Effects of moisture upon nonlinear dynamics in a simple atmospheric model

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Introduction

Question: *How can precipitation and moist convection affect dynamics at synoptic scales?*

At present, large-scale precipitation and its effect on latent heat release are not resolved but **parameterized** in GCM's.

Standard scheme (Betts-Miller):

$$P = \frac{q - q_s}{\tau} H(q - q_s)$$

where q is the specific humidity, q_s its saturation threshold and τ a relaxation time.

The **threshold effect** induced by the Heaviside function $H(\cdot)$ is fundamentally **nonlinear**. In consequence, no traditional linear wave solution exists anymore. Furthermore, precipitation front can be formed at the interface $P = 0/P > 0$.

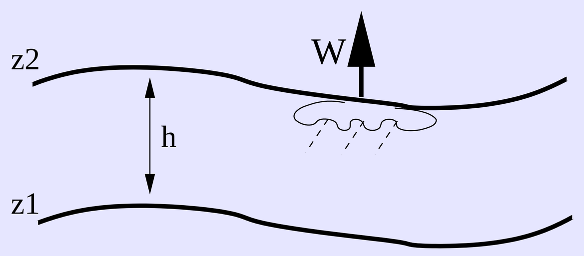
Objectives: derive a **simple moist** atmospheric model and analyse the effects of moisture induced by this specific nonlinearity.

Moist Convective RSW Model

- **RSW-type derivation**: vertical averaging of primitive equations between 2 material surfaces
  - $-w_2 = \frac{dz_2}{dt} + W$ where $W \equiv$ convective parameterization
  - $-w_1 = \frac{dz_1}{dt}$
- **Moisture:** equation for bulk humidity Q including precipitation (for simplicity, Q_s =const).

$$P = \frac{Q - Q_s}{\tau} H(Q - Q_s)$$

- **Closure:** mass flux $\propto$ latent heat release $W = \beta P$ ($\beta > 0$).



Moist Convective RSW model

$$\begin{cases} \partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -g\nabla h - f\hat{\boldsymbol{z}} \times \boldsymbol{v} \\ \partial_t h + \nabla \cdot (\boldsymbol{v} h) = -\beta P \\ \partial_t Q + \nabla \cdot (\boldsymbol{v} Q) = -P \end{cases}$$

Properties

1. *Hyperbolic* system for $\tau \neq 0$ but *piecewise hyperbolic* system for $\tau \to 0$.
2. Linearization of the hydrodynamic part of the model with $\boldsymbol{v} = (u, 0)$, $f = 0$ and $h \to -\theta$, gives the equations used by Gill [1] and Majda *et al.* [2-4].
3. Mass is not conserved.
4. Precipitation always *dissipates* total energy of the (isolated) system.

$$\partial_t E = -\beta \int d\boldsymbol{x} \left(\frac{\boldsymbol{v}^2}{2} + gh \right) P$$

5. Moist enthalpy, $m = h - \beta Q$, is always *conserved*.

Method of characteristics

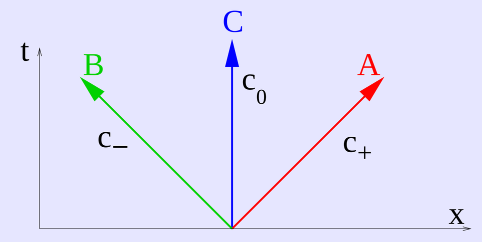
- **Hyperbolic system for** $\tau \neq 0$

For $\boldsymbol{v} = (u, 0)$ and $f = 0$,

- 3 characteristics: $c_\pm = u \pm \sqrt{gh}$ and $c_0 = u$.
- corresponding Riemann variables $r_\pm = u \pm 2\sqrt{gh}$ and $r_0 = \frac{Q}{h}$ are **modified** by P.

$$\frac{dr_0}{dt} = -\left(1 - \frac{\beta Q}{h}\right) P$$

$$\frac{dr_-}{dt} = +\beta\sqrt{\frac{g}{h}} P \qquad \frac{dr_+}{dt} = -\beta\sqrt{\frac{g}{h}} P$$



(for small perturbations)

- **Piecewise hyperbolic system for** $\tau \to 0$

For $P_{\tau \to 0} = -Q_s \nabla \cdot \boldsymbol{v} > 0$ ($\equiv$ CISK-parameterization), the system becomes

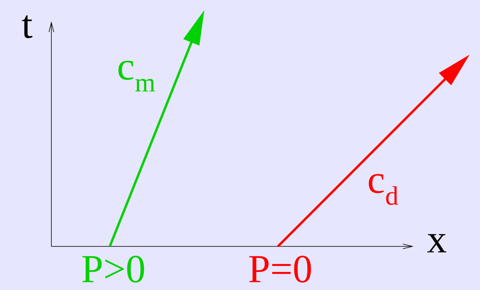
$$\begin{cases} \boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -g\nabla h - f\boldsymbol{k} \times \boldsymbol{v} \\ h_t + \nabla \cdot (\boldsymbol{v} h) = \beta Q_s \nabla \cdot \boldsymbol{v} \end{cases}$$

For $\boldsymbol{v} = (u, 0)$ and $f = 0$,

- 2 characteristics: $c^m_\pm = u \pm \sqrt{g(h - \beta Q_s)}$.
- corresponding Riemann variables $r^m_\pm = u \pm 2\sqrt{g(h - \beta Q_s)}$ are **invariant**.

The moist relative characteristic velocity is *weaker* than the dry one:

$$c_m = \sqrt{g(h - \beta Q_s)} < c_d = \sqrt{gh}$$

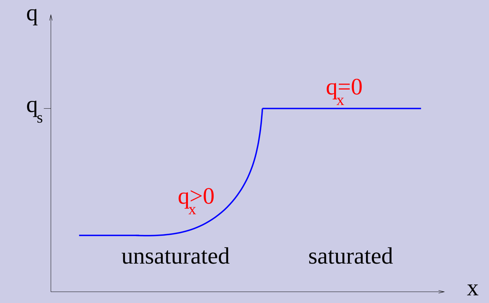


(for small perturbations)

References: [1] **Gill A. E.** *Studies of moisture effetcs in simple atmospheric models: the stable case*, Geophys. Astrophys. Fluid Dyn., v. 19, p. 119-152 (1982). [2] **Frierson D. M. W., Majda A. J., and Pauluis O. M.** *Large scale dynamics of precipitation fronts in the tropical atmosphere: a novel relaxation limit*, Commun. Math. Sci., v. 2, p. 591-626 (2004). [3] **Stechman S. N. and Majda A. J.** *The structure of precipitation fronts for finite relaxation time*, Theor. Comput. Fluid Dyn., v. 20, p. 337-404 (2006). [4] **Pauluis O. M., Frierson D. M. W., and Majda A. J.** *Precipitation fronts and the reflection and transmission of tropical disturbances*, Quart. J. Roy. Meteor. Soc., v. 134, p. 913-930 (2008). [5] **Bouchut F.** *Efficient numerical finite-volume schemes for shallow-water models*, in *Nonlinear dynamics of Rotating Shallow Water: Methods and Advances*, V. Zeitlin, ed. Elsevier, p.189 - 264, (2007).

Fronts propagation

Fronts are considered as **weak discontinuities** ($\equiv$only in the gradients). Example:



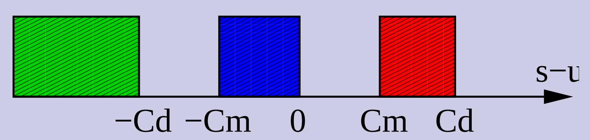
They propagates with a velocity s following the **Rankine-Hugoniot conditions**.

For $\boldsymbol{v} = (u, 0)$ and $f = 0$,

$$\begin{cases} (u - s)[u_x] - g[h_x] = 0 \\ (u - s)[h_x] - h[u_x] = -\beta[P] \\ (u - s)[Q_x] + Q[u_x] = -[P] \end{cases}$$

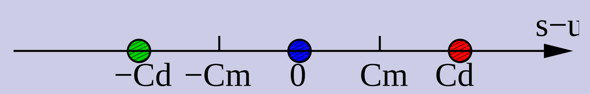
where $[a] = a_+ - a_-$.

- If $\tau \to 0$, precipitation is discontinuous ($[P] \neq 0$) and 3 types of precipitation fronts exist.



This confirms the linear results of Majda *et al.* [2-4].

- If $\tau \neq 0$, precipitation is continuous ($[P] = 0$) and 3 types of non-precipitating fronts exist.



Scattering of a simple wave by a moisture front

Numerical model: Finite volume method (Bouchut [5])
$\Rightarrow$ based on the hyperbolic structure of the equations.

Emergence of a precipitating region in the scattering area where precipitation plays a role of a **dissipative reflector** (Fig.1).

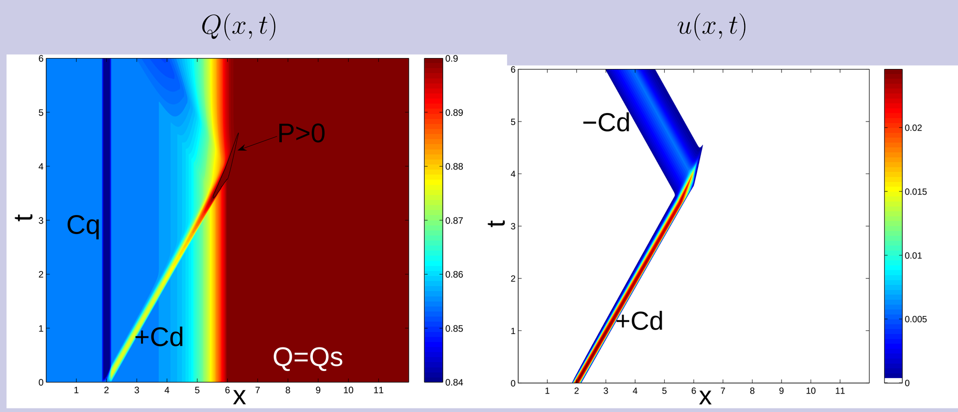


Fig.1 Small localized parabolic disturbance propagating from an unsaturated region to a moisture front.

Confirmation of prediction for $\tau \to 0$ (Fig.2):

- precipitating region limited by two precipitation fronts (s_1,s_2),
- existence of **moist** characteristics c_m.

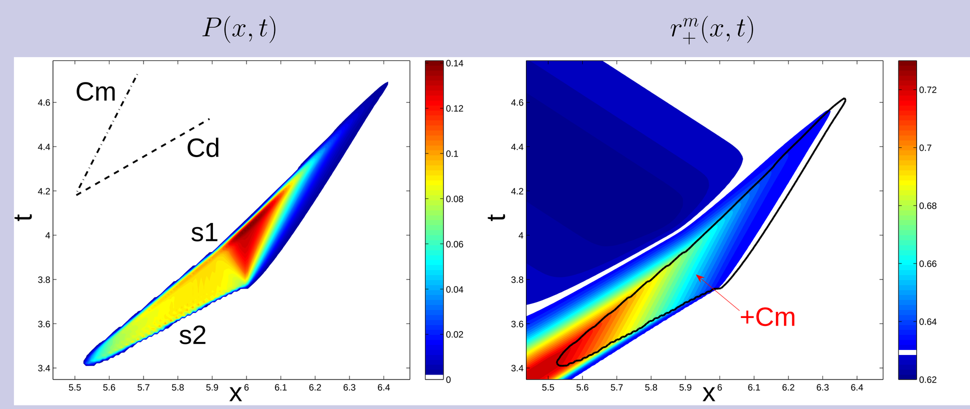


Fig.2 Same as Fig.1: zoom in the precipitating region. ($\tau = \gamma\Delta t$, $\gamma = 5$ and Δt =timestep)

Nonlinear stage of the evolution of a simple wave

Comparison of wave breaking in dry and moist cases:

When precipitation occurs, a **plateau** appears between the precipitation front (s_P) and the shock front (s_B) (Fig.3).

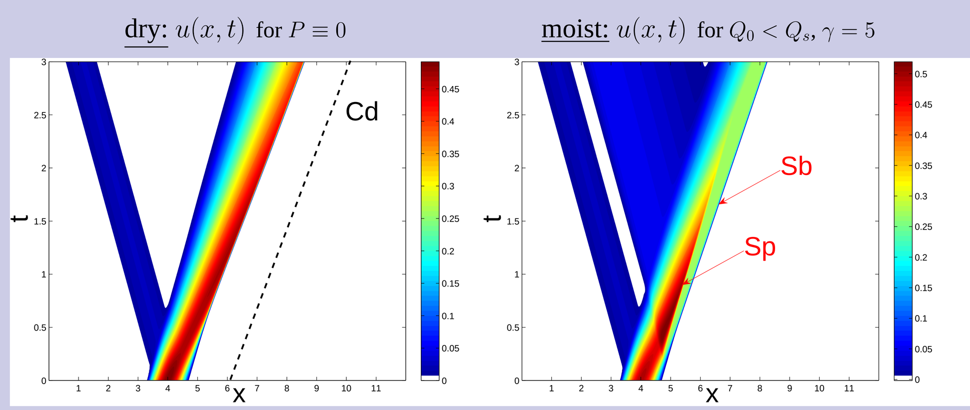


Fig. 3 Propagation of a localized parabolic disturbance of significant amplitude.

Particular moist case:
propagation in an uniformly **saturated** domain ($Q_0 = Q_s$).

For rapid and slow relaxations, precipitation directly occurs and **prevents** breaking process (Fig.4).

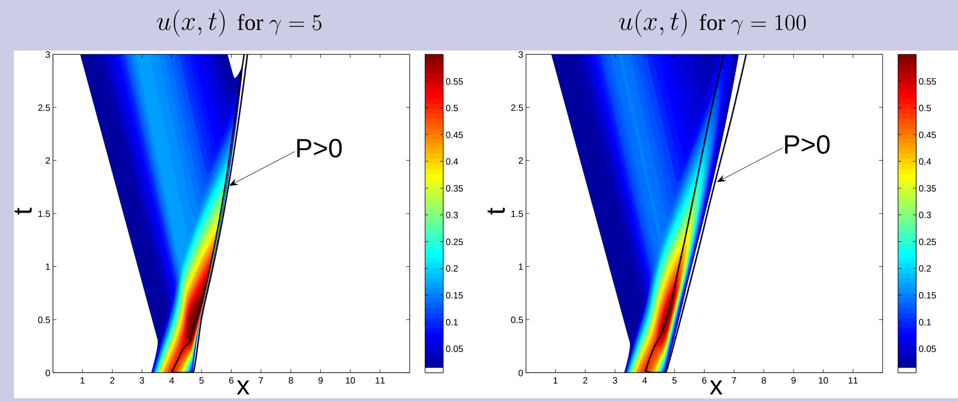


Fig. 4 Propagation of a localized parabolic disturbance of significant amplitude in an uniformly saturated domain.

Conclusions

This simple **fully nonlinear** "moist-convective" model allows to **generalize the previous linear results** [1-3] on precipitation fronts, the role of precipitation as dissipative reflector and moist characteristics.

The finite volume scheme with moist relaxation correctly reproduces the analytical results and appears to be an **appropriate numerical tool** to analyse fully nonlinear effects, and e.g. to show that precipitation modifies (and can prevent) breaking.

Work in progress: rotation effects, 2D dynamics, topography effects and 2-layer version of the model.