MEC 654
Polytechnique-UPMC-Caltech
Year 2014-2015

Turbulence

chapitre 5

understanding the cascade

5.1 cascade : a local mechanism

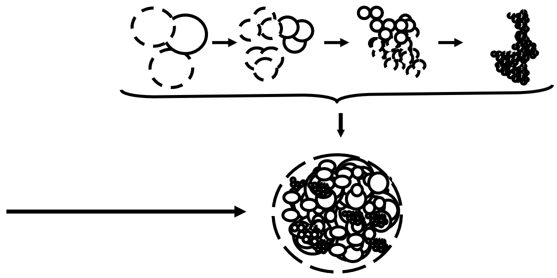
• the cascade does not develop in space

✓ this is a local process

✓ all « flow structures » are nested

cascade model

physical space



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5.2 energy is not conserved

• what does remain constant in the cascade ?

✓ this is the rate of the energy transfer per unit mass from scale to scale :

structure of scale l :

$$\varepsilon_l \sim \frac{\text{energy}}{\text{lifetime}} \sim \frac{u_l^2}{\tau_l} \sim \frac{u_l^2}{l/u_l} = \frac{u_l^3}{l_0} = \varepsilon_0 \quad \Rightarrow \quad \begin{cases} \text{velocity} & u_l \sim \varepsilon_0^{1/3}\, l^{1/3} \\ \text{energy} & u_l^2 \sim \varepsilon_0^{2/3}\, l^{2/3} \\ \text{life time} & \tau_l \sim l/u_l \sim \varepsilon_0^{-1/3}\, l^{2/3} \end{cases}$$

as l decreases, each « flow structures » contain less and less energy, but transfer energy more and more rapidly

• how does the « flow structure's » Reynolds evolve ?

$$\boxed{\mathrm{Re}_l = u_l l/\nu \sim l^{4/3}}$$

the « structure's » Reynolds number decreases during the cascade process. When $\mathrm{Re}_l \sim 1$, the cascade ends because viscosity succeeds in transforming kinetic energy into heat.

5.3 the smallest turbulence scales

- **what is the smallest scale produced by the cascade process ?**

✓ (l_ν, u_ν) such that : $\left.\begin{array}{l} u_{l_\nu}^3 / l_\nu \sim \varepsilon_0 \\ \mathrm{Re}_l = \dfrac{u_\nu l_\nu}{\nu} \approx 1 \end{array}\right\} \Rightarrow \boxed{l_\nu = \left(\dfrac{\nu^3}{\varepsilon_0}\right)^{1/4} \equiv \eta}$

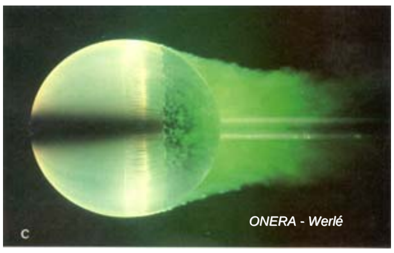
Kolmogorov's scale

✓ otherwise $\varepsilon_0 \sim \dfrac{u_0^3}{l_0} \Rightarrow \boxed{\dfrac{l_0}{\eta} \sim \left(\dfrac{u_0\, l_0}{\nu}\right)^{3/4} = \mathrm{Re}_0^{3/4}}$

$\Rightarrow$ in a cascade, the smallest turbulent scale η is $\mathrm{Re}_0^{3/4}$ times smaller that the largest

5.4 turbulence : a « costly » mechanism

- **the « cost » of turbulence**



$\mathrm{Re} = \dfrac{U_0 D}{\nu} \approx 300\,000$

$\left.\begin{array}{l} \nu_{air} \sim 10^{-5}\, m^2/s \\ l_0 \sim D = 30cm \\ U_0 \sim 10ms^{-1} \end{array}\right\}$ football

Kolmogorov' scale $\boxed{\dfrac{\eta}{D} \sim \left(\dfrac{\nu}{U_0 D}\right)^{3/4} = \mathrm{Re}^{-3/4}} \Rightarrow \boxed{\eta \approx 8\mu m}$

spatial mesh for numerical simulations $\boxed{N_{\text{points}} \sim \left(\dfrac{D}{\eta}\right)^3 = \mathrm{Re}^{9/4}} \Rightarrow \boxed{N_{\text{points}} \approx 210^{12} \text{ !!}}$

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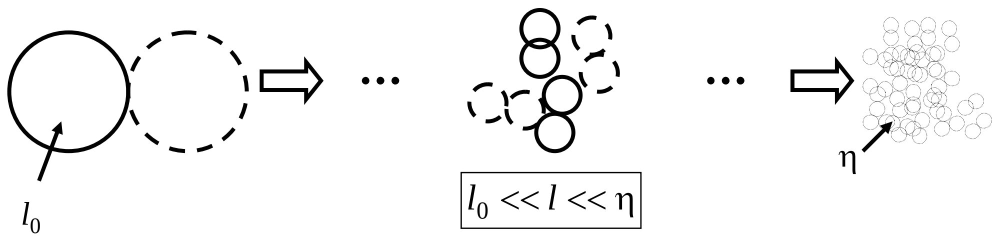
5.5 turbulence in Fourier space

• **cascade**

energy injection | energy transfer | dissipation

rate $\varepsilon_0 \sim u_0^3 / l_0$ | $\varepsilon_l \sim u_l^3 / l \sim \varepsilon_0$ | ε_0



$\boxed{l_0 << l << \eta}$

independant on l_0 and ν

⇨ inertial regime (non-viscous)

⇨ no prescribed scale

5.5 turbulence in Fourier space (…)

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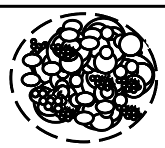
• **physical space**



kinetic energy $\langle u^2 \rangle = \sum_l u_l^2$

• **Fourier space** $\hat{u}_\kappa = \sum_l u_l e^{i\kappa l}$ kinetic energy $\langle u^2 \rangle = \sum_\kappa \hat{u}_\kappa^* \hat{u}_\kappa$

(Parseval)

• **energy spectrum** $E_\kappa = \hat{u}_\kappa^* \hat{u}_\kappa \rightleftarrows u_l^2$

• **inertial regime** $\kappa_0 \left(\sim \frac{1}{l_0} \right) << \kappa_l \left(\sim \frac{1}{l} \right) << \kappa_\eta \left(\sim \frac{1}{\eta} \right)$

• **in the inertial regime one must have :** $\boxed{E_\kappa = f\left(\varepsilon_0, \kappa\right)}$

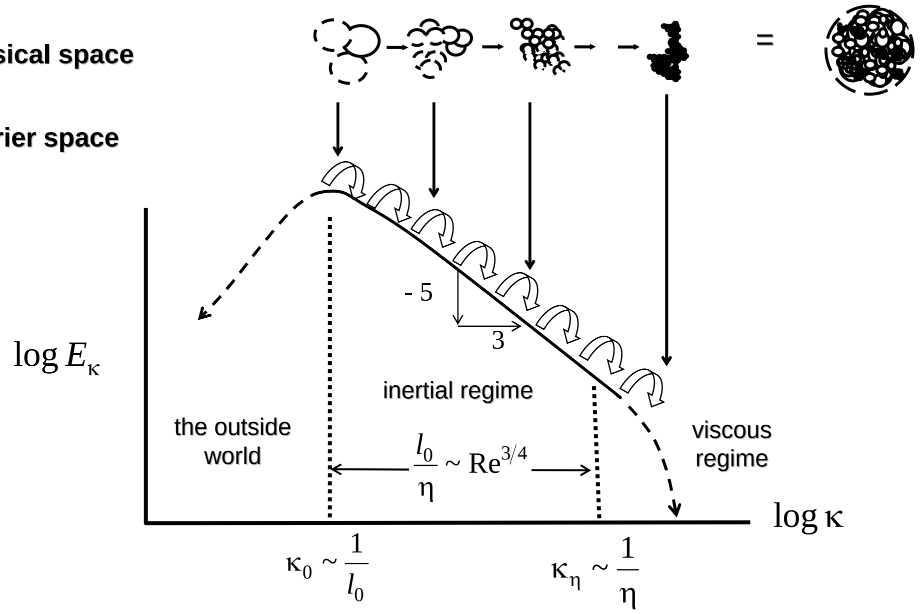
dimensions

$$\left.\begin{array}{l} \left[E_k\right] = L^3/T^2 \ (*) \\ \left[\varepsilon_0\right] = L^2/T^3 \\ \left[\ \kappa\ \right] = L^{-1} \end{array}\right\} \Rightarrow \boxed{E_\kappa \sim \varepsilon_0^{2/3}\ \kappa^{-5/3}}$$

(*) Parseval : $\langle u^2 \rangle = \int_0^\infty E(k)\, dk \ \Rightarrow \ \left[\langle u^2 \rangle\right] = L^2/T^2 = \left[E(k)\right] \times L^{-1} \ \Rightarrow \ \left[E(k)\right] = L^3/T^2$

5.5 turbulence in Fourier space (…)

- physical space
- Fourier space



chapitre 6
miscellaneous

6.1 turbulence and rheology

6.2 turbulence and singularities

6.3 turbulence and fractal

6.4 turbulence and applications

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6.1 turbulence and rheology

- **observation**

✓ turbulence changes the **dynamical properties** of flows

✓ but it does not change, in any way, the **physical properties** of fluid

- **question**

✓ conversely : do the physical properties of fluid change turbulence ?

- **answer**

✓ yes

- **back to chapter 4**

✓ the case of a pipe

back to

chapitre 4

turbulence : a paradox

4.1 dissipation
4.2 dissipation : the case of a sphere
4.3 a paradox
4.4 dissipation : the case of a pipe
4.5 the Richardson-Kolmogov cascade

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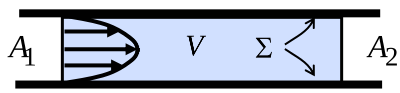
4.4 dissipation : the case of a pipe

we skipped this case last Monday

4.4 dissipation : the case of a pipe

$$\begin{cases} 0 = \dfrac{1}{V} \iint_S \overline{\phi_{e_k}}^T . \underline{n}\, dS - \varepsilon_V \\ \phi_{e_k} = -\left(\dfrac{p}{\rho} + e_k \right) \underline{u} + 2\nu \underline{\underline{d}} . \underline{u} \quad \text{- power of surfacic efforts} \\ \varepsilon_V \quad \text{- mean dissipation rate per unit mass} \end{cases}$$

• **pipe** $S = \Sigma \cup A_1 \cup A_2$



$$\Rightarrow \quad \varepsilon_V = \underbrace{\cancel{\frac{1}{V} \iint_\Sigma \overline{\phi_{e_k}}^T . \underline{n}\, ds}}_{\text{no slip}} + \frac{1}{V} \iint_{A_1 \cup A_2} \overline{\phi_{e_k}}^T . \underline{n}\, ds$$

✓ supposing the **velocity field** statistically homogenous in the direction parallel to the pipe axis

$$\Rightarrow \varepsilon_V = -\frac{1}{V} \iint_{A_1 \cup A_2} \frac{1}{\rho} \overline{p\, \underline{u}}^T . \underline{n} dS \; - \underbrace{\cancel{\frac{1}{V} \iint_{A_1 \cup A_2} \overline{e_k\, \underline{u}}^T \underline{n}\, dS + \frac{1}{V} \iint_{A_1 \cup A_2} 2\nu \overline{\left(\underline{\underline{d}} . \underline{u} \right)}^T . \underline{n}\, dS}}_{\text{statistical homogeneity along the pipe axis}}$$

$$\Rightarrow \boxed{\varepsilon_V = -\frac{1}{V} \iint_{A_1 \cup A_2} \frac{1}{\rho} \overline{p\, \underline{u}}^T . \underline{n}\, dS}$$ energy of the pressure force is transformed into heat

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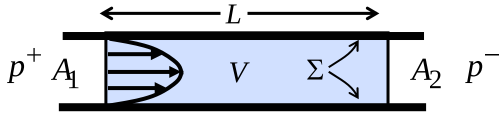
4.4 dissipation : the case of a pipe (…)



$$\boxed{\varepsilon_V = -\frac{1}{V}\iint_{A_1\cup A_2}\frac{1}{\rho}\,\overline{p\,\underline{u}}^T.\underline{n}\,dS}$$

- **approximation : introducing**
  - ✓ the mean surfacic pressures $p^+ = \frac{1}{A_1}\iint_{A_1}\overline{p}^T dS$, $p^- = \frac{1}{A_2}\iint_{A_2}\overline{p}^T dS$
  - ✓ the mean bulk velocity $\overline{U_b}^T = \frac{1}{A_1}\iint_{A_1}\overline{\underline{u}}^T.\underline{n}\,dS = \frac{1}{A_2}\iint_{A_2}\overline{\underline{u}}^T.\underline{n}\,dS = const.$

$$\left.\begin{array}{r} A_1 = A_2 = A \\ V = A\times L \end{array}\right\} \Rightarrow \boxed{\varepsilon_V \approx \frac{p^+ - p^-}{L}\,\overline{U_b}^T}$$

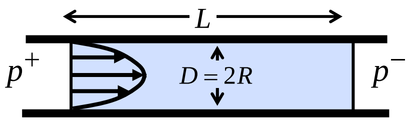
mean pressure power per unit mass

pressure gradient   bulk velocity

$$\Rightarrow \boxed{\rho V \varepsilon_V \approx (\,p^+ - p^-\,)\times\overline{U_b}^T\times A}$$

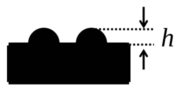
mean pressure power

4.4 dissipation : the case of a pipe (…)



$$\boxed{\rho V \varepsilon_V \approx (\,p^+ - p^-\,)\times\overline{U_b}^T\times A}$$

- **dimensional analysis** $p^+ - p^- = \mathcal{G}\left(\overline{U_b}^T, D, L, h, \rho, \eta\right)$



roughness

$$\Rightarrow \boxed{\frac{p^+ - p^-}{\rho(\,\overline{U_b}^T\,)^2} = \overline{\lambda}^T(\,\mathrm{Re}, \frac{h}{D}, \frac{L}{D}\,)} \qquad \mathrm{Re} = \frac{\rho\,\overline{U_b}^T D}{\eta} \qquad \frac{h}{D} \qquad \frac{L}{D} \qquad (*)$$

pipe friction coefficient | Reynolds | roughness | pipe aspect ratio

- **conclusion**

$$\boxed{\rho V \varepsilon_V \approx \underbrace{\overline{\lambda}^T(\,\mathrm{Re}, \frac{h}{D}, \frac{L}{D}\,)}_{\text{viscosity, roughness}} \times \underbrace{\rho(\,\overline{U_b}^T\,)^3 D^2}_{\text{injected power}}}$$

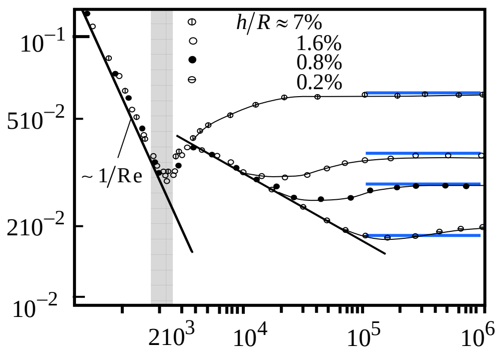
- **experiments provide :** $\lim_{L/D\to\infty}\overline{\lambda}^T(\,\mathrm{Re}, \frac{h}{D}, \frac{L}{D}\,)$

(*) a relationship involving 7 parameters and 3 fondamental dimensions (length L, time T, weight M) implies a relation between (7-3)=4 independant non-dimensional parameters (see the Vaschy-Buckingham « Π » theorem)

4.4 dissipation : the case of a pipe (…)

- **experiments on pipes**

$\lim_{L/D\to\infty} \overline{\lambda}^T$



$$\lim_{\substack{\mathrm{Re}\to\infty\\ L/D\to\infty}} \overline{\lambda}^T = const.(h/R)$$

$$\mathrm{Re} = \frac{\rho U_0 D}{\eta}$$

. Nikuradse, J. Stromungsgesetze in rauhen rohren. Forsch. Arb. Ing. Wes. 36, 1933

- **so we observe that :**

$$\lim_{\substack{\mathrm{Re}\to\infty\\ L/D\to\infty}} (\rho V \varepsilon_V) \approx const.(h/R) \times \underbrace{\rho(\overline{U_b}^T)^3 D^2}_{\text{injected power}}$$

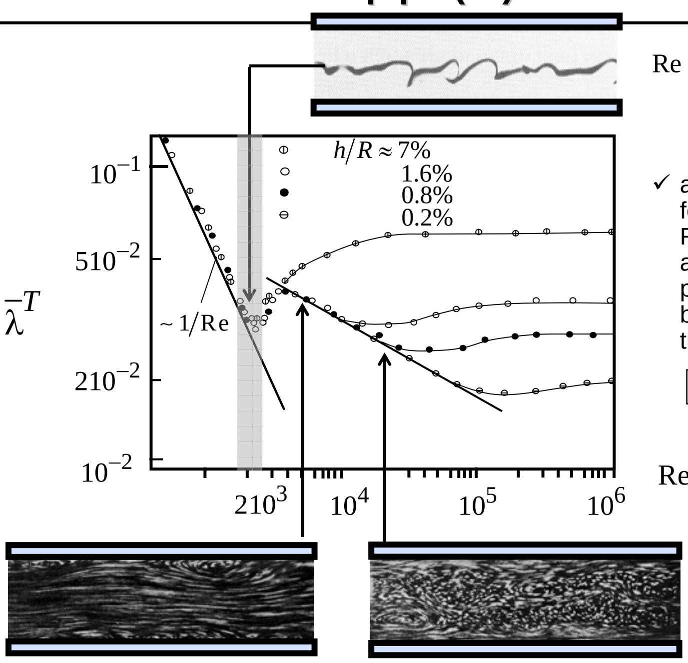
- **which shows again that :**

at large Reynolds numbers, the power absorbed by viscous friction becomes independ on viscosity !!

4.4 dissipation : the case of a pipe (…)

Re = 2 000

$\overline{\lambda}^T$



Re = 5 000

Re = 20 000

✓ a concensus for the critical Reynolds above which a pipe flow becomes turbulent is :

$\boxed{\mathrm{Re} \approx 2000}$

$$\mathrm{Re} = \frac{\rho U_0 D}{\eta}$$

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4.4 dissipation : the case of a pipe (…)

✓ a concensus for the critical Reynolds above which a pipe flow becomes turbulent is :

$$\boxed{\mathrm{Re} \approx 2000}$$

➢ **exemple**

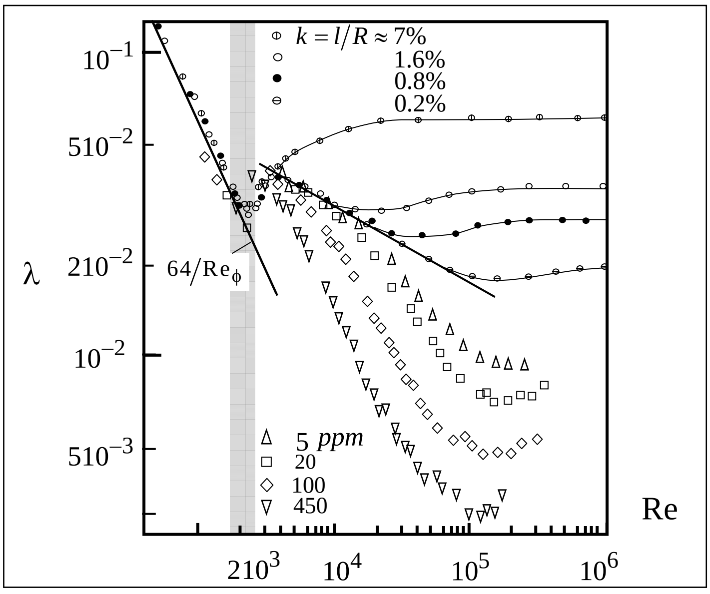
✓ domestic hydraulic network pipe

diameter $D \approx 10^{-2}\ m$

✓ turbulent regime :

$$\mathrm{Re} = \frac{U\ D}{\nu} > 2000 \quad \Rightarrow \quad U > \frac{2\,10^{3} \times 10^{-6}}{10^{-2}} = 20\ cm.s^{-1}$$

6.1 turbulence and rheology



. Nikuradse, J. Stromungsgetze in rauhen rohren. Forsch. Arb. Ing. Wes. 36, 1933
. Virk, PhD 1966

- polyethylene oxyd in solution
- dissipation may decrease by 80%
- this should result from an increase in the viscocity
- the fluid is non newtonian, its structure being anisotropic
- an elongation viscosity appears that can be more than a thousand times larger than the shear viscosity η
- Note : this suggests that **laminar-turbulent transition** (still around Re=2000) and **turbulence** are two different problems

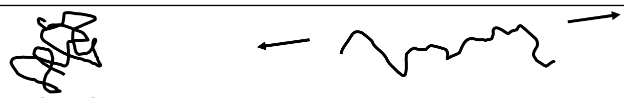
6.1 turbulence and rheology (…)

- **exemple : pipeline**

Trans-Alaska Pipeline System (TAPS)

www.alyeska-pipe.com

- ✓ one of the word's longest pipe (800 miles)
- ✓ **non-newtonien** fluid application : Drag Reduction Agent (DRA)
- ✓ liquid gaz doped with DRA : long polymeric hydrocarbon chain
- ✓ mass flow is increased from 1.44 to 2.136 millions of barels / day !!
- ✓ mecanism :



steady polymer

polymer sheared by turbulence

6.2 turbulence and singularities

- **dissipation function** $\epsilon(\underline{x},t) = {}^t\underline{\underline{\tau}} : \nabla \underline{u} > 0$

- **incompressible and homogeneous newtonian fluid with constant viscosity**

$$\epsilon(\underline{x},t) = 2\eta\, \underline{\underline{d}} : \underline{\underline{d}}$$

- **other form** (chap.3, §3.4)

$$\epsilon(\underline{x},t) = \overbrace{\eta\, \underbrace{|\nabla \underline{u}|^2}_{\text{gradients}}}^{\varepsilon_1 \text{ pseudo-dissipation}} + \eta\, div(\nabla \underline{u} . \underline{u})$$

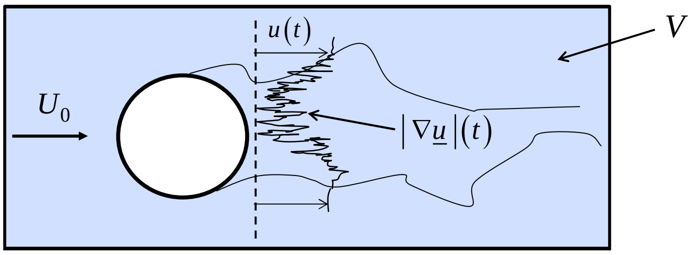
- **averaging in a volume**

$$\langle \epsilon \rangle_V \equiv \frac{1}{V} \iiint_V \epsilon(\underline{x},t)\, dV = \frac{1}{V} \iiint_V \eta |\nabla \underline{u}|^2\, dV + \underbrace{\iint_S \eta(\nabla \underline{u} . \underline{u}) . \underline{n}\, dS}_{\substack{=0 \\ \text{on conveniently} \\ \text{chosen boundary} \\ \text{(e.g. ideal flow)}}}$$

⇨ turbulence : a « gradient factory »

6.2 turbulence and singularities (…)

• **sphere**



• **dissipation** $\varepsilon_V = \overline{\langle \epsilon \rangle_V}^T = \frac{1}{T}\int_T \left[\frac{1}{V}\iiint_V \frac{1}{T}\int_T \eta |\nabla \underline{u}|^2 \, dV \right] dt$

$$\boxed{\lim_{\mathrm{Re}\to\infty} \left(\rho V \varepsilon_V \right) = const.\times \rho U_0^3 D^2}$$

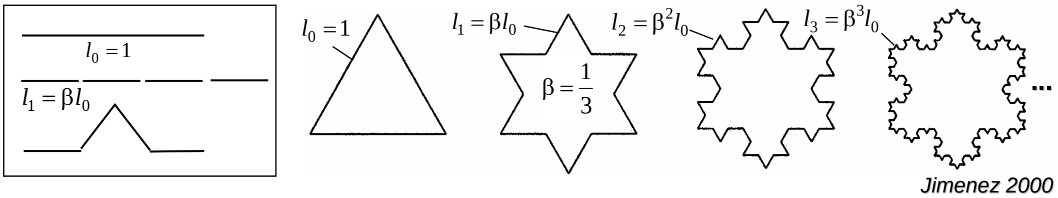
• **singularities** $\lim_{\eta\to 0} \{ \underbrace{\eta}_{\to 0} \times \frac{1}{V}\iiint_V \left[\frac{1}{T}\int_T \underbrace{|\nabla \underline{u}|^2}_{\to\infty} dt \right] dv \} = const.$

$\Longrightarrow$ $\underline{u}$ develops singularities

6.3 turbulence and fractal

• **fractal : a continuous non-differentiable object**

• **example : Koch's flake**



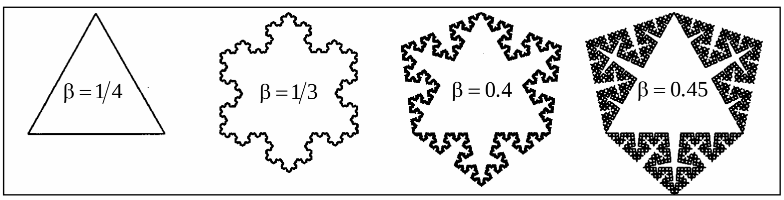
Jimenez 2000

✓ after *n* iterations :

➢ number of sides : $N_n = 3\times(4)^n$

➢ total length : $L_n = 3\times(4\beta)^n$

$\Longrightarrow$ $\boxed{\lim_{n\to\infty} L_n = \infty \quad \text{if} \quad \beta > \frac{1}{4}}$



$\Longrightarrow$ an infinite line covering a finite surface

6.3 turbulence and fractal (…)

- **fractals in nature**

increase the exchange surface for a given volume

examples

object	flux
• lung + bloodstream	• $O^2 \leftrightarrow CO^2$
• foliage	• $CO^2 \leftrightarrow O^2$
• lightning	• electrons
• roads	• goods
• **turbulence in fluids**	• **momentum**

6.3 turbulence and fractal (…)

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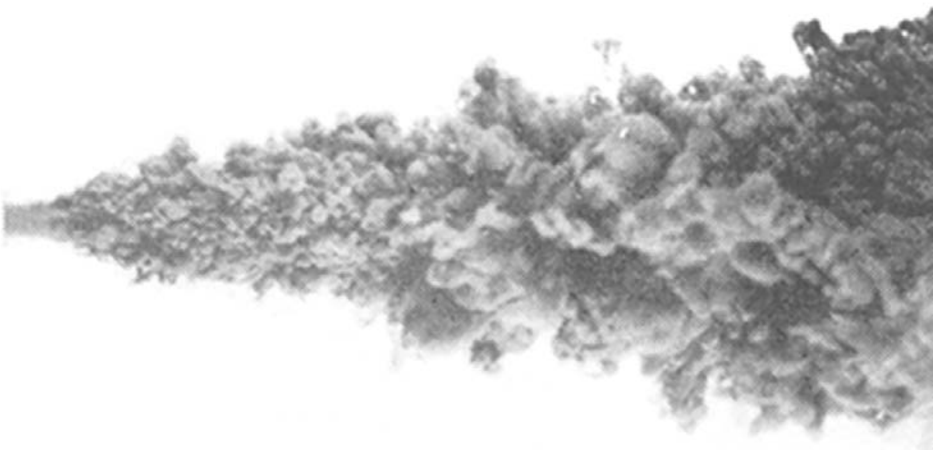
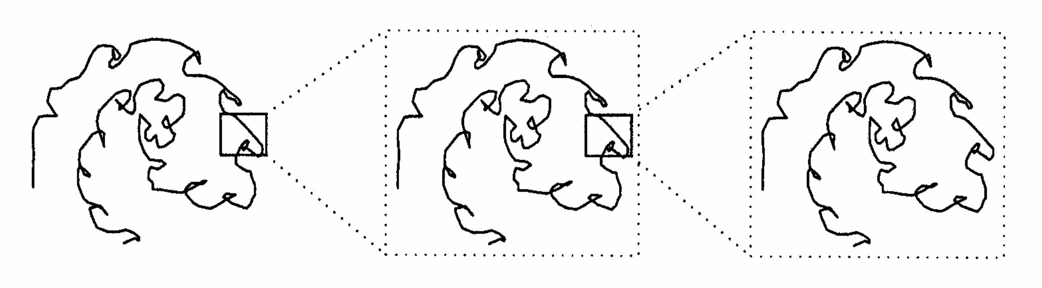
- **a turbulent jet : a fractal object ?**



6.3 turbulence and fractal (…)

• **fractal is self similar**

✓ interface fluid / fluid



Jimenez 2000

scale invariance

6.3 turbulence and fractal (…)

• **fractal, self similarity and scale invariance**

self similarity

a fonction $u(l)$ of a variable l is said to be self similar if, for any real number λ the ratio between $u(l)$ and $u(\lambda l)$ does not depend on l

scale invariance

a self similar fonction $u(l)$ satisfies a power scaling law $u(l) \sim l^{p}$ (see annex)

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annex – self similarity and scale invariance : demonstration

✓ putting : $\boxed{u(\lambda\, l) = A(\lambda, l)\, u(l)}$

✓ supposing : $\dfrac{\partial}{\partial l} A(\lambda, l) = 0$

$$\Longleftrightarrow \quad \frac{\partial}{\partial l}\left(\frac{u(\lambda l)}{u(l)} \right) = \frac{1}{u(l)} \frac{\partial u(\lambda l)}{\partial l} - \frac{u(\lambda l)}{u^2(l)} \frac{du(l)}{dl}$$

$$= A(\lambda, l)\left[\frac{\lambda}{u(\lambda l)} \frac{du(\lambda l)}{d(\lambda l)} - \frac{1}{u(l)} \frac{du(l)}{dl} \right] = 0$$

$$\Longleftrightarrow \quad \lambda \frac{d \ln u(\lambda l)}{d(\lambda l)} = \frac{d \ln u(l)}{dl} \quad \Longleftrightarrow \quad \boxed{\frac{d \ln u(\lambda l)}{d \ln u(l)} = 1}$$

✓ check that $\forall p \in \mathbb{R}$, $\boxed{u(l) \sim l^p}$ is a solution :

$$\frac{d \ln (\lambda l)^p}{d \ln l^p} = \frac{d \ln (\lambda l)}{d \ln l} = \cancel{\frac{d \ln \lambda}{d \ln l}} + \frac{d \ln l}{d \ln l} = 1$$

6.3 turbulence and fractal (…)

• **fractal, self similarity and scale invariance**

self similarity

a fonction $u\,(l)$ of a variable l is said to be self similar if, for any real number λ the ratio between $u\,(l)$ and $u\,(\lambda\, l)$ does not depend on l

scale invariance

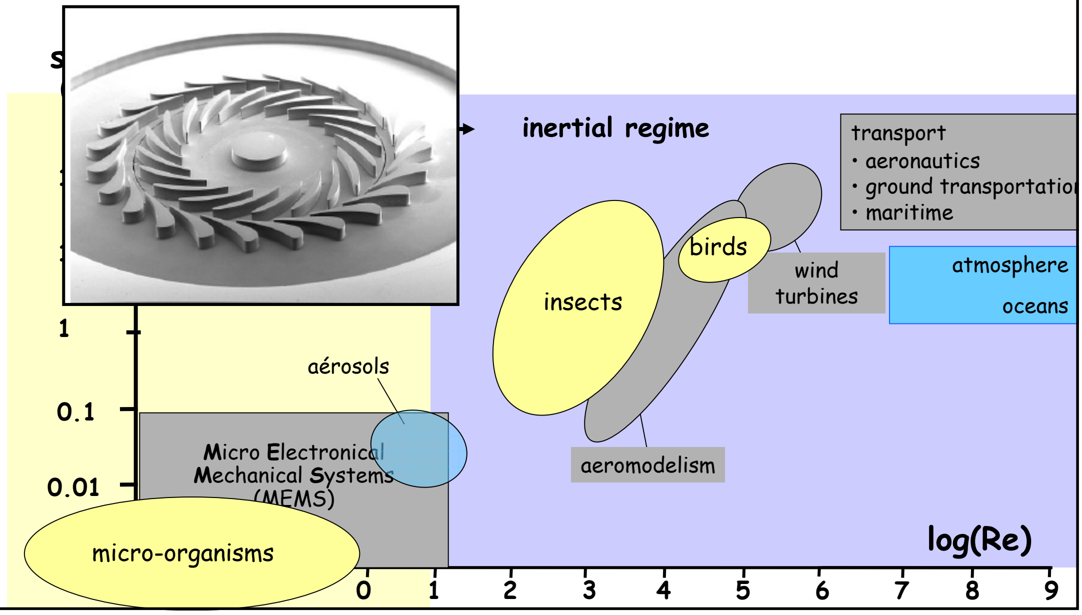
a self similar fonction $u\,(l)$ satisfies a power scaling law $u(l) \sim l^p$ (see annex)

• **the Richardson-Kolmogorov cascade**

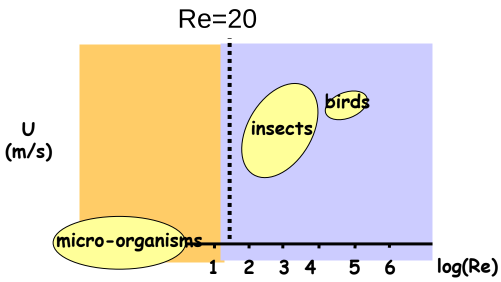
✓ we found that the energy cascade is characterized by a power law $u(l) \sim l^p,\ p = \frac{1}{3}$:
so this is a self similar process, which means « identical wathever the scale »

✓ finally, note that the self-similarity of a physical process is conceivable only in a bounded domain of the variables considered. In the cascade, this concerns the inertial regime, which comprises scales much smaller than the outer scales determined by the turbulence generating process and much larger than the smallest scales where viscosity smoothes the variables and resetablishes their analytical character

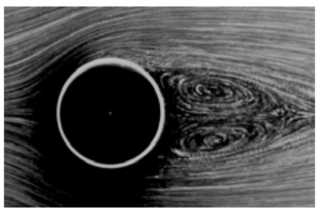
6.4 turbulence and applications



6.4 turbulence and applications (...)



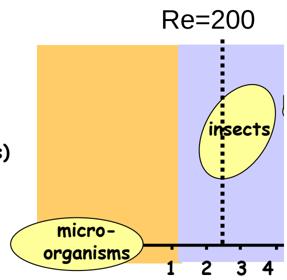
sphere



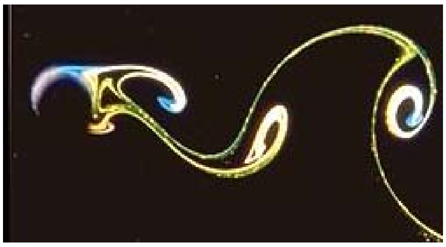
a « death » region :

the fluid remains sticked to the body steadily. There is no possibility to exploit a reaction to a momentum variation of the fluid. So, one cannot move and live in this Reynolds number range.

6.4 turbulence and applications (...)



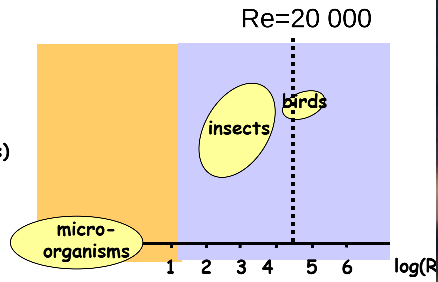
sphere



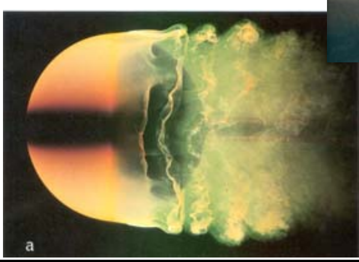
vortices

some fluid momentum is ejected. The body reacts and can move. Insects live here. This is not yet a turbulent regime.

6.4 turbulence and applications (...)

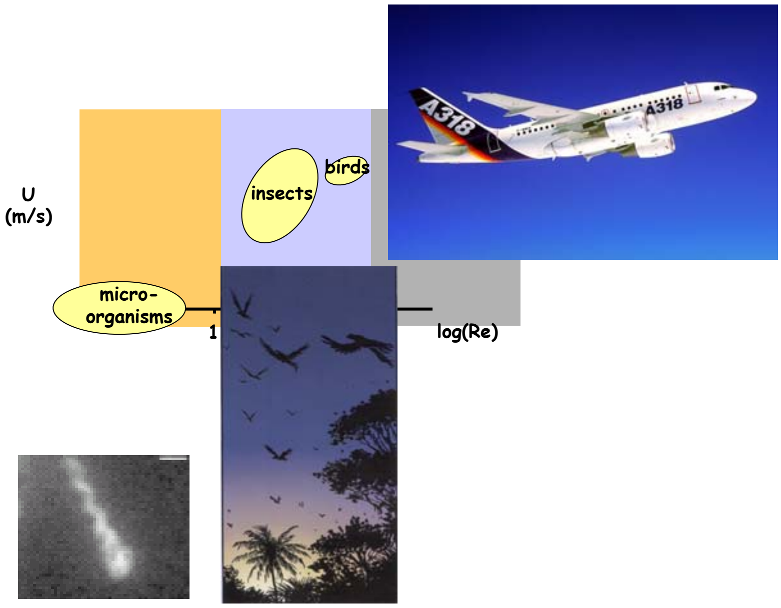


sphere

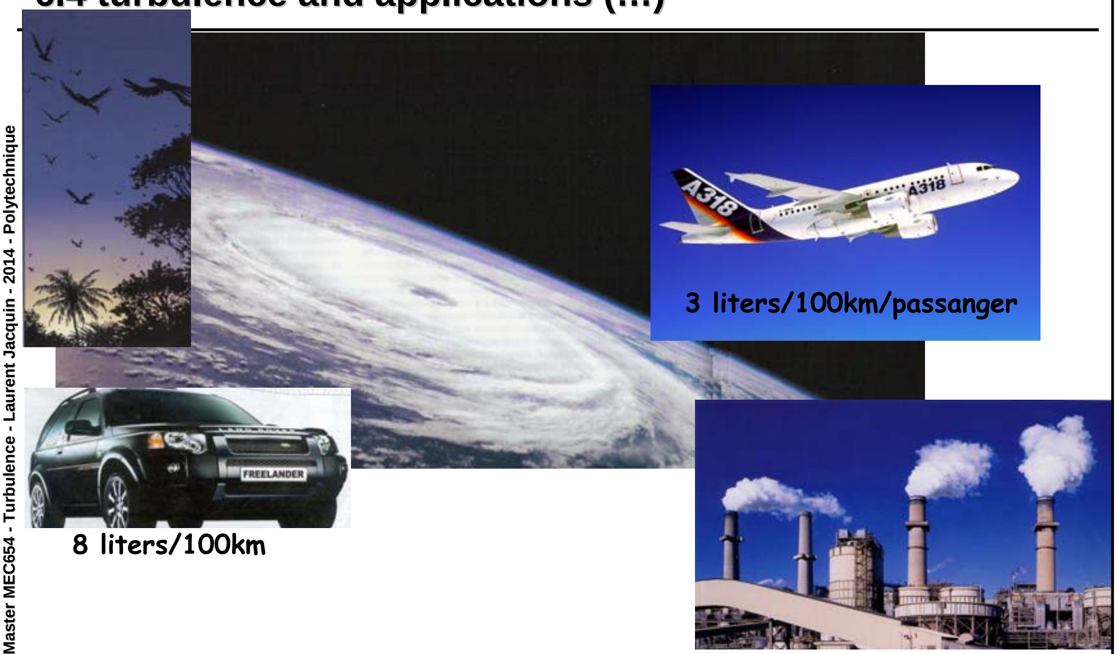


vortices and turbulence

6.4 turbulence and applications (...)



6.4 turbulence and applications (...)



6.5 turbulence and art

https://www.youtube.com/watch?v=PMerSm2ToFY