# The Propagation and Dissipation of Equatorial Kelvin Waves in the TTL

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# Introduction

In the tropical tropopause layer, we observe stationary zonal structure and transient variation on many timescales. Here, we explore the link between the slowly varying background flow and transient variations in the temperature structure of the tropical tropopause layer (see margins, left at 125 hPa, right at 50 hPa). These waves are identified as Kelvin waves (Box A). using linear Kelvin wave theory (Box B) we can show how the background flow should affect the waves. In the TTL in boreal winter, we observe the waves to disappear at fixed longitude (Box C). Theory predicts that the background flow will cause waves to stop when it reaches a critical velocity (strong westerly), and this is apparent in both the position of the maximum. Interannual variability (Box D), links changes in background velocity mainly due to the QBO with mixing in the TTL (Box E). Regions of low Richardson number (white regions, Figure 2) also correspond well with Kelvin waves.

### B Linear Kelvin wave theory

A Kelvin wave is a solution of the linearised primitive equations about a constant background flow,  $\!\mathcal{U}_{\!\!\!}$ . From this simple solution, we can derive the dispersion relation.

$$\omega - k\bar{u} = k\sqrt{gh}$$

Here, k is the horizontal wavenumber, and h is the equivalent depth. This should hold approximately in the real atmosphere. The phase speed (the speed at which crests propagate) relative to the ground can also be calculated from this solution. It is simply

 $c = \omega/k = \bar{u} + \sqrt{qh}$ 



Figure 2. The effects of an increasing background velocity on a travelling Kelvin wave. If the background velocity is sufficient to reduce the phase speed to zero (so  $\bar{u} = u_{\rm crit} = -\sqrt{gh}$  in (2)), the amplitude increases and the wave must stop.

## A Identification of waves

Travelling disturbances in the temperature structure (buoyancy frequency) at 125 hPa can be shown to be Kelvin waves. These disturbances are clearly visible in the signal on the left.



Figure 1. The contours in this diagram show the significance of peaks in an approximation of the power spectral density (PSD). The PSD is estimated using Welch's method with a window size of 64 days. All Kelvin wave activity happens on a much shorter timescale. The black lines show dispersion relations of Kelvin waves with different vertical scales (equivalent depths in metres). See Box B for the full dispersion relation (equation 1). This method was taken from Wheeler and Kiladis 1999 [1,2].

#### References

M. Wheeler and G. Kiladis. J. Atmos. Sci., 1999.
M. Wheeler and G. Kiladis. J. Atmos. Sci., 2000.
M. Fujiwara et al. Geophys. Res. Lett. 2003.



Figure 3. How moller Diagram of buoyancy frequency anomaly (shading, red is high/ stable and blue is low/unstable). The eastward propagating (Kelvin) waves are clearly visible. Regions of low Richardson number (Ri < 1 are shown in white. The black line on main plot shows the location of the maximum westerly background velocity. The black line to the right shows the velocity at this minimum. Also shown are the critical velocities for equivalent depths of 25m and 50m (Box B).

# D Interannual Variability

(1)

(2)



Figure 4. The minimum of the time averaged velocity at 125 hPa (solid black line) and the zonal average velocity at 50 hPa (dashed line) are shown (all quantities averaged over Dec to Feb for each year).

The number of grid points at which Hi < 1 (see Box E) in Dec to Feb within  $\pm 50$  deg. of the position of the minimum background velocity in each year is shown in red.

## E Mixing and the Kelvin-Helmholtz Instability

Vertical mixing in the atmosphere can occur through several different mechanisms. The Kelvin-Helmholtz instability is a shear instability, and is characterised by the Richardson number, a balance between shear and static stability.

$$Ri = \frac{N^2}{(\partial u/\partial z)^2}$$
 Static stability Shear instability

Usually, Ri < 0.25 is considered the critical region for instability. However, this figure comes from studying an idealised problem. We have also considered using Ri < 1. Low Richardson number events are investigated in Figures 3 (Box C) and 4 (Box D) on both short and long timescales.

#### Summary

We find good agreement between the critical velocity (Box B) and the location and timing of waves stopping (Box C and Hovmoller diagram at 125 hPa, left margin), showing that the background flow determines the location of this stopping. We find that near this critical velocity, wave amplitudes increase and an associated increase in low Richardson number events. This may be a significant mechanism for mixing in the TTL (as also observed, e.g. [3]). We see correlation between the background velocity (due mainly to QBO) and mixing on interannual scales (Box D).

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Data

ECMWF Interim temperature and velocity fields from 1989 to 2009 interpolated onto standard pressure levels were used. The 125 hPa level is used throughout.