Supplementary Material

Relative dispersion in generalized two-dimensional turbulence
by A. Foussard et al.

March 6, 2017

This document discusses how the turbulence characteristics evolve during the time evolution of the dispersion of the particle pairs.

1 Energy decay

Figure 1 presents the time evolution of total generalized enstrophy

$$Z_g(t) = \int\int K^{2\alpha}|\widehat{\psi}(k,l,t)|^2 dkdl$$

(with $K = \sqrt{k^2+l^2}$ the wavenumber modulus and $\widehat{\psi}$ the Fourier transform of the streamfunction) and of the total kinetic energy

$$E(t) = \int\int K^2|\widehat{\psi}(k,l,t)|^2 dkdl.$$

Here we use the timescale of the simulation, i.e. $t = 0$ corresponds to the beginning of the simulation starting from random initial conditions, $t = 40$ is the time at which particles are seeded in the flow, and $t = 80$ is the end of the simulation.

For each simulation, we observe the decrease of total kinetic energy and total generalized enstrophy. The total decrease is at most 40% at the end of the simulation (Fig. 1). Note that during the time interval of particle release ($t = 40$ to $t = 80$), the total kinetic energy only decreases by 17%. For the viscous runs, the kinetic energy decreases more rapidly than for the other simulations. For the generalized enstrophy (i.e. total tracer variance), an intriguing result is that all the runs decay at the same rate during all the time integration, despite the fact that we did not rescale the timescale. Only the two viscous runs have a faster decay

Figure 2 presents the time evolution of total relative enstrophy

$$Z(t) = \int\int K^4|\widehat{\psi}(k,l,t)|^2 dkdl$$

as a function of time. For $\alpha < 2$, the enstrophy increases with time up to $t = 40$. After this time, the enstrophy has a slow decay for all the simulations, motivating the release of the particles at that time. Between $t = 40$ and $t = 80$ the enstrophy decays by almost 35% for the non-viscous runs.

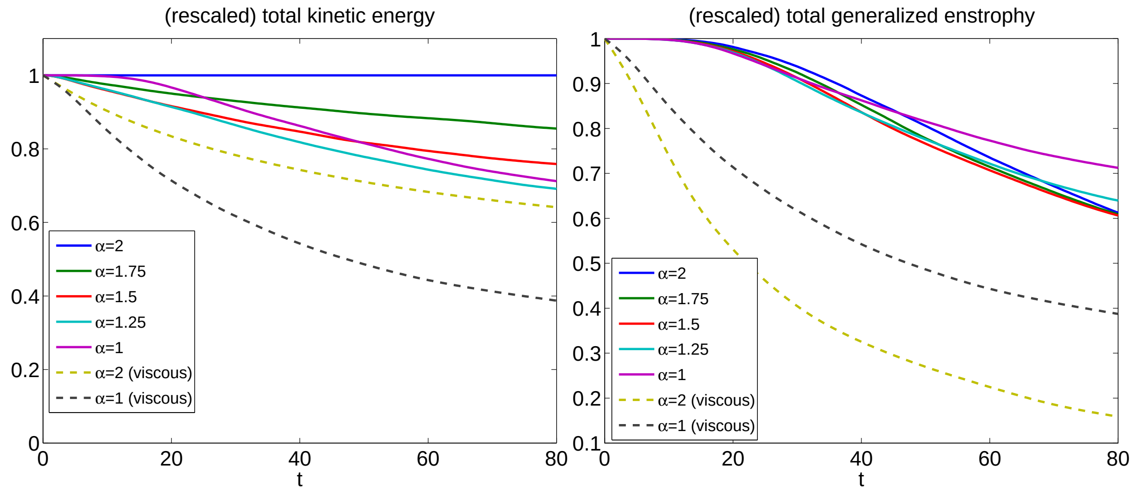

Figure 1: Total kinetic energy (a) and total generalized enstrophy (b) as a function of time; both quantities are rescaled with respect to their initial value.

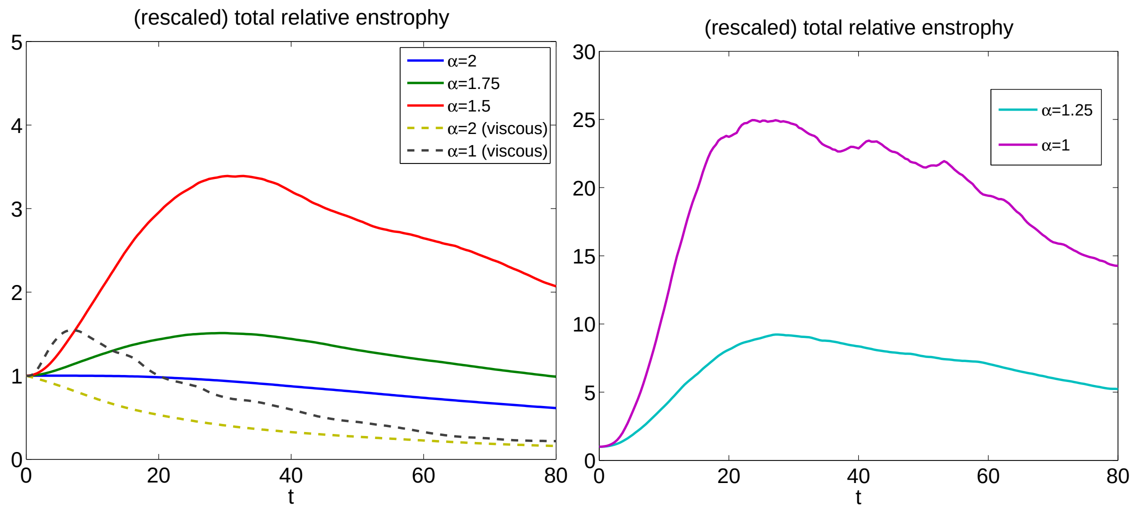

Figure 2: Total relative enstrophy rescaled with respect to its initial value as a function of time.

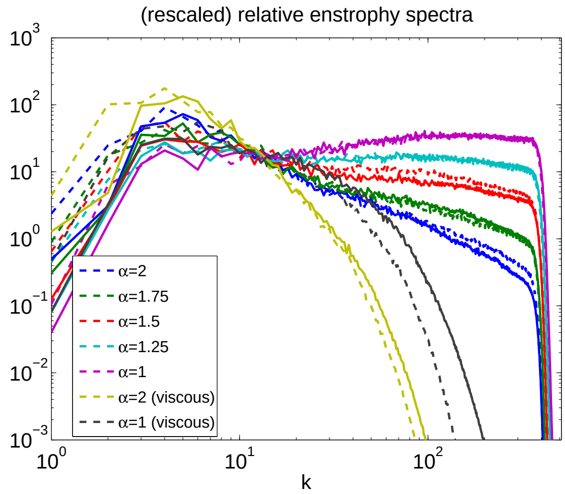

Figure 3: Relative enstrophy spectra (rescaled such that they share the same amplitude at $k = 15$). Continuous curves: $t = 40$ (time of particle release). Dashed curves: $t = 80$ (end of the simulation.)

2 Evolution of the spectra

Figure 3 presents the relative enstrophy spectrum at $t = 40$ (time of particle release) and $t = 80$ (end of the simulation). These spectra have been rescaled in order to see how the shape is modified between the beginning of the particle release and the end of the simulation. One can see that the spectra slopes do not change much for $k > 10$, except for the viscous run with $\alpha = 1$ for which the smallest scales contain less relative enstrophy at the end of the simulation. For $k < 10$, we see that there is a general tendency for the peak to shift towards smaller wavenumbers.

The fact that the spectra do not change much over the period during which particles are tracked suggests that the dispersion statistics should not be too strongly affected by the free decay of the simulations.

3 Physical fields

Figures 4 and 5 present the tracer field at the beginning of the particle release and at the end of the simulation. It is difficult to isolate differences in the overall organisation of the tracer fields but some general remarks can be made. For the non-viscous runs and for each value of α, one observes that small-scale eddies are still present at the end of the simulation. For the viscous simulations, there is a tendency to have larger coherent vortices with less filaments in between. For $\alpha = 2$ and 1.75, there are maybe more small-scale vortices at the end of the simulation than at particle release time. In general, for all the simulations, there are less large-scale vortices, in agreement with the shift of the energy peak towards smaller scales. For $\alpha \leq 1.5$, it is difficult to assess if there is a tendency for an increase or decrease of the population of small scale vortices.

From these different diagnostics, it seems to us that the total energy decrease of these freely-decaying runs does not have a large effect in the time interval during which we examine particle pair separations.

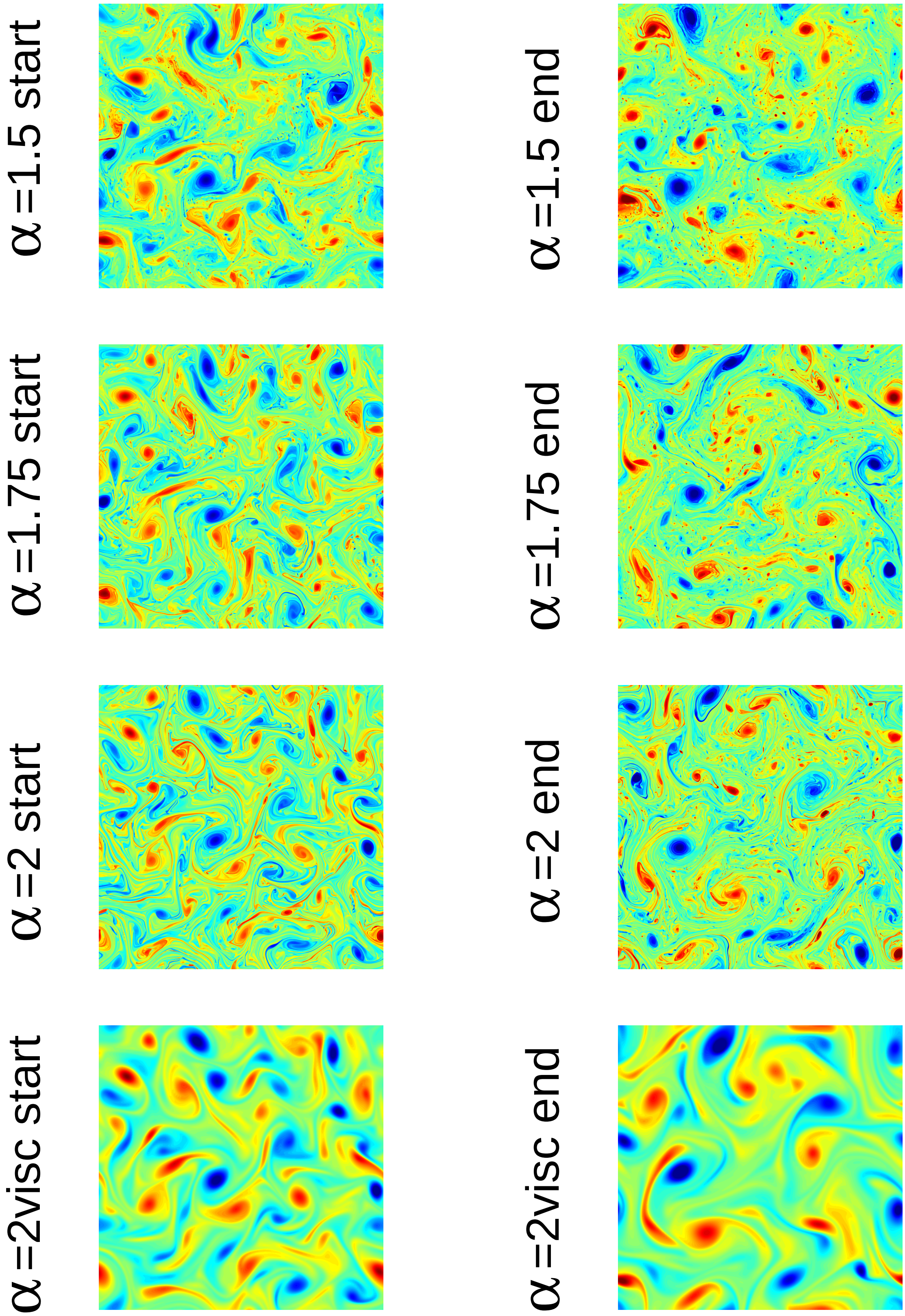

Figure 4: Tracer fields at time of particle release ($t = 40$) and at final time ($t = 80$) for different α (2 (viscous), 2, 1.75 and 1.5). Values have been normalized by $\langle q^2 \rangle^{1/2}$.

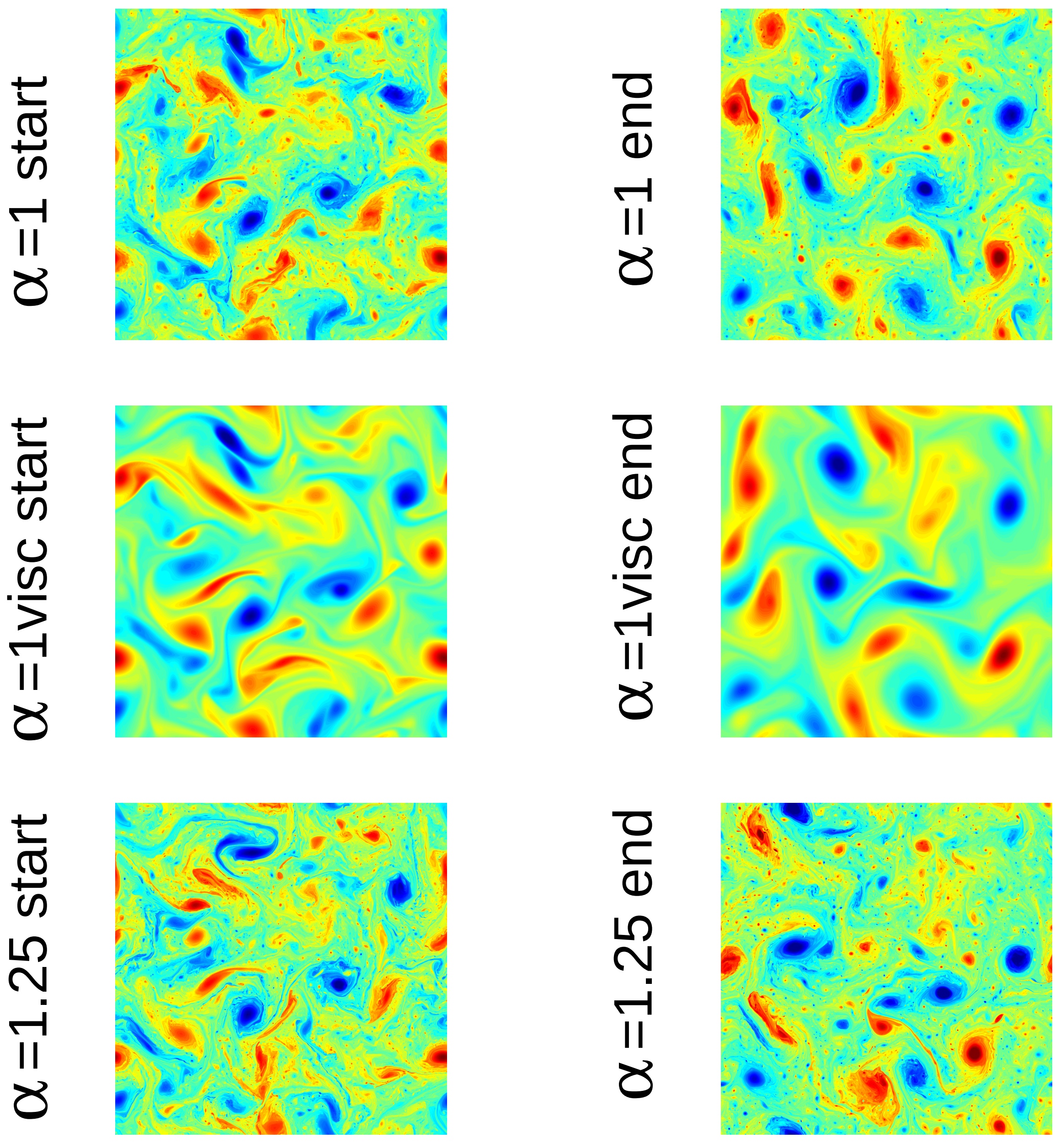

Figure 5: Tracer fields at time of particle release (t = 40) and at final time (t = 80) for different α (1.25, 1 (viscous), 1).Values have been normalized by $\langle q^2 \rangle^{1/2}$.