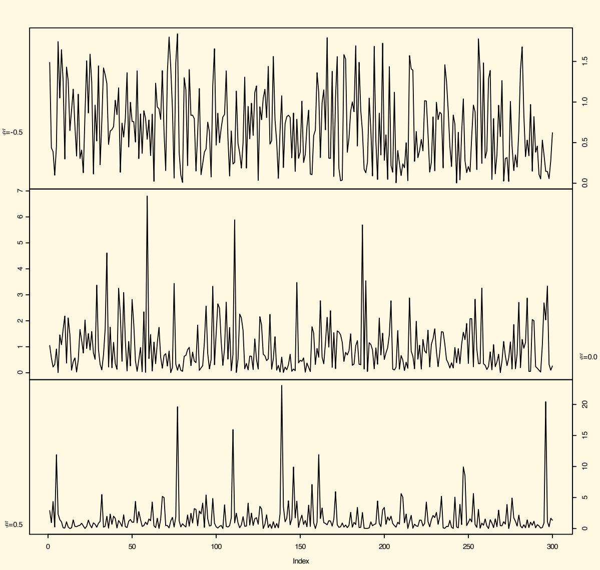
Exponential $F(x) = 1 - \exp(-x)$ for $x > 0$. Then $\sigma_u = 1$ and $\xi = 0$

Uniform $F(x) = x$ for $0 < x < 1$. Then $\sigma_u = 1$ and $\xi = -1$

Stability property (home work E)

If the exceedance $(\mathbf{R} - u | \mathbf{R} > u)$ follows a GPD(σ_u, ξ) then the higher exceedance $(\mathbf{R} - v | \mathbf{R} > v)$ also follows GPD$(\sigma_u + (v - u)\xi, \xi)$

GPD : “From Bounded to Heavy tails”



Estimating the GPD parameters estimates $(\hat{\sigma}_u, \hat{\xi})$

- Maximum likelihood estimation
- Methods of moments type (PWM and GPWM)
- Exhaustive tail-index approaches

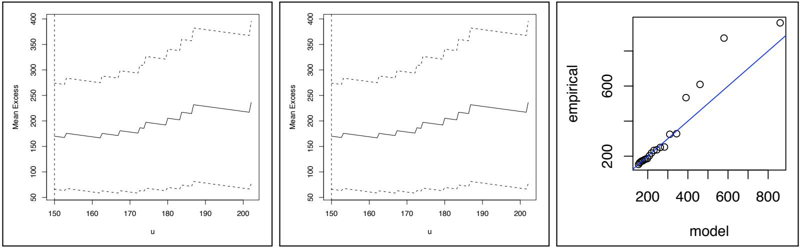
Taking advantages of the stability property

- Mean Excess function

$$\mathbb{E}(\mathbf{R} - u | \mathbf{R} > u) = \frac{\sigma_u + u\xi}{1 - \xi}$$

- the scale parameter varies linearly in the threshold u
- the shape parameter ξ is fixed wrt the threshold u

GPD diagnostics & models selection for our Crete data



$\hat{\xi} = 0.56\ (0.37)$

GPD

GPD return level z_p

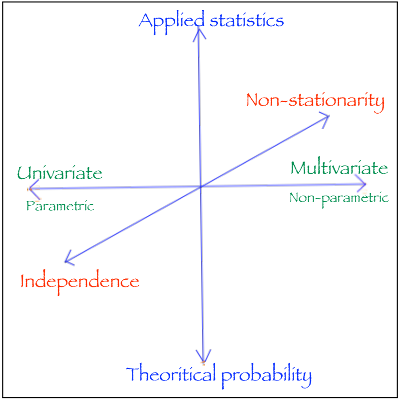
$$z_p = u + \frac{\sigma_u}{\xi}\left(\left[\frac{p}{\mathbb{P}(R > u)}\right]^{-\xi} - 1\right)$$

Estimating the return level z_p

$$\hat{z}_p = u + \frac{\hat{\sigma}_u}{\hat{\xi}}\left(\left[\frac{p \times n}{N_u}\right]^{-\hat{\xi}} - 1\right)$$

A first summary

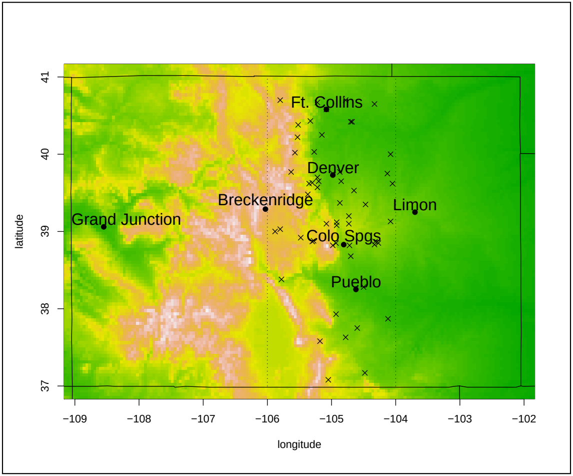
- So far, we have assumed an idd model
- Asymptotic probability suggests GEV and GPD
- Maximum likelihood approach provides asymptotic parameters and return levels uncertainties
- Goodness of fit and model selection



Outline

►► **Main Part**

Daily precipitation (April-October, 1948-2001, 56 stations)



Precipitation in Colorado's front range

Data

- 56 weather stations in Colorado (semi-arid and mountainous region)
- Daily precipitation for the months April-October
- Time span = 1948-2001
- Not all stations have the same number of data points
- Precision : 1971 from 1/10th of an inche to 1/100

D. Cooley, D. Nychka and P. Naveau, (2007). Bayesian Spatial Modeling of Extreme Precipitation Return Levels. *Journal of The American Statistical Association (in press).*

Our main assumptions

- Process layer : The scale and shape GPD parameters $(\xi(x), \sigma(x))$ are random fields and result from an unobservable latent spatial process
- Conditional independence : precipitation are independent given the GPD parameters

Our main variable change

$$\sigma(x) = \exp(\phi(x))$$

Hierarchical Bayesian Model with three levels

$$\begin{aligned}\mathbb{P}(\text{process, parameters}|\text{data}) \quad &\propto \quad \mathbb{P}(\text{data}|\text{process, parameters}) \\ &\quad\times\mathbb{P}(\text{process}|\text{parameters}) \\ &\quad\times\mathbb{P}(\text{parameters})\end{aligned}$$

Process level : the scale and shape GPD parameters $(\xi(x), \sigma(x))$ are hidden random fields

Our three levels

A) **Data layer** $:= \mathbb{P}(\text{data}|\text{process, parameters}) =$

$$\mathbb{P}_\theta\{\mathbf{R(x_i)} - u > y | \mathbf{R(x_i)} > u\} = \left(1 + \frac{\xi_i \, y}{\exp \phi_i}\right)^{-1/\xi_i}$$

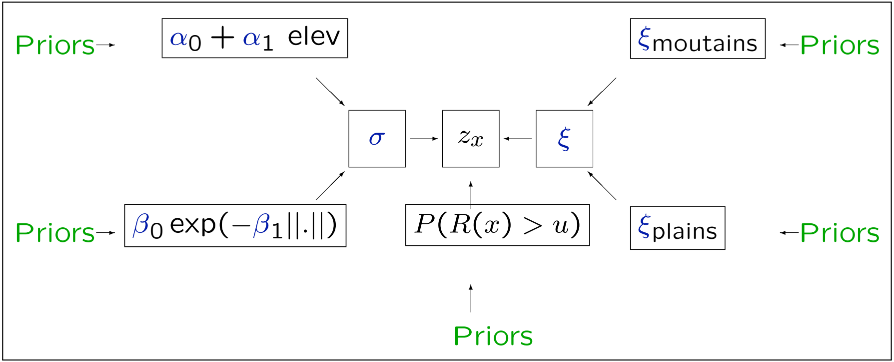
B) **Process layer :=** $\mathbb{P}(\text{process}|\text{parameters})$:

$$\text{e.g. } \phi_i = \alpha_0 + \alpha_1 \times \text{elevation}_i + \text{MVN}\left(0, \beta_0 \exp(-\beta_1 ||x_k - x_j||)\right)$$

$$\text{and } \xi_i \quad = \quad \begin{cases} \xi_{\text{moutains}}, \text{ if } x_i \in \text{mountains} \\ \xi_{\text{plains}}, \text{ if } x_i \in \text{plains} \end{cases}$$

C) **Parameters layer (priors) :=** $\mathbb{P}(\text{parameters})$:
e.g. $(\xi_{\text{moutains}}, \xi_{\text{plains}})$ Gaussian distribution with non-informative mean and variance

Bayesian hierarchical modeling

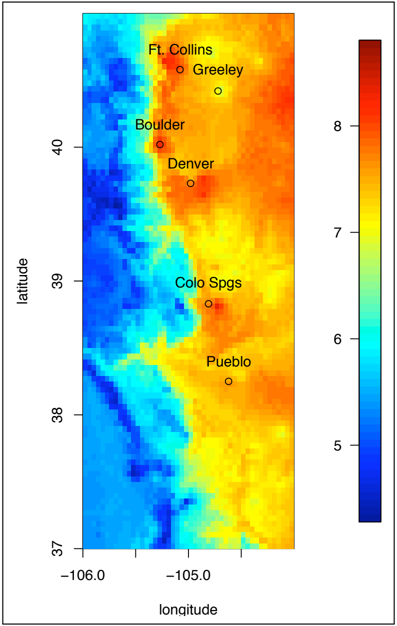


Model selection

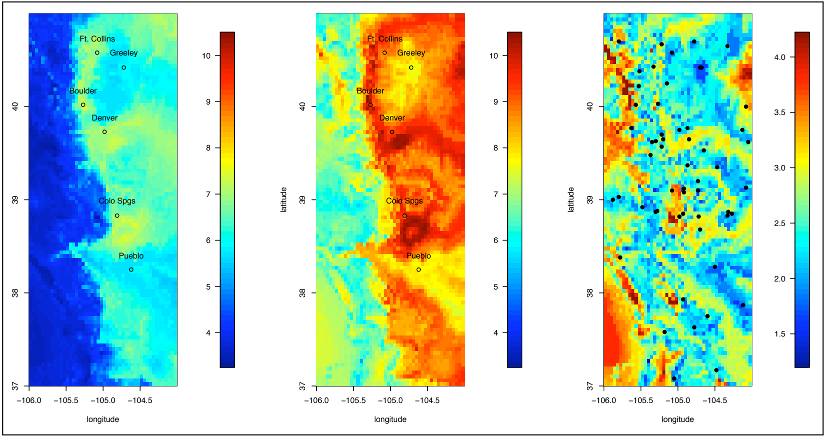
Baseline model		$\bar{D}$	p_D	*DIC*
Model 0:	$\phi = \phi$ $\xi = \xi$	73,595.5	2.0	73,597.2
Models in latitude/longitude space		$\bar{D}$	p_D	*DIC*
Model 1:	$\phi = \alpha_0 + \epsilon_\phi$ $\xi = \xi$	73,442.0	40.9	73,482.9
Model 2:	$\phi = \alpha_0 + \alpha_1(\text{msp}) + \epsilon_\phi$ $\xi = \xi$	73,441.6	40.8	73,482.4
Model 3:	$\phi = \alpha_0 + \alpha_1(\text{elev}) + \epsilon_\phi$ $\xi = \xi$	73,443.0	35.5	73,478.5
Model 4:	$\phi = \alpha_0 + \alpha_1(\text{elev}) + \alpha_2(\text{msp}) + \epsilon_\phi$ $\xi = \xi$	73,443.7	35.0	73,478.6
Models in climate space		$\bar{D}$	p_D	*DIC*
Model 5:	$\phi = \alpha_0 + \epsilon_\phi$ $\xi = \xi$	73,437.1	30.4	73,467.5
Model 6:	$\phi = \alpha_0 + \alpha_1(\text{elev}) + \epsilon_\phi$ $\xi = \xi$	73,438.8	28.3	73,467.1
Model 7:	$\phi = \alpha_0 + \epsilon_\phi$ $\xi = \xi_{\text{mtn}}, \xi_{\text{plains}}$	**73,437.5**	**28.8**	**73,466.3**
Model 8:	$\phi = \alpha_0 + \alpha_1(\text{elev}) + \epsilon_\phi$ $\xi = \xi_{\text{mtn}}, \xi_{\text{plains}}$	73,436.7	30.3	73,467.0
Model 9:	$\phi = \alpha_0 + \epsilon_\phi$ $\xi = \xi + \epsilon_\xi$	73,433.9	54.6	73,488.5

NOTE: Models in the climate space had better scores than models in the longitude/latitude space. $\epsilon_\cdot \sim \text{MVN}(0, \Sigma)$, where $[\sigma]_{i,j} = \beta_{\cdot,0} \exp(-\beta_{\cdot,1} \|\mathbf{x}_i - \mathbf{x}_j\|)$.

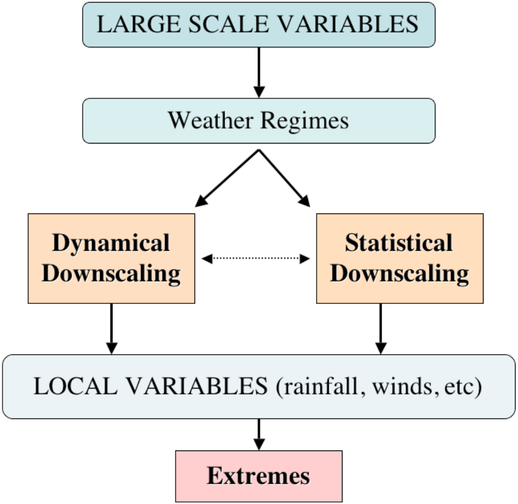
Return levels posterior mean



Posterior quantiles of return levels (.025, .975)



Downscaling of rainfalls



Vrac and Naveau, (2007). Stochastic downscaling of precipitation : From dry events to heavy rainfalls. *Water Resource Research (in press)*

Our data

- **Local scale :** R_t = Daily precipitation recorded at 37 stations 1980-1999 (DJF)
- **Large scale :** X_t = NCEP geopotential height, Q and DT at 850mb
- **Weather regimes :** S_t = Four regimes of precipitation

Our objective :

What is the precipitation probability distribution of R_t given the large and regional scale characteristics, $\boldsymbol{X}_t$ and S_t ?

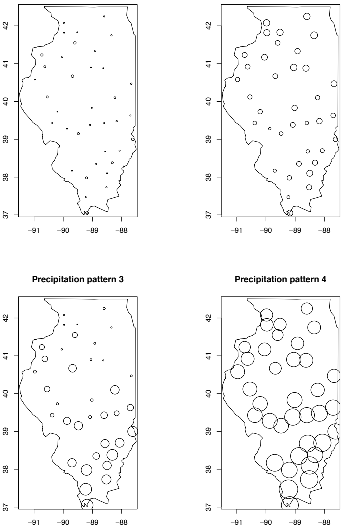
Subsidiary questions :

- What is the precipitation distribution at a given site ?
- What are meaningful regional patterns ?
- How to connect the different scales ?

Our strategy

A GPD latent process (hidden markov process + logistic model) that depends on X_t et de S_t

Illinois rainfall patterns



How to switch from one precipitation pattern to the other ?

Markov chains

$$\mathbb{P}(S_t = j | S_{t-1} = i) \propto \gamma_{ij}$$

..., but these transition probabilities are independent of the atmospheric variables $\boldsymbol{X}_t$ like Q

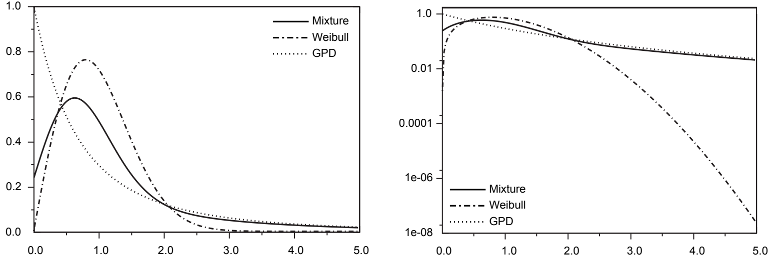
Non-homogeneous Markov chains

$$P(S_t = j | S_{t-1} = i, \boldsymbol{X}_t) \propto \gamma_{ij} \exp\left[-\frac{1}{2}(\boldsymbol{X}_t - \mu_{ij})\Sigma^{-1}(\boldsymbol{X}_t - \mu_{ij})'\right]$$

where

- Σ = atmospheric variables covariance
- μ_{ij} = atmospheric variables means

Precipitation density : non-homogeneous Mixture Model

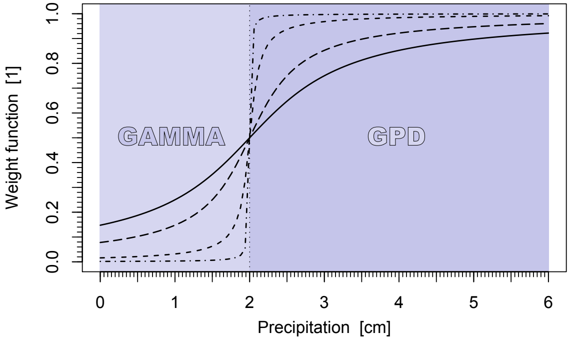


Mixture Distribution

GPD and Gamma (Weibull after Frigessi et al. (2003))

$$f_{\text{mix}}(r) = \frac{1}{Z}\Big(\underbrace{\big(1 - w_{\mu,\tau}(r)\big)}_{\text{Gamma weight}} \cdot \underbrace{f_{\Gamma(\alpha,\beta)}(r)}_{\text{Gamma pdf}} + \underbrace{w_{\mu,\tau}(r)}_{\text{GPD weight}} \cdot \underbrace{f_{G(\sigma,\xi)}(r, u = 0)}_{\text{GPD}}\Big)$$

Weight Function

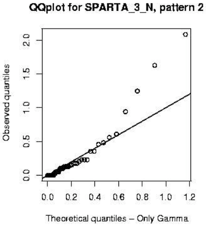


Weight Function

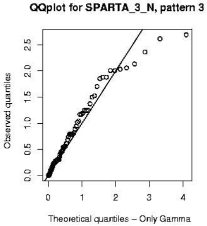
Dynamic mixture model for unsupervised tail estimation without threshold selection (Frigessi et al., 2002)

$$w_{m,\tau}(r) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{r-\mu}{\tau}\right)$$

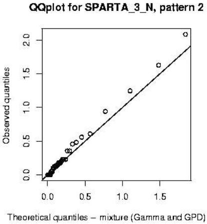
QQ plots for the Spartan station

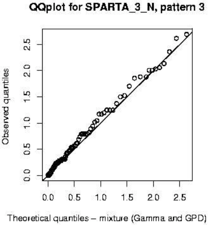


(a)



(b)





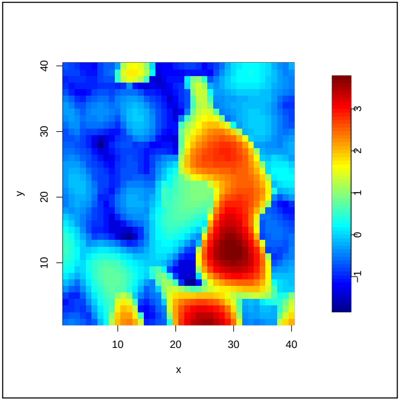
Model selection

- **(0)** gamma + GPD : parameters vary with location and precipitation pattern,
- **(i)** only gamma : parameters vary with location and precipitation pattern,
- **(ii)** gamma + GPD with one ξ parameter per pattern
- **(iii)** same as (ii) with τ set to be equal to 0,
- **(iv)** gamma + GPD with one common ξ for all stations and all patterns,
- **(v)** same as (iv) with τ set to be equal to 0.
- **(iii)*** same as model (iii) but only gamma distributions for pattern 1.

Model selection

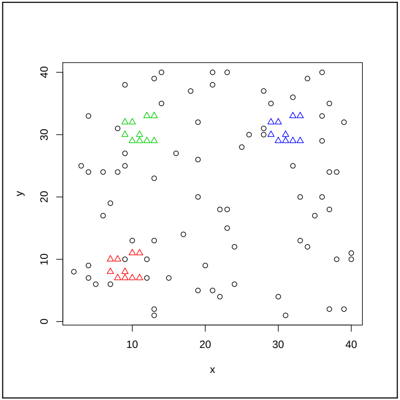
Station	Model (0) $p = 24n$	Model (i) $p = 8n$	Model (ii) $p = 20n + 4$	Model (iii) $p = 16n + 4$	Model (iv) $p = 20n + 1$	Model (v) $p = 16n + 1$	Model (iii)* $p = 12n + 5$
Aledo	AIC=-796.52	AIC=-816.58	AIC=-795.76	AIC=-809.79	AIC=**-819.46**	AIC=-816.18	AIC=-816.79
Aurora	AIC=-1137.47	AIC=-1149.99	AIC=-1256.53	AIC=-1293.89	AIC=**-1358.48**	AIC=-1152.51	AIC=-1299.89
Fairfield	AIC=14.36	AIC=103.07	AIC=22.45	AIC=22.37	AIC=**-76.81**	AIC=-10.21	AIC=16.37
Sparta	AIC=277.10	AIC=372.92	AIC=235.65	AIC=228.35	AIC=231.91	AIC=251.44	AIC=**222.35**
Windsor	AIC=-1014.80	AIC=-920.68	AIC=-1016.25	AIC=-1017.59	AIC=**-1069.99**	AIC=-1028.91	AIC=-1023.59
All five stations	AIC=-4433.18	AIC=-4422.27	AIC=-4479.50	AIC=-4515.13	AIC=-4425.06	AIC=-4423.78	AIC=**-4553.13**

Spatial Statistics for Maxima



How to describe the **spatial dependence** as a function of the **distance** between two points ?

Spatial Statistics for Maxima



How to perform **spatial interpolation** of extreme events ?

Spatial Statistics for Maxima

A few Approaches for modeling spatial extremes

- **Max-stable processes** : Adapting asymptotic results for multivariate extremes
  Schlather & Tawn (2003), Naveau et al. (2007), de Haan & Pereira (2005)
- **Bayesian or latent models** : spatial structure **indirectly** modeled via the EVT parameters distribution
  Coles & Tawn (1996), Cooley et al. (2005)
- **Linear filtering** : Auto-Regressive spatio-temporal heavy tailed processes,
  Davis and Mikosch (2007)
- **Gaussian anamorphosis** : Transforming the field into a Gaussian one
  Wackernagel (2003)

Max-stable processes

Max-stability in the univariate case with an unit-Fréchet margin

$$F^t(tx) = F(x), \text{ for } F(x) = \exp(-1/x)$$

Max-stability in the multivariate case with unit-Fréchet margins

$$F^t(tx_1, \ldots, tx_m) = F(x_1, \ldots, tx_m), \text{ for } F_i(x_i) = \exp(-1/x_i)$$

A central question

$$\mathbb{P}\left[M(x) < u, M(x+h) < v\right] = ??$$

Bivariate case for Maxima from an asymptotic point of view

If one assumes unit Fréchet margins then the distribution of the vector $(M(x), M(x+h))$ goes to

$$F(u,v) = \exp\left[-V_h(u,v)\right]$$

where

$$V_h(u,v) = 2\int_0^1 \max\left(\frac{w}{u}, \frac{1-w}{v}\right)\, dH_h(w)$$

with $H_h(.)$ a distribution function on $[0,1]$ such that $\int_0^1 w\, dH_h(w) = 0.5$.

Home work : check that $F(u,v)$ is bivariate max-stable

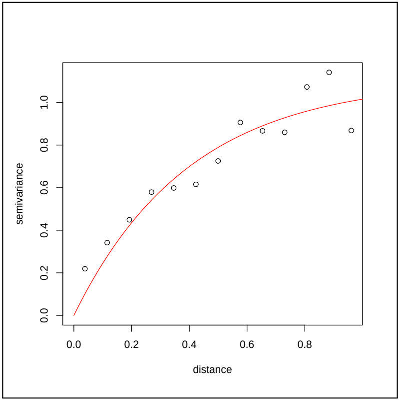
Bivariate case $(M(x), M(x+h))$

Complex non-parametric structure

$$V_h(u,v) = 2\int_0^1 \max\left(\frac{w}{u}, \frac{1-w}{v}\right) dH_h(w)$$

How to estimate $V_h(u,v)$?

Geostatistics : Variograms



Complex non-parametric structure

$$\gamma(h) = \frac{1}{2}\mathbb{E}|Z(x+h) - Z(x)|^2$$

- Finite if **light** tails
- Capture **all** spatial structure if $\{Z(x)\}$ **Gaussian** fields
- but not well adapted for extremes

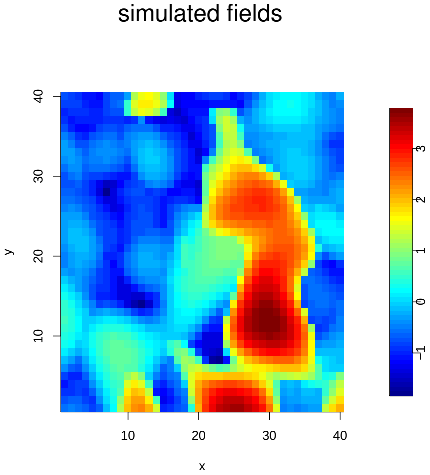
A madogram type

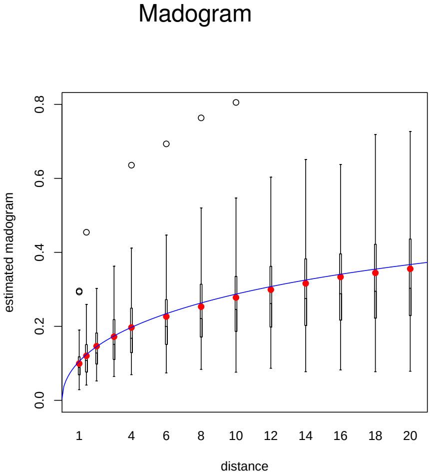
$$\nu_h = \frac{1}{2}\mathbb{E}\left|F(M(x+h)) - F(M(x))\right|$$

Properties

- Defined for light & heavy tails
- nice link with EVT but only gives $V_h(1,1)$

$$\nu_h = \frac{1}{2}\mathbb{E}\left|F(M(x+h)) - F(M(x))\right|$$





The $\lambda-$madogram

$$\nu_h(\lambda) = \frac{1}{2}\mathbb{E}\left|F^{\lambda}(M(x+h)) - F^{1-\lambda}(M(x))\right|$$

Properties

- Defined for light & heavy tails
- Called a λ-Madogram
- Nice links with extreme value theory
- $\nu_h(0) = \nu_h(1) = 0.25$

A fundamental relationship

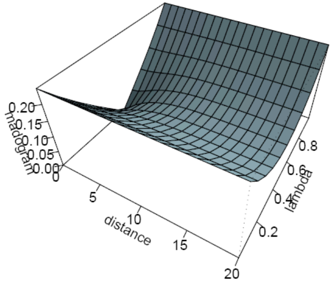
Home work

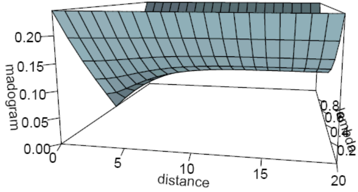
$$\nu_h(\lambda) = \frac{V_h(\lambda, 1-\lambda)}{1 + V_h(\lambda, 1-\lambda)} - c(\lambda), \textit{ with } c(\lambda) = \frac{3}{2(1+\lambda)(2-\lambda)}$$

Conversely,

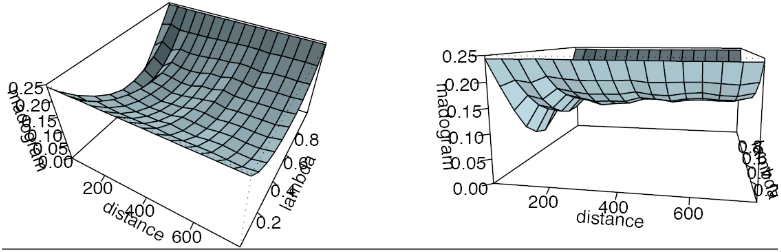
$$V_h(\lambda, 1-\lambda) = \frac{c(\lambda) + \nu_h(\lambda)}{1 - c(\lambda) - \nu_h(\lambda)}$$

The $\lambda-$madogram



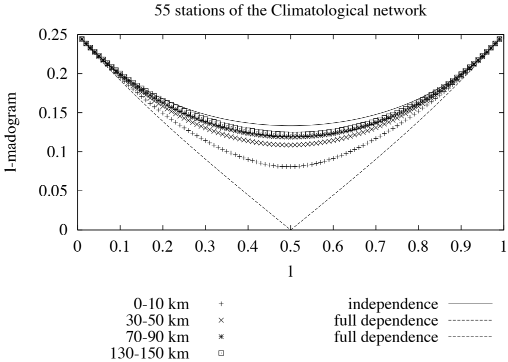


30-year maxima of daily precipitation in Bourgogne



146 stations of maxima of daily precipitation over 1970-1999 in Bourgogne

54-year maxima of daily precipitation in Belgium



55 stations of precipitation maxima over 1951-2005 in Belgium

Conclusions

- An asymptotic probabilistic concept
- A statistical modeling approach
- Identifying clearly assumptions
- Assessing uncertainties
- Goodness of fit and model selection